



## ON HYPERGEOMETRIC SERIES ASSOCIATED WITH THE GENERALIZED ZETA FUNCTIONS

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### Abstract

In this work we will introduce and study a generalized zeta function of the general Hurwitz – Lerch zeta functions. Here, we aim to derive basic properties of generalized zeta function for the general Hurwitz – Lerch zeta functions include some integral representations for several general Hurwitz – Lerch zeta functions and fractional derivative. A number of known and new results which introduced are generalization of a known results introduced by Shy-Der and Srivastava [6].

**Key words:** Hurwitz-Lerch zeta functions; Mellin-Barnes integrals; hypergeometric functions; hypergeometric-type generating functions.

## 1. Introduction

For  $\text{Re}(z) > 1$ , the Riemann zeta function  $\zeta(z)$  is defined as follows [1,p.88]

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}, \quad (1.1)$$

which can, except for a simple pole at  $z = 1$  with its residue 1. be continued moromorphically to the whole complex  $z$ -plane. The Hurwitz (or generalized) zeta function denoted usually by  $\zeta(z, a)$  and so for  $\text{Re}(z) > 1$ ,

$$\zeta(z, a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^z}, \quad (1.2)$$

can just as  $\zeta(z)$ . be continued moromorphically to the whole complex  $z$ -plane except for a simple pole at  $z = 1$  with its residue 1. When  $a = 1$ , (1.2) reduces to (1.1). In terms of Hurwitz-Lerch zeta function  $\Phi(x, z, a)$  defined by [5,p.121] and [1]:

$$\Phi(x, z, a) = \sum_{n=0}^{\infty} \frac{x^n}{(a+n)^z}, |x| < 1, a \neq 0, -1, -2, \dots \quad (1.3)$$

Equivalently, it has the integral expression

$$\Phi(x, z, a) = \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-at} (1 - xe^{-t})^{-1} dt, \quad (1.4)$$

provided  $\text{Re}(a) > 0$  and either  $|x| \leq 1$  and  $\text{Re}(z) > 0$  or  $x = 1$  and  $\text{Re}(z) > 1$ .

So that obviously

$$\Phi(1, z, a) = \zeta(z, a). \quad (1.5)$$

The zeta function in (1.3) has since been extended by Goyal and Laddha [3,p.100(1.5)], in the form :

$$\Phi_{\mu}^*(x, z, a) = \sum_{n=0}^{\infty} \frac{(\mu)_n}{(a+n)^z} \frac{x^n}{n!}, \quad (1.6)$$

where  $|x| < 1, \text{Re}(a) > 0, \mu \geq 1$ .

Equivalently , it has the integral expression :

$$\Phi_{\mu}^{*}(x, z, a) = \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-at} (1 - xe^{-t})^{-\mu} dt, \quad (1.7)$$

provided that  $\mu \geq 1, \operatorname{Re}(a) > 0, |x| \leq 1$  and either  $\operatorname{Re}(z) > 0$  or  $\operatorname{Re}(z) > \operatorname{Re}(\mu)$  according to  $x \neq 1$  or  $x = 1$ .

In [6] Shy-Der and Srivastava introduced the general family of the Hurwitz-Lerch zeta function in the form:

$$\Phi_{\mu, \nu}^{(\rho, \sigma)}(x, z, a) = \sum_{n=0}^{\infty} \frac{(\mu)_{\rho n}}{(\nu)_{\sigma n}} \frac{x^n}{(a+n)^z}, \quad (1.8)$$

where  $\mu \in \mathbb{C}, a, \nu \in \mathbb{C} \setminus \mathbb{Z}_0^-, \rho, \sigma \in \mathbb{R}^+, \rho > \sigma$  when  $x, z \in \mathbb{C}, \rho = \sigma, z \in \mathbb{C} \mid |x| < 1, \rho = \sigma$

and  $\operatorname{Re}(z - \mu + \nu) > 1$  when  $|x| = 1$ .

Equivalently, it has the following integral expressions :

$$\Phi_{\mu, \nu}^{(\rho, \sigma)}(x, z, a) = \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-at} {}_2\psi_1 \left[ \begin{matrix} (\mu; \rho), (1; 1); \\ (\nu; \sigma); \end{matrix} x^k e^{-t} \right] dt. \quad (1.9)$$

In this paper we will introduce and study a generalized zeta function of the general Hurwitz – Lerch zeta function  $\Phi_{\mu, \nu}^{(\rho, \sigma)}(x, z, a)$ . This work aims to derive basic properties of generalized zeta function for the general Hurwitz – Lerch zeta functions include some integral representations for several general Hurwitz – Lerch zeta functions and fractional derivative. A number of known and new results which introduced are generalization of a known results introduced in [6].

In section 2 we will introduce analogous definition of the general family of Hurwitz-Lerch zeta function denoted by  $\Phi_{\mu, \nu, \lambda}^{(\rho, \sigma)}(x, z, a)$  with a known special cases. Some integral representation for  $\Phi_{\mu, \nu, \lambda}^{(\rho, \sigma)}(x, z, a)$  are discussed in section 3 and fractional derivative in section 4.

## 2. The Function $\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a)$

In this section we will give analogous definition of the function  $\Phi_{\mu,\nu}^{(\rho,\sigma)}(x, z, a)$

which is defined by (1.3) in its form :

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = (\lambda)^{-z} \Phi_{\mu,\nu}^{(\rho,\sigma)}\left(x, z, \frac{a}{\lambda}\right) = \sum_{n=0}^{\infty} \frac{(\mu)_{\rho n}}{(\nu)_{\sigma n}} \frac{x^n}{(a + \lambda n)^z}, \quad (2.1)$$

$\mu \in \mathbb{C}, \lambda, a, \nu \in \mathbb{C} \setminus \mathbb{Z}_0^-, \rho, \sigma \in \mathbb{R}^+, \rho > \sigma$  when  $x, z \in \mathbb{C}, \rho = \sigma$  and

when  $|x| < 1, \rho = \sigma$  and  $\operatorname{Re}(z - \mu + \nu) > 1$  when  $|x| = 1$ .

Clearly, we have

$$\Phi_{\nu,\nu,1}^{(\sigma,\sigma)}(x, z, a) = \Phi_{\nu,\nu,1}^{(0,0)}(x, z, a) = \Phi(x, z, a), \quad (2.2)$$

$$\Phi_{\mu,1,1}^{(1,1)}(x, z, a) = \Phi_{\mu}^*(x, z, a). \quad (2.3)$$

Obviously, when  $\lambda = 1$ , (2.1) reduce to (1.8). Further, on putting

$x = a = 1$  in (2.2), it reduces to (1.1). Next, on putting  $x = 1$  and  $\mu = 1$  in (2.3), it reduces to (1.2).

## 3. Integral representations

By using the Fox-Wright extension of the generalized hypergeometric  ${}_pF_q$  function with p numerator and q denominator parameters, defined by:

$${}_p\Psi_q \left[ \begin{matrix} (\alpha_1; A_1), (\alpha_2; A_2), \dots, (\alpha_p; A_p); \\ (\beta_1; B_1), (\beta_2; B_2), \dots, (\beta_q; B_q); \end{matrix} ; z \right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p (\alpha_j)_{A_{jn}}}{\prod_{j=1}^q (\beta_j)_{B_{jn}}} \frac{z^n}{n!},$$

where the argument  $z$ , the complex parameters  $\alpha_j (j = 1, \dots, p)$  and  $\beta_j (j = 1, \dots, q)$ ,

and the positive real parameters  $A_j (j = 1, \dots, p)$  and  $B_j (j = 1, \dots, q)$  are so constrained

that the series in (3.1) converges absolutely [1,p.183] and Eulerian integral formula of second kind [1] :

$$(n+a)^{-z} = \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-(n+a)t} dt, (\min\{Re(z), Re(n+a)\} > 0).$$

it is easy to derive the following integral representation.

**Theorem 1:** Let  $Re(a) > 0, Re(z) > 0$  when  $|x| < 1 (x \neq 1)$   $Re(z) > 1$  when

$x = 1$ . Then

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = \frac{(\lambda)^{-z}}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-\frac{a}{\lambda}t} {}_2F_1 \left[ \begin{matrix} (\mu; \rho), & (1; 1); \\ & (\nu; \sigma); \end{matrix} \right] x e^{-t} dt. \quad (3.1)$$

**Proof:** By starting from the left-hand side of formula (3.1), then in view of definition

(2.1), it is easily seen that

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = (\lambda)^{-z} \Phi_{\mu,\nu}^{(\rho,\sigma)} \left( x, z, \frac{a}{\lambda} \right).$$

Upon using the integral formula (1.9), we are led finally to the right-hand side of formula (3.1).

In terms of the Gauss hypergeometric function  ${}_2F_1$  ( see [7] ):

$${}_2F_1(a, b; c; x) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{x^n}{n!}, \quad c \neq 0, -1, -2, \dots,$$

a special cases of (3.1), occur when  $\rho = \sigma = \lambda = 1$  as follows :

$$\Phi_{\mu,\nu,1}^{(1,1)}(x, z, a) = \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} e^{-at} {}_2F_1[\mu, 1; \nu; x e^{-t}] dt. \quad (3.2)$$

By using the familiar Euler transformation formula [1,p.64]

$${}_2F_1(\alpha, \beta; \gamma; x) = (1-x)^{\gamma-\alpha-\beta} {}_2F_1(\gamma-\alpha, \gamma-\beta; \gamma; x),$$

$$(|\arg(1-x)| \leq \pi - \varepsilon, (0 < \varepsilon < \pi), \gamma \notin \mathbb{Z}_0^-),$$

we get equivalently integral formula for (3.2) in form :

$$\Phi_{\mu,\nu,1}^{(1,1)}(x, z, a) = \frac{1}{\Gamma(z)} \int_0^\infty t^{z-1} e^{-at} (1-xe^{-t})^{\nu-\mu-1} {}_2F_1[\nu-\mu, \nu-1; \nu; xe^{-t}] dt . \quad (3.3)$$

For  $\nu=1$ , (3.3) reduces to the well- known result of Goyal and Laddha [3]

$$\Phi_{\mu,1,1}^{(1,1)}(x, z, a) = \frac{1}{\Gamma(z)} \int_0^\infty t^{z-1} e^{-at} (1-xe^{-t})^{-\mu} dt = \Phi_\mu^*(x, z, a) . \quad (3.4)$$

**Theorem 2:** Let  $\text{Re}(\mu) > 0, \rho \in \mathbb{R}^+$  . Then

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = \frac{(\lambda)^{-z}}{\Gamma(\mu)} \int_0^\infty t^{\mu-1} e^{-t} \Phi_{\mu,\nu}^{(0,\sigma)}\left(xt^\rho, z, \frac{a}{\lambda}\right) dt , \quad (3.5)$$

and

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = \frac{(\lambda)^{-z}}{\Gamma(\mu)} \int_0^1 \left(\text{Log} \frac{1}{t}\right)^{\mu-1} \Phi_{\mu,\nu}^{(0,\sigma)}\left(x \left(\text{Log} \frac{1}{t}\right)^\rho, z, \frac{a}{\lambda}\right) dt . \quad (3.6)$$

**Proof:** Denote, for convenience, the right-hand side of formula (3.5) by  $I$ . Then it is easily seen that

$$\begin{aligned} I &= (\lambda)^{-z} \frac{1}{\Gamma(\mu)} \int_0^\infty t^{\mu-1} e^{-t} \Phi_{\mu,\nu}^{(0,\sigma)}\left(xt^\rho, z, \frac{a}{\lambda}\right) dt . \\ &= \sum_{n=0}^\infty \frac{(\mu)_{0n} x^n}{\Gamma(\mu)(\nu)_{\sigma n} (a + \lambda n)^z} \int_0^\infty t^{\mu+\rho n-1} e^{-t} dt . \end{aligned}$$

Upon using the Hankel's formula [6, p.13(2)]

$$\frac{1}{\Gamma(z)} = \frac{1}{2\pi i} \int_{-\infty}^{(0+)} e^t t^{-z} dt , \quad |\arg(t)| \leq \pi, z \in \mathbb{C} . \quad (3.7)$$

and definition (2.1) , we are led finally to the left-hand side of formula (3.5).

In the same manner one can derive the formula (3.6).

In their special cases, when  $\rho = \sigma = \nu = \lambda = 1$ , the integral representations

(3.5) and (3.6) reduce to the following forms:

$$\Phi_\mu^*(x, z, a) = \frac{1}{\Gamma(\mu)} \int_0^\infty t^{\mu-1} e^{-t} \Phi_{\mu,1}^{(0,1)}(xt, z, a) dt , \quad (3.8)$$

and

$$\Phi_{\mu}^{*}(x, z, a) = \frac{1}{\Gamma(\mu)} \int_0^1 (\text{Log } \frac{1}{t})^{\mu-1} \Phi_{\mu,1}^{(0,1)} \left( x (\text{Log } \frac{1}{t}), z, a \right) dt, \quad (3.9)$$

respectively.

**Theorem 3:** Let  $0 < \text{Re}(\mu) < \text{Re}(\nu)$ ,  $\sigma \in \mathbb{R}^+$ . Then

$$\Phi_{\mu,\nu,\lambda}^{(\sigma,\sigma)}(x, z, a) = \frac{(\lambda)^{-z} \Gamma(\nu)}{\Gamma(\nu-\mu)\Gamma(\mu)} \int_0^{\infty} \frac{t^{\mu-1}}{(1+t)^{\nu}} \Phi \left( \frac{xt^{\sigma}}{(1+t)^{\sigma}}, z, \frac{a}{\lambda} \right) dt. \quad (3.10)$$

**Proof:** Denote, for convenience, the right-hand side of formula (3.10) by  $I$ . Then it is easily seen that

$$I = (\lambda)^{-z} \frac{\Gamma(\nu)}{\Gamma(\nu-\mu)\Gamma(\mu)} \int_0^{\infty} \frac{t^{\mu-1}}{(1+t)^{\nu}} \Phi \left( \frac{xt^{\sigma}}{(1+t)^{\sigma}}, z, \frac{a}{\lambda} \right) dt.$$

Now, in view of integral formula

$$\Phi_{\mu,\nu}^{(\rho,\sigma)}(x, z, a) = \frac{\Gamma(\nu)}{\Gamma(\nu-\mu)\Gamma(\mu)} \int_0^{\infty} \frac{t^{\mu-1}}{(1+t)^{\nu}} \Phi \left( \frac{xt^{\sigma}}{(1+t)^{\sigma}}, z, a \right) dt,$$

$$(0 < \text{Re}(\mu) < \text{Re}(\nu), \sigma \in \mathbb{R}^+),$$

and the definition (2.1), we obtain the left-hand side of formula (3.10).

Obviously, for  $\sigma = \nu = \lambda = 1$ , (3.10) reduces to the following result :

$$\Phi_{\mu}^{*}(x, z, a) = \frac{\sin(\pi\mu)}{\pi} \int_0^{\infty} \frac{t^{\mu-1}}{(1+t)^{\nu}} \Phi \left( \frac{xt}{(1+t)}, z, a \right) dt. \quad (3.11)$$

Furthermore we derive the following result .

**Theorem 4:** Let  $\nu \in \mathbb{C} / \mathbb{Z}_0^-, \sigma \in \mathbb{R}^+$ ,  $|\arg(w)| < \pi$ . Then

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = \frac{(\lambda)^{-z} \Gamma(\nu)}{2\pi i} \int_{-\infty}^{(0+)} w^{-\nu} e^w \Phi_{\mu,\nu}^{(\rho,0)} \left( xw^{-\sigma}, z, \frac{a}{\lambda} \right) dw. \quad (3.12)$$

**Proof:** Denote, for convenience, the right-hand side of formula (3.12) by  $I$ . Then in

view of (1.8), it is easily seen that

$$I = \sum_{n=0}^{\infty} \frac{(\mu)_{\rho n} x^n}{(a + \lambda n)^z} \frac{\Gamma(\nu)}{2\pi i} \int_{-\infty}^{(0+)} w^{-(\nu+\sigma n)} e^w dw .$$

Upon using the formula (3.7) and definition (2.1), we are led finally to left-hand side of formula (3.12).

Obviously, for  $\rho = \sigma = \nu = \lambda = 1$ , (3.12) reduces to the following result :

$$\Phi_{\mu}^*(x, z, a) = \frac{1}{2\pi i} \int_{-\infty}^{(0+)} w^{-1} e^w \Phi_{\mu,1}^{(1,0)}(xw^{-1}, z, a) dw , \quad (3.13)$$

$$(|\arg(w)| < \pi) .$$

Furthermore, in its limit case when  $\rho \rightarrow 0$ , (3.12) would simplify immediately to the following formula :

$$\Phi_{\mu,\nu,\lambda}^{(0,\sigma)}(x, z, a) = \frac{(\lambda)^{-z} \Gamma(\nu)}{2\pi i} \int_{-\infty}^{(0+)} w^{-\nu} e^w \Phi\left(xw^{-\sigma}, z, \frac{a}{\lambda}\right) dw , \quad (3.14)$$

$$(\nu \in \mathbb{C} / \mathbb{Z}_0^-, \sigma \in \mathbb{R}^+, |\arg(w)| \leq \pi) .$$

If we replace the Hankel's loop in (3.12) to (3.14) by a suitable Mellin – Barnes contour integral we get :

**Theorem 5 :** Let  $\operatorname{Re}(\nu) > 0, \sigma, \tau \in \mathbb{R}^+$ . Then

$$\Phi_{\mu,\nu,\lambda}^{(\rho,\sigma)}(x, z, a) = \frac{(\lambda)^{-z} \Gamma(\nu)}{2\pi i} \int_{\tau-i\infty}^{\tau+i\infty} w^{-\nu} e^w \Phi_{\mu,\nu}^{(\rho,0)}\left(xw^{-\sigma}, z, \frac{a}{\lambda}\right) dw , \quad (3.15)$$

$$\Phi_{\mu}^*(x, z, a) = \frac{1}{2\pi i} \int_{\tau-i\infty}^{\tau+i\infty} w^{-1} e^w \Phi_{\mu,1}^{(1,0)}(xw^{-1}, z, a) dw , \quad (3.16)$$

and

$$\Phi_{\mu,\nu,\lambda}^{(0,\sigma)}(x, z, a) = \frac{(\lambda)^{-z} \Gamma(\nu)}{2\pi i} \int_{\tau-i\infty}^{\tau+i\infty} w^{-\nu} e^w \Phi\left(xw^{-\sigma}, z, \frac{a}{\lambda}\right) dw . \quad (3.17)$$

**Proof :** Denote, for convenience, the right-hand side of formula (3.15) by  $I$ . Then it is easily seen that



$$I = \frac{(\lambda)^{-z} \Gamma(\nu)}{2\pi i} \int_{\tau-i\infty}^{\tau+i\infty} w^{-\nu} e^w \Phi_{\mu,\nu}^{(\rho,0)} \left( xw^{-\sigma}, z, \frac{a}{\lambda} \right) dw .$$

Now, in view of integral formula[23]

$$\Phi_{\mu,\nu}^{(\rho,\sigma)}(x, z, a) = \frac{\Gamma(\nu)}{2\pi i} \int_{\tau-i\infty}^{\tau+i\infty} w^{-\nu} e^w \Phi_{\mu,\nu}^{(\rho,0)}(xw^{-\sigma}, z, a) dw ,$$

$$(\operatorname{Re}(\nu) > 0, \sigma, \tau \in \mathbb{R}^+),$$

and definition (2.1), we are led finally to the left-hand side of formula (3.15).

In the same manner one can derive formulas (3.16) and (3.17).

#### 4. Fractional Derivative

We recall the Riemann - Liouville fractional derivative operator  $D_x^\mu$  [2,p.181] and [4]

$$D_x^\mu f(x) = \begin{cases} \frac{1}{\Gamma(-\mu)} \int_0^x (x-t)^{-\mu-1} f(t) dt, (\operatorname{Re}(\mu) < 0) \\ \frac{d^m}{dx^m} D_x^{\mu-m} f(x) \quad , \quad m-1 \leq \operatorname{Re}(\mu) < m (m \in \mathbb{N}) \end{cases} \quad (4.1)$$

Clearly , we have :

$$D_x^\mu \{x^\alpha\} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha-\mu+1)} x^{\alpha-\mu}, \quad (\operatorname{Re}(\alpha) > -1, \alpha - \mu \geq 0). \quad (4.2)$$

It is easy to derive the following results.

**Theorem 5:** Let  $\operatorname{Re}(\mu) > 0, \sigma \in \mathbb{R}^+$ . Then

$$D_x^{\mu-\nu} \left\{ x^{\mu-1} (\lambda)^{-z} \Phi \left( x^\sigma, z, \frac{a}{\lambda} \right) \right\} = \frac{\Gamma(\mu)}{\Gamma(\nu)} x^{\nu-1} \Phi_{\mu,\nu,\lambda}^{(\sigma,\sigma)}(x^\sigma, z, a) . \quad (4.3)$$

**Proof:** Denote, for convenience, the left-hand side of formula (4.3) by  $F^l$ . Then in view of (1.3), it is easily seen that

$$F^l = D_x^{\mu-\nu} \left\{ (\lambda)^{-z} \sum_{n=0}^{\infty} \frac{x^{\mu+\sigma n-1}}{(a+n)^z} \right\}.$$

Upon using the relation (4.2), we are led finally to right -hand side of formula (4.3).

In particular , when  $\lambda = 1$ , (4.3) would reduce to the following result :

$$D_x^{\mu-\nu} x^{\mu-1} \Phi(x^\sigma, z, a) = \frac{\Gamma(\mu)}{\Gamma(\nu)} x^{\nu-1} \Phi_{\mu,\nu}^{(\sigma,\sigma)}(x^\sigma, z, a) . \quad (4.4)$$

On putting  $\sigma = \nu = \lambda = 1$  in (4.3) , formula (4.3) reduces to the interesting result

$$\Phi_\mu^*(x, z, a) = \frac{1}{\Gamma(\mu)} D_x^{\mu-1} x^{\mu-1} \Phi(x, z, a) . \quad (4.5)$$

When  $\nu = \lambda = 1$  in (4.3) , we arrive to the following interesting result

$$\Phi_{\mu,1,1}^{(\sigma,\sigma)}(x^\sigma, z, a) = \frac{1}{\Gamma(\mu)} D_x^{\mu-1} x^{\mu-1} \Phi(x^\sigma, z, a) . \quad (4.6)$$

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