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Discriminating efficient DMUs using MAJ-FRH

Dariush Akbarian[1], **Hossein Dibachi**

Department of Mathematics, Arak Branch, Islamic Azad University, Arak, Iran.

[1] Corresponding author,

E-mail address: d akbarian@yahoo.com, d-akbarian@iau-arak.ac.ir

Abstract

Free Replication Hull (FRH) is a special case of the CCR DEA problem in which the convexity axiom is excluded from the PPS of the CCR model and the intensity variables are restricted to the whole numbers. In this paper we propose MAJ-FRH, similar to the ranking method in data envelopment analysis proposed by Mehrabian et al. [6] to discriminate FRH-efficient DMUs.

Keywords: Data envelopment analysis; Free Replication Hull (FRH); Ranking.

1. Introduction

Free Replication Hull (FRH), first proposed by Tulkens [10] is, in fact, an special case of CCR model in which the convexity axiom is relaxed and the intensity variables are restricted to the whole numbers. One important issue in Data envelopment analysis (DEA) is to discriminate between DEA-efficient DMUs. Izadikhah et al. [2] proposed a ranking method for all DMUs based on changing reference set and investigated the impact of efficient DMUs on the vitual and on influences of other efficient DMUs. Rakhshan et al. [1] proposed a fully ranking based on combining DEA and AHP and using Tchebycheff norm simultaneously. One of the popular ranking method is the super-efficiency DEA model and developed by Andersen and Petersen [4]. The main idea of this approach is to evaluate a DMU after this performer itself is excluded from the reference set. The problem with super-efficiency DEA is that under certain conditions infeasibility occurs which limits the applicability of the technique (see [6] for details). In order to prevail over this drawback, Mehrabian [3] provided a modification to the dual formulation. The model is referred to MAJ. In some cases, the MAJ model is infeasible (see Saati et al.[8]); so, Saati et al. [8] provided a modification to the MAJ model to ensure feasibility. Sun et al. [7] proposed a method for discriminating Free Disposal Hull (FDH) efficient DMUs based on the MAJ model. The number of efficient DMUs in FDH is significantly more than those in FRH. So, discriminate efficient DMUs in FRH is simpler than FDH. Also, unlike FDH model, FRH allows evaluated unit compares with multiplied combinations of other DMUs. It may be shows the advantage of using the FRH instead of FDH in evaluation DMUs. As far as we know, no specific studies on the discrimination of FRH-efficient DMUs problem have been proposed. In this paper we propose an alternative method for discriminating FRH-efficient DMUs based on the MAJ method. A comparison of cross-efficiency measure (CEM) by Doyle and Green [5] is presented. This paper is organized as follows. Section 2 we briefly review the FRH model and the MAJ method for ranking DMUs. In section 3 we introduce the MAJ-FRH method for ranking FRH-efficient DMUs and provide some examples. The paper is concluded in Section 4.

2. Literature review

2.1 FRH

The FRH model first, proposed by Tulkens [10] in which all integer combinations of the DMUs are considered. Consider a set of *n* DMUs which is associated with *m* inputs and *s* outputs.

Particularly, DMU_j $(j = 1,\ldots,n)$ consumes amount x_{ij} of input i and produces amount y_{rj} output r. Let $X_j = \left(x_{1j},\ \ldots,\ x_{mj}\right)$ in which $X_j \geq 0 \ \wedge\ X_j \neq 0$ and $Y_j = \left(y_{1j},\ \ldots,\ y_{rj}\right)$ in which $Y_j \geq 0 \ \wedge\ Y_j \neq 0$. The production possibility set (PPS) of FRH model define as follows:

$$T_{FRH} = \left\{ (X,\ \mathrm{Y}) \middle| X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j,\ \lambda_j \in \{0,1,2,3\ \ldots\},\ \mathrm{j} \in \mathrm{J} \right\}$$

The input-oriented FRH model, corresponds to DMU_p, $(p \in J)$, is given by (1):

$$\begin{aligned}
& \min\ \theta - \varepsilon \left(\sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+ \right) \\
& s.t.\ \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip},\quad i = 1,\ldots,m \\
& \qquad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp},\quad r = 1,\ldots,s \qquad\qquad (1) \\
& \qquad \lambda_j \in \{0,\ 1,\ 2,\ 3,\ \ldots\},\ j \in J \\
& \qquad s_i^- \geq 0, \qquad\qquad\qquad i = 1,\ldots,m \\
& \qquad s_r^+ \geq 0, \qquad\qquad\qquad r = 1,\ldots,s \\
& \qquad \theta \qquad\qquad\qquad\qquad\ free
\end{aligned}$$

where ε is non-Archimedean small and positive number and s_i^- and s_r^+ $i = 1,\ldots,m,\ \ r = 1,\ldots,s$ are called slack variables belong to ℝ≥0. Note that s_i^- represents input excesses; also s_r^+ represent output shortfalls. Also θ *is* real numbers. This is a mixed integer programming problem. DMU_p is said to be *strong efficient* (*FRH-efficient*) if and only if: $\theta^* = 1$ and $s_i^- = 0, s_r^+ = 0$. The superscript ($*$) indicates optimality. Also, we use the GAMs software to run the FRH models.

2.2 MAJ

One problem with AP model is infeasibility. Therefore, Mehrabian et al. [3] suggested MAJ model in order to ensure feasibility. Their model corresponding DMU_p is as follows:

$$
\begin{aligned}
&min\ \ \omega_p + 1 \\
&s.t. \quad \sum_{j \in J-\{p\}} \lambda_j x_{ij} \le x_{ip} + \omega_p, \quad i = 1,\dots,m \\
&\qquad \sum_{j \in J-\{p\}} \lambda_j y_{rj} \ge y_{rp}, \qquad\qquad r = 1,\dots,s \qquad\qquad (2) \\
&\qquad \lambda_j \ge 0, \qquad\qquad\qquad\qquad j \in J-\{p\} \\
&\qquad \omega_p \qquad\qquad\qquad\qquad\qquad free
\end{aligned}
$$

But as mentioned earlier, the MAJ model may be infeasible. Therefore, Saati et al. [8] provided the revised MAJ model as follows:

$$
\begin{aligned}
&min\ \ \omega_p + 1 \\
&s.t. \quad \sum_{j \in J-\{p\}} \lambda_j x_{ij} \le x_{ip} + \omega_p, \quad i = 1,\dots,m \\
&\qquad \sum_{j \in J-\{p\}} \lambda_j y_{rj} \ge y_{rp} - \omega_p, \quad r = 1,\dots,s \qquad\qquad (3) \\
&\qquad \lambda_j \ge 0, \qquad\qquad\qquad\qquad j \in J-\{p\} \\
&\qquad \omega_p \qquad\qquad\qquad\qquad\qquad free
\end{aligned}
$$

3. Main result

In this section we propose MAJ-FRH model by relaxing the convexity assumption in model (3). MAJ-FRH model formulated as follows:

$$
\begin{aligned}
& min\ \ \omega_p + 1 \\
& s.t. \quad \sum_{j \in J - \{p\}} \lambda_j x_{ij} \le x_{ip} + \omega_p, \quad i = 1, \dots, m \\
& \qquad \sum_{j \in J - \{p\}} \lambda_j y_{rj} \ge y_{rp} - \omega_p, \quad r = 1, \dots, s \qquad (4) \\
& \qquad \lambda_j \in \{0,\ 1, 2, 3, \dots\}, \quad j \in J - \{p\} \\
& \qquad \omega_p \qquad\qquad\qquad\qquad free
\end{aligned}
$$

As far as we know, no specific studies on the discrimination power of FRH have been proposed.

Therefore, this paper contributes a FRH based upon (modified) MAJ (Saati et al. [8]) for discriminating between FRH-efficient DMUs using MAJ-FRH model (4) without infeasibility problem.

Example 1. Consider five DMUs A, B, C, D and E each consume two input to produce one output as given in Table (1). Running the DEA model (1) will result three FRH-efficient DMUs as B, C and D. The results of ranking of these DMUs by using CEM and MAJ-FRH are summarized in Table (2). As it can be seen the CEM provides a full ranking of DMUs. The MAJ-FRH cannot provide full ranking of FRH-efficient DMUs and leads three ties between FRH-efficient DMUs B, C and D and two ties between A and E.

Example 2. A set of three DMUs taken from [7] is given in Table (3). All DMUs are FRH-efficient by model (1). The results of ranking of these DMUs by using CEM and MAJ-FRH are reported in Table (4). In this case CEM and MAJ-FRH cannot provide a full ranking of DMUs.

Example 3. In this Example we evaluate by CEM and MAJ-FRH methods the data set listed in Table (5) and are taken from Table 1 in [9]. All DMUs are FRH-efficient DMUs. The results of ranking of these DMUs using CEM and MAJ-FRH are summarized in Table (6). As it can be seen, the CEM and MAJ-FRH can provide a full ranking of DMUs. However, MAJ-FRH method leads to two ties between DMUs OS7 and OS22 and between OS12 and OS13.

4. Conclusion

As far as we know; there is no any research has been proposed to the discrimination of the FRH-efficient DMUs. In this paper we propose a variant super efficiency model for ranking FRH-efficient DMUs based on MAJ method ([3]). Unlike the most of the other super efficiency methods, our proposed approach is always feasible. We examined our method by three examples.

From the results of numerical comparisons, we may conclude that MAJ-FRH method provide more reasonable ranking when the number of DMUs, input and output are increased. Moreover,

one may use AP and/or 0-1 LP methods in order to discrimination of the FRH-efficient DMUs. This needs further research.

Tables

Table 1: Example 1. DMUs' data (extracted from [4]).

DMU	A	B	C	D	E
Input1	2	2	5	10	10
Input1	12	8	5	4	6
Output	1	1	1	1	1

Table 2: Result of Example 1.

DMU	CEM		MAJ-FRH	
	Score	Rank	Score	Rank
A	0.65	4	1	4
B	0.85	1	2	1
C	0.84	2	2	2
D	0.73	3	2	3
E	0.75	5	1	5

Table 3: Example 2. DMUs' data.

DMU	DMU1	DMU2	DMU3
Input1	4	1	8
Input2	2	5	1
Output1	0	1	0
Output2	1	1	1

Table 4: Result of Example 2.

DMU	CEM Score	MAJ-FRH Score
DMU1	infeasible	2
DMU2	infeasible	2
DMU3	infeasible	2

Table 5: Example 3. DMU's Data

DMU	Inputs						Outputs				
	C[a]	I[a]	P[a]	M&C[a]	Other JA activities[a]	Other operating costs[b]	C[b]	I[b]	P[b]	M&C[b]	Other JA activities[b]
OS 1	55	28	80	43	57	821.6297	392.3953	62.0127	471.1391	19.1946	51.9546
OS 2	12	4	3	2	3	69.0808	66.3004	10.9712	6.1344	0.2477	0.1782
OS 3	2	1	2	1	1	12.9246	4.4534	3.7658	7.487	0.3754	0.0904
OS 4	7	4	30	5	9	218.006	33.7826	11.8225	188.9325	4.637	3.1162
OS 5	19	4	45	3	9	211.7006	120.0685	11.26	77.132	1.6226	35.3613
OS 6	10	5	32	14	12	219.3919	61.8237	13.6316	150.3821	4.8328	19.4298
OS 7	9	6	13	7	10	106.9018	35.4512	13.7407	73.5423	3.3843	0.6542
OS 8	9	5	8	4	5	103.0289	61.2949	13.1706	45.2847	1.4578	1.39176
OS 9	5	3	9	6	5	109.9591	55.5173	7.0557	61.6572	4.4775	0.5266
OS10	17	9	52	22	37	363.3056	120.7935	26.1731	257.0555	17.3159	6.7286
OS11	2	2	5	2	3	31.5761	13.4334	4.6068	16.3876	1.5393	0.2503
OS12	8	4	31	7	9	180.0819	40.087	9.7788	139.1302	11.0481	2.6673
OS13	11	3	25	14	11	238.2405	59.0392	12.8907	189.5601	9.6284	3.3184
OS14	5	4	16	5	10	109.6939	34.7642	10.4361	79.2585	2.5592	0.4648
OS15	9	4	25	10	10	165.6027	37.1657	12.55	134.2577	8.9578	0.9486
OS16	9	2	1	2	14	169.0845	82.0091	11.6801	6.9266	103.2209	5.1423
OS17	5	2	17	4	4	121.8967	30.3236	6.6783	103.4982	3.4092	0.7909
OS18	6	2	5	3	4	61.818	35.5225	6.0714	31.5346	2.9075	1.8022
OS19	4	1	6	2	2	56.9165	17.4308	4.2854	42.6173	2.5145	0.7661
OS20	9	5	24	9	16	177.5242	49.9451	14.3873	122.1197	4.5403	9.6991
OS21	5	2	10	4	4	111.2907	38.5934	6.7844	72.4829	6.4178	1.089
OS22	5	2	16	5	5	111.4448	28.3405	6.9281	86.3661	3.8395	3.2509
OS23	15	7	43	13	21	254.6729	61.2704	18.6946	198.2142	12.2306	8.161
OS24	8	3	15	4	9	146.5831	56.924	8.1121	98.4952	2.4588	2.2893
OS25	7	3	27	8	9	143.4506	36.8097	8.9171	109.8258	5.698	3.5025
OS26	36	13	60	30	31	372.6933	130.9099	30.8536	268.4286	11.7677	5.3655
OS27	46	14	124	45	55	765.9635	188.4216	42.5586	570.2482	27.8539	57.5849
OS28	23	6	37	23	43	497.7266	80.8111	20.4679	309.5052	86.8802	49.6043
OS29	20	14	87	14	32	445.9886	93.2598	30.4755	357.1496	6.7055	16.0935
OS30	15	6	24	10	18	165.6646	50.3537	14.647	114.4559	4.5443	12.1747
OS31	10	4	24	9	18	205.9418	60.3521	14.3967	139.6452	3.951	7.56
OS32	27	7	51	26	33	268.7522	72.7858	28.2966	204.6602	7.7147	12.0527

[a] Indicates the number of employees.
[b] Indicates 10 million yen. C = credit, I = insurance, P = purchasing, M&C = marketing and consultation.

Table 6: Result of example 3.

DMU	CEM		MAJ-FRH	
	Score	Rank	Score	Rank
OS1	0.77	17	13.3980	5
OS2	0.64	32	21.8711	4
OS3	0.81	10	7.4870	13
OS4	0.90	1	12.0575	6
OS5	0.76	22	23.0448	3
OS6	0.82	4	9.0000	9
OS7	0.74	28	3.0000	26
OS8	0.77	19	6.9302	14
OS9	0.77	21	6.2480	16
OS10	0.78	15	2.8346	28
OS11	0.76	25	4.5266	21
OS12	0.76	26	4.0000	23
OS13	0.82	6	4.0000	23
OS14	0.82	8	4.1391	22
OS15	0.77	18	5.2743	18
OS16	0.79	14	92.1728	1
OS17	0.86	2	9.6451	8
OS18	0.75	27	8.6557	10
OS19	0.82	5	7.7065	11
OS20	0.81	11	4.6460	20
OS21	0.82	7	3.7318	24
OS22	0.83	3	3.0000	26
OS23	0.76	23	2.8494	27
OS24	0.78	16	4.9704	19
OS25	0.76	24	2.6212	29
OS26	0.66	31	2.7407	30
OS27	0.77	20	10.3857	7
OS28	0.82	9	34.3434	2
OS29	0.80	13	6.2851	15
OS30	0.73	29	7.5982	12
OS31	0.81	12	3.2927	25
OS32	0.72	30	5.8358	17

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