SCIREA Journal of Mathematics

http://www.scirea.org/journal/Mathematics

October 17, 201

Volume 1, Issue1, October 2016

Bayesian inference & ranking of news TV channel using method of paired comparison

Syed Adil Hussain (**Corresponding Author**)

Lecturer, Department of Statistics, University of Gujrat, Gujrat, Pakistan

Email: syed.adil@uog.edu.pk

Taha Hasan Associate Professor, Department of Statistics, Islamabad Model College for Boys, F-10/4, Islamabad, Pakistan.

Email: taha.qau@gmail.com

Abstract

This research considers the Bayesian inference & ranking of the domestic News TV channels using method of paired comparisons. Results for analysis are computed in C language and programs' coding are executed for seven parametric model's inference. Furthermore, to ensure the appropriateness of the model, the goodness of fit criteria is used as used by Aslam (2002). The model ensures good fit for the news TV channels. The research aims to play a dynamic role in ranking of domestic News TV channels by providing the preferences probabilities which reflects ranking of each of the factors.

Keywords: Paired comparison method; Bayesian Inference (Posterior Means, Posterior Predictive Probabilities and modem goodness of fit) of Bradley-Terry, Non informative prior; Uniform distribution.

1. Introduction

The paired combine experimentations of a certain object are done by a judge or a panel of judges. One of object from each pair is picked according to preference by judge or panel of judges. Such experiment bases on consumer preference such as personal rating, and psychological investigation. The example of paired comparison experiment is sport tournament in which matches represent pair and the one who win the match represent the preference objects. To find the best object among all is the purpose of this experiment, to rank the object in order of merit, to decide whether there is any noticeable difference between the object, or to test the judge's perception. The result of paired comparison may be difficult to intercept, the judges prefer the first object to second, the second object to third but again he prefer the third object which leads the analysis towards bias in estimates.

1.3 History of private news channel in Pakistan

Owned of Geo News is Jang Group/test transmission of Geo News Channel was started on 14 Aug 2002, but in Oct 2002 a regular transmission was started. For many month illegal closures faced on Geo News Channel but still popular News Channel. Express News Channel launched on 1st January, 2008. On 5TH Feb, 2009 'Express News 24/7 launched. Express News Channel one of the most popular and demanding News Channel in Pakistan. This Channel has almost grabbed the world in few years and awareness about all current issues and affairs. Business recorder group is owned of the Aaj News Channel. In 23rd March it was started. This channel is also one of the most demanding Channels among all other in viewers. One of the channels with maximum rating is Sama. Sama Slogan was given on 25TH of December 2007. Without any doubt Sama News is one the most Favorite News Channel among all country. It is also seen in other countries.

1.3.1 News Channels in Pakistan

In 1964 introduce Television in Pakistan. In anticipation of delayed 1990's only a handful of TV channels were obtainable for audience. President of Pakistan Mr. Pervaiz musharaf allow to operate in 2000 and it guide to a new production of TV channels. All the time News Channel is important. Though for some people, News listing are a somewhat boring activity but if you updated from all affairs then these News Channel will become favorite of yours. Now a day many kinds of news channels are watching like Geo News, Sama News, Ary News, Exprees News, Dunya News, dawn News and Aaj News are most popular News Channel in Pakistan. Geo News has the largest yearly cumulative reach 58.7, Ary news has the 2^{nd} largest yearly cumulative reach 35.4, Aaj News has 31.2, Dawn News has yearly cumulative reach 10.1, Dunya News has yearly cumulative reach 12, and Express News has yearly cumulative reach 25.

1.4 The Paired Comparisons Models

The method of paired comparisons leads us towards an unexpected amount of model building. It provides us the representation of the experimental process by the probabilistic choice models. Suppose there are *t* objects to be compared in pairs by the judges and let d_{ij} be the indicator random variable that can take the value 0 for θ_i and 1 for θ_j and d_{ji} can take the value 1 for θ_i and 0 for θ_j .We assume throughout the experiment that all the comparisons are statistically independent except that $d_{ij} + d_{ji} = 1$, then Preference probability that $\theta_i \rightarrow \theta_j$ will be

$$\Pr(d_{ij}=1) = \xi_{ij} \qquad 3.1$$

And Preference probability that $\theta_j \rightarrow \theta_i$

$$\Pr(d_{ji}=1) = \xi_{ji} \qquad 3.2$$

More generally with replication and ordered effect

$$\xi_{ij} = \xi_{ijkl} \qquad 3.3$$

This means that θ_i is preferred on θ_j at k^{th} comparison by l^{Th} judge. Here ξ_{ij} and ξ_{ji} are probabilities under the following condition

$0 < \theta_{ij}\,,\ \theta_{ji} < 1$ 3.1.4

1.5 The Linear Model for Paired Comparisons

The linear models for paired comparisons are mostly important. In a paired comparisons experimentation, the object θ_i is preferred on θ_j if the merit $w_i > w_j$ and the object θ_j is preferred on θ_i if the merit $w_j > w_i$. Let $\zeta_j = w_j - \eta_j$ and $\zeta_j = w_j - \eta_j$ for which $\mathrm{i} = 1, 2...., \mathrm{m}$ and $\mathrm{j} = 1, 2...., \mathrm{m}$ that the objects have merit or worth η_i and η_j. If every pair $\left(\zeta_i - \zeta_j\right)$ has the same bi-variate distribution then $\left(\zeta_j - \zeta_i\right)$ will also have the same distribution, now

$$\Pr\left\{\left(\zeta_i - \zeta_j\right) < v\right\} = H(v) \qquad 3.4$$

It follows that

$$\omega_{ij} = \Pr\{(w_i - w_j) > 0\} \qquad 3.5$$

$$= \Pr\left\{\left(\zeta_i - \zeta_j\right) > -\left(\eta_i - \eta_j\right)\right\}$$

$$\omega_{ij} = H\left(\eta_i - \eta_j\right) \qquad 3.6$$

Whenever the preference probabilities can be expressed in terms of a symmetrical cumulate density function, the term w_i satisfies to be a linear model. This model is the generalization of the Thurstone-Mosteller model (1927) for which w_j is assumed to be normal $N\left(\eta_i, \sigma^2\right)$, equi correlated with common correlation co efficient.

$$\omega_{ij} = \mathrm{H}(\eta_i - \eta_j) = \int_{-(\eta'_i - \eta'_j)}^{\infty} z(v)\,dv \qquad 3.7$$

Where $\eta'_i = \dfrac{\eta_i}{\left[2\sigma^2(1-\rho)\right]^{1/2}}$ 3.8

And $z(v) = \dfrac{1}{\sqrt{2\pi}} e^{-1/2 v^2}$ 3.9

1.6 The Bradley-Terry (1952) Model for Paired Comparisons

Bradley-Terry (1952) developed the basic model for paired comparison after Zermelo (1929) consideration that states, “The objects or treatments have merit η_i and η_j when judge on some characteristic and may be represent by the continuous random variable with the following limit.

$$w_i, (-\infty < w_i < +\infty)." \qquad 3.10$$

$$H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\theta_i - \ln\theta_j)}^{\infty} \sec h^2(w/2) dw \qquad 3.11$$

Where $i \neq j$ and $i = 1, 2, ..., m$ 3.12

$$\omega_{ij} = H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\theta_i - \ln\theta_j)}^{\infty} \sec h^2(w/2) dw \qquad 3.13$$

This model implies that the difference between two underlying variables $\left(\xi_i - \xi_j\right)$ has a logistic density with parameter $(\ln\theta_i - \ln\theta_j)$ and is formulated as

$$\omega_{ij} = \frac{\theta_i}{\theta_i + \theta_j} \qquad 3.14$$

Where ω_{ij} denotes the preference probability for object i when θ_i and θ_j are compared.

The Bradley-Terry model (1952) is used for the parameters of factors affecting terrorism in Pakistan. The probability of observed results in s^{th} recurrence of pair of parameters is given as follow.

$$P_{ijs} = \left[\frac{e^{2\gamma_i}}{e^{2\gamma_i} + e^{2\gamma_j}} \right]^{x_{ijs}} \left[\frac{e^{2\gamma_j}}{e^{2\gamma_i} + e^{2\gamma_j}} \right]^{x_{jis}}$$

Notations of the Model

We consider the following notations for the analysis of the model.

$x_{i.ij}$ = Number of times i^{th} object is preferred on j^{th} object.

$x_{j.ij} =$ Number of times j^{th} object is preferred on i^{th} object.

$r_{ij} = x_{i.ij} + x_{j.ij}$ = Total number of comparisons between i^{th} and j^{th} objects

$n_i =$ Total number of times i^{th} object is preferred to any other objects.

1.7 Likelihood Function of the Model

The likelihood function of the model for seven factors affecting terrorism in Pakistan is gives below takes the following form;

$$l(x;\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7) \propto \frac{\prod_{i=1}^{7}(e^{2\theta_i})^{n_i}}{\prod_{i<j}^{7}(e^{2\theta_i}+e^{2\theta_j})^{r_{ij}}} \qquad (4.1)$$

Where $n_i = \sum_{j\neq i}^{m} x_{i.ij}$ and $r_{ij} = x_{i.ij} + x_{j.ij}$ be the total number of comparisons between these objects.

1.8 Uniform Prior

Bayesian analysis for paired comparison models is not simple because no prior distribution is assumed to be the conjugate prior of data from paired comparisons models. Non-Informative Uniform prior is assumed for the analysis. The joint uniform prior for parameters space $\underset{\sim}{\theta}$ is given below;

$$P(\underset{\sim}{\theta}) \propto 1 \qquad (4.2)$$

Here $\underset{\sim}{\theta} = (\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7)$

With $0 < \theta_i < 1$

2. Bayesian Inference

Bayesian inference of two data sets is performed. The target population of this study is the judges/experts/faculty of public/private sector universities in Pakistan. Sampled population of this study is faculty (judges/experts') of University of Gujrat. Experts' preferences for factors of terrorism and ranking of Pakistan News TV Channels are obtained from faculty members selected randomly. Simple random sampling is used. Distinct departments' faculty is selected in the sample and questionnaire included the tables of pair-wise preferences for the factors influencing terrorism in Pakistan and ranking of Pakistan news TV Channels. A sample of size 100 is selected from the sampled population. A Complete list of faculty members is available from SSC department of University of Gujrat. Programs are designed in C++ language and one of them is given in appendix.

2.1 Posterior Means for News TV Channels

The general integral formula for ϑ_i is given as below.

$$E(\vartheta_i) = \int_{\vartheta_1=0}^{1} \int_{\vartheta_1=0}^{1-\vartheta_1} \int_{\vartheta_3=0}^{1-\vartheta_1-\vartheta_2} \int_{\vartheta_4=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3} \int_{\vartheta_5=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4} \int_{\vartheta_6=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4-\vartheta_5} \frac{(\vartheta_i).\prod_{i=1}^{7}(\vartheta_i)^{n_i}}{M\left(\prod_{i<j}^{7}(\vartheta_i+\vartheta_j)^{r_{ij}}\right)} d\vartheta_6 d\vartheta_5 d\vartheta_4 d\vartheta_3 d\vartheta_2 d\vartheta_1 \quad (4.6)$$

Here $\vartheta_7 = 1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4-\vartheta_5-\vartheta_6$ is the constraint on the numerical integration.

The results of posterior means are computed and organized in Table. 3 for as below;

Table.5

GO (Geo News)	DN (Duniya News)	EX (Express News)	SM (Sama News)	DN (Dawn News)	AR (ARY News)	AJ (Aaj News)
0.06000	0.11199	0.12226	0.15431	0.12729	0.23041	0.19374

Table.6 Ranking of the News Channel in Pakistan

News Channel	Expected probabilities (ϑ_i)	Rank
Geo News	0.06000	(7)
Duniya News	0.11199	(6)

News Channel	Expected probabilities (ϑ_i)	Rank
Express News	0.12226	(5)
Sama News	0.15431	(3)
Dawn News	0.12729	(4)
ARY News	0.23041	(1)
Aaj News	0.19374	(2)

2.3 Posterior Predictive Probabilities for News TV Channels

Predictive probabilities for seven News TV channels are computed for each pair of factors using a program designed for seven objects in C++. The predictive probability for i^{th} & j^{th} objects is given follow $for, i < j$ as;

$$\mathrm{P}_{(ij)} = \frac{1}{M} \int_{\vartheta_1=0}^{1} \int_{\vartheta_2=0}^{1-\vartheta_1} \int_{\vartheta_3=0}^{1-\vartheta_1-\vartheta_2} \int_{\vartheta_4=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3} \int_{\vartheta_5=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4} \int_{\vartheta_6=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4-\vartheta_5} \mathrm{P}(\underset{-}{\vartheta}|\underset{-}{x}).\omega_{ij} d\vartheta_6 d\vartheta_5 d\vartheta_4 d\vartheta_3 d\vartheta_2 d\vartheta_1, \qquad (4.8)$$

Here $\mathrm{P}(\underset{-}{\vartheta}|x) = \dfrac{\prod_{i=1}^{7} (e^{2\vartheta_i})^{n_i}}{M\left(\prod_{i<j}^{7} (e^{2\vartheta_i} + e^{2\vartheta_j})^{r_{ij}}\right)}$ be the posterior distribution.

$\omega_{ij} = \dfrac{\vartheta_i}{\vartheta_i + \vartheta_j}$ be the preference probabilities of ‘i’ factor over ‘j’ factor.

Here $\vartheta_7 = 1 - \vartheta_1 - \vartheta_2 - \vartheta_3 - \vartheta_4 - \vartheta_5 - \vartheta_6$ will be the constraint on the numerical integration.

Table.8 Posterior Predictive Probabilities ForTerrorism:

P(ij)	Estimate	P(ji)=1- P(ij)	Estimate
P(12)	0.318893	P(21)	0.681107
P(13)	0.271797	P(31)	0.72821
P(14)	0.251345	P(41)	0.748655

P(ij)	Estimate	P(ji)=1- P(ij)	Estimate
P(15)	0.265147	P(51)	0.734853
P(16)	0.187265	P(61)	0.81274
P(17)	0.20524	P(71)	0.79476
P(23)	0.443484	P(32)	0.556516
P(24)	0.41893	P(42)	0.58107
P(25)	0.435502	P(52)	0.564498
P(26)	0.331447	P(62)	0.668553
P(27)	0.357005	P(72)	0.642995
P(34)	0.475443	P(43)	0.524557
P(35)	0.492019	P(53)	0.507981
P(36)	0.384342	P(63)	0.615658
P(37)	0.411068	P(73)	0.588932
P(45)	0.516576	P(54)	0.483424
P(46)	0.406639	P(64)	0.593361
P(47)	0.434452	P(74)	0.565548
P(56)	0.391602	P(65)	0.608398
P(57)	0.41867	P(75)	0.581329
P(67)	0.528465	P(76)	0.471835

2.4 Appropriateness of the Model for Factors of Terrorism

We use Chi-Square method to test the hypothesis about the goodness of fit of the model for factors of terrorism. We used the criteria by Aslam (2002) as follow;

The null and alternate hypotheses are as follow

H_0; The model is good fit of the data

$\overline{H}_o$; The model does not fit the data

We calculate the expected frequencies by the following formula

$x_{ij} = r_{ij}(\omega_{ij})$

For all $i < j$

The level of significance is 5%

The test statistic follows the Chi-Square distribution as

$$\chi^2 = \sum_{i<j=1}^{m} \left\{ \frac{(x_{ij} - x_{ij})^2}{x_{ij}} + \frac{(x_{ji} - x_{ji})^2}{x_{ji}} \right\}$$

$d.f = m(m-2) = 35$

We calculate the Chi-Square test statistic value as

χ^2_{cal} = 25.28484196

The table value is $\chi^2_{(0.05,35)}$ =49.5679

Since the χ^2_{cal} doesn't not fall in the critical region that is

$\chi^2_{cal} \not\geq \chi^2_{(0.05,35)}$

So as a conclusion we have no evidence to reject the null hypothesis. We conclude that the model good fits the data.

2.5 Interpretations of ranking of Pakistan News TV Channels using Bayesian Paradigm

This study revealed the preferences of faculty members (experts) for the ranking of domestic News TV Channels with predictive inference. For Pakistan News TV channels, ARY News ranks first among experts. For the analysis of data we used paired comparison using Bayesian inferences via non-informative uniform prior. For this purpose C language is used for programming and results are generated for each program about estimation of parameters for posterior means and posterior predictive probabilities. In this study we also discussed the results News Channels according the preference of faculty members. The faculty member's more preferred the News Channel is ARY News that has the maximum probability of the preference and the second rank goes to the Aaj News that probability is less than one so give the second rank and the future third, fourth, fifth, sixth and seventh order is given to according Sama News, Dawn News, Express News, Duniya News, Geo News respectively. The more preferred News Channel in Pakistan is ‘ARY News’ most faculty members preferred that Channel and have a strong position and that less preferred Channel is ‘Geo News’ most faculty members are not preferred that Channel and have a weak position.

3. Recommendations

In this research paired comparisons methodology is considered. It can be further generalized to the multiple comparisons experimentations. In our study we analyzed the preferences of news TV channels for which the experts’ opinions were collected from faculty. In future research may be conduct from the policy makers (Establishment), researchers (scientists) or other responsible Govt. officers. Ranking of factors of terrorism within Pakistan is done and can be attained for different countries facing terrorism.

Appendix

```
/* Program to find The Posterior Means for the Bradley-Terry Model using Uniform Prior by Quadrature
Method (T=7)*/
# include <stdio.h>
# include <math.h>
# include <conio.h>
# define pi 3.142857
 void main()
{
int n=50,a12=14,a13=11,a14=14,a15=12,a16=13,a17=15,a23=21,a24=22,a25=20,a26=20,a27=24,
a34=20,a35=22,a36=18,a37=20,a45=23,a46=18,a47=20,a56=12,a57=14,a67=26;
double t1,t2,t3,t4,t5,t6,t7,t8,p12,p13,p14,p15,p16,p17,p18,p23,p24,p25,p26,p27,p28,p34,p35,p36,p37,p38,
p45,p46,p47,p48,p56,p57,p58,p67,p68,p78,fun,integ,integc,integt1,integt2,integt3,integt4,integt5,
integt6,integt7,ip=0.01,dl=0.05;
clrscr();
printf("Start of Program...\n");
// Finding the Normalizing Constant
integ=0.0;
printf("\nCalculating NC...");
for (t1=ip;  t1<=1.0-dl; t1+=dl)
for (t2=ip;  t2<=1.0-t1-dl;  t2+=dl)
for (t3=ip;  t3<=1.0-t1-t2-dl;  t3+=dl)
```

```
for (t4=ip;  t4<=1.0-t1-t2-t3-dl;  t4+=dl)
for (t5=ip;  t5<=1.0-t1-t2-t3-t4-dl;  t5+=dl)
for (t6=ip;  t6<=1.0-t1-t2-t3-t4-t5-dl;  t6+=dl)

{
t7=1.0-(t1+t2+t3+t4+t5+t6);
p12=t1/(t1+t2);p13=t1/(t1+t3);p14=t1/(t1+t4);p15=t1/(t1+t5);p16=t1/(t1+t6);p17=t1/(t1+t7);p18=t1/(t1+t8);
p23=t2/(t2+t3);p24=t2/(t2+t4);p25=t2/(t2+t5);p26=t2/(t2+t6);p27=t2/(t2+t7);p28=t2/(t2+t8);p34=t3/(t3+t4);
p35=t3/(t3+t5);p36=t3/(t3+t6);p37=t3/(t3+t7);p38=t3/(t3+t8);p45=t4/(t4+t5);p46=t4/(t4+t6);p47=t4/(t4+t7);
p48=t4/(t4+t8);p56=t5/(t5+t6);p57=t5/(t5+t7);p58=t5/(t5+t8);p67=t6/(t6+t7);p68=t6/(t6+t8);p78=t7/(t7+t8);
//printf("\nPij's:%f%f%f%f%f%f%f%f%f%f%f%f%f%f%f",p12,p13,p14,p15,p16,p23,p24,p25,p26,p34,p35,p36,p
45,p46,p56);
fun=pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*pow(p14,a14)*pow(1.0-p14,n-
a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-
a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-
a25)*pow(p26,a26)*pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-
a34)*pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*pow(1.0-p37,n-
a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*pow(p47,a47)*pow(1.0-p47,n-
a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-
a67);
//printf("\nIndividual Function:%10.5g",fun);
integ+=fun*pow(dl,7);
//printf("  Iteration #: %d, Individual NC:%10.5g",integ);
//getch();
}
printf("\nNC found to be:%12g",integ);
// Verifying the Normalizing Constant & Estimating the Parameters
integc=0.0;  integt1=0.0;  integt2=0.0;  integt3=0.0;  integt4=0.0;   integt5=0.0;  integt6=0.0;  integt7=0.0;
printf("\n\nEstimating Parameters...");
for (t1=ip;  t1<=1.0-dl; t1+=dl)
for (t2=ip;  t2<=1.0-t1-dl;  t2+=dl)
for (t3=ip;  t3<=1.0-t1-t2-dl;  t3+=dl)
for (t4=ip;  t4<=1.0-t1-t2-t3-dl;  t4+=dl)
for (t5=ip;  t5<=1.0-t1-t2-t3-t4-dl;  t5+=dl)
for (t6=ip;  t6<=1.0-t1-t2-t3-t4-t5-dl;  t6+=dl)
{
t7=1.0-(t1+t2+t3+t4+t5+t6);
p12=t1/(t1+t2);p13=t1/(t1+t3);p14=t1/(t1+t4);p15=t1/(t1+t5);p16=t1/(t1+t6);p17=t1/(t1+t7);p18=t1/(t1+t8);
p23=t2/(t2+t3);p24=t2/(t2+t4);p25=t2/(t2+t5);p26=t2/(t2+t6);p27=t2/(t2+t7);p28=t2/(t2+t8);p34=t3/(t3+t4);
p35=t3/(t3+t5);p36=t3/(t3+t6);p37=t3/(t3+t7);p38=t3/(t3+t8);p45=t4/(t4+t5);p46=t4/(t4+t6);p47=t4/(t4+t7);
p48=t4/(t4+t8);p56=t5/(t5+t6);p57=t5/(t5+t7);p58=t5/(t5+t8);p67=t6/(t6+t7);p68=t6/(t6+t8);p78=t7/(t7+t8);
fun=(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
```

```
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integc+=fun*pow(dl,7);
fun=t1*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt1+=fun*pow(dl,7);
fun=t2*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt2+=fun*pow(dl,7);
fun=t3*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt3+=fun*pow(dl,7);
fun=t4*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
```

```
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt4+=fun*pow(dl,7);
fun=t5*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt5+=fun*pow(dl,7);
fun=t6*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt6+=fun*pow(dl,7);
fun=t7*(1.0/integ)*pow(p12,a12)*pow(1.0-p12,n-a12)*pow(p13,a13)*pow(1.0-p13,n-a13)*
pow(p14,a14)*pow(1.0-p14,n-a14)*pow(p15,a15)*pow(1.0-p15,n-a15)*pow(p16,a16)*
pow(1.0-p16,n-a16)*pow(p17,a17)*pow(1.0-p17,n-a17)*pow(p23,a23)*pow(1.0-p23,n-a23)*
pow(p24,a24)*pow(1.0-p24,n-a24)*pow(p25,a25)*pow(1.0-p25,n-a25)*pow(p26,a26)*
pow(1.0-p26,n-a26)*pow(p27,a27)*pow(1.0-p27,n-a27)*pow(p34,a34)*pow(1.0-p34,n-a34)*
pow(p35,a35)*pow(1.0-p35,n-a35)*pow(p36,a36)*pow(1.0-p36,n-a36)*pow(p37,a37)*
pow(1.0-p37,n-a37)*pow(p45,a45)*pow(1.0-p45,n-a45)*pow(p46,a46)*pow(1.0-p46,n-a46)*
pow(p47,a47)*pow(1.0-p47,n-a47)*pow(p56,a56)*pow(1.0-p56,n-a56)*pow(p57,a57)*
pow(1.0-p57,n-a57)*pow(p67,a67)*pow(1.0-p67,n-a67);
integt7+=fun*pow(dl,7);}
integt7=1.0-integt1-integt2-integt3-integt4-integt5-integt6;
printf("\nInc     NC     GO DN  EX  SM  DN  AR  AJ ");
printf("\n%6.3f%9.2g%4.2g",dl,integ,integc);
printf("%11.5f%13.5f%10.5f%10.5f%11.5f%11.5f%11.5f",integt1,integt2,integt3,integt4,integt5,integt6,integt
7);
printf("\nProgram ended, Press Enter to exit...");
getch();}
```

References

[1] Bramley, T. and B. Black (2008). *Maintaining performance standards: aligning raw score scales on different tests via a latent trait created by rank-ordering examinees' work*. Third International Rasch Measurement conference, University of Western Australia, Perth.

[2] Newton, P., J. Baird, et al. (2007). *Techniques for monitoring the comparability of examination standards.* Qualifications and Curriculum Authority London.

[3] Halai, N. (2012). "*An analysis of higher education commission rankings 2012*." Quality of private universities in Pakistan 22(7): 775 - 786.

[4] (2014). "*Assessing Internal Democracy of Major Political Parties of Pakistan*. " PILDAT.

[5] Aslam, M. (2002). "*Bayesian analysis for paired comparison models allowing ties and not allowing ties*." PAKISTAN JOURNAL OF STATISTICS-ALL SERIES- 18(1): 53-70.

[6] Aslam, M. (2003). "*An application of prior predictive distribution to elicit the prior density*." Journal of Statistical Theory and applications 2(1): 70-83.

[7] Bernardo, J. M. (2003). *Bayesian Statistics 7: Proceedings of the Seventh Valencia International Meeting,* Oxford University Press.

[8] Frackowiak, R. S., K. J. Friston, et al. (2004). "*Human brain function*." San Diego, CA.

[9] Gelman, A. (2006). "*Prior distributions for variance parameters in hierarchical models (comment on article by Browne and Draper)."* Bayesian analysis 1(3): 515-534.

[10] Gelman, A., J. B. Carlin, et al. (2014). *Bayesian data analysis*, Taylor & Francis.

[11] Hoijtink, H., I. Klugkist, et al. (2008). *Bayesian evaluation of informative hypotheses.* Springer Science & Business Media.

[12] Bramley, T. and T. Gill (2010). "*Evaluating the rank‐ordering method for standard maintaining*." Research Papers in Education 25(3): 293-317.