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The numerical–asymptotic solution of spatial model of the aerobic water treatment

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Abstract

In this paper, the mathematical model of biological water treatment by spatial filter was designed. The algorithm of numerical-asymptotic approximation of the solution of the corresponding model problem was developed. It describes a system of nonlinear singularly perturbed differential equations, such as "convection-diffusion mass transfer" for area, bounded by the four surfaces of the current and two equipotential surfaces. On this basis, a computer experiment of optimization parameters of the purification process was conducted. It includes the loading time of protective action, the size of the filter etc., the results of which confirmed a known fact that the performance of the filter greatly depends on the choice of its shape.

Keywords: spatial modeling task, biological filter, asymptotic-numerical solution, singularly perturbed problem.

1. Introduction

Methods for biological wastewater treatment have several significant advantages over other methods due to the ability of microorganisms to adapt to adverse conditions (high concentrations and toxicity, a complex mixture of pollutants) [1-8]. The finding of cost-effective and environmentally acceptable methods of cleaning industrial and household and municipal waste water was and remains very relevant in major cities. A growing number of urban residents, extensive infrastructure, intensive functioning of food, microbiological, pharmaceutical and other industries lead to daily growth of wastewater contaminated with organic matter. Why is increasing the need for water purification.

Currently, the design of sewage treatment is based on the results of large domestic and foreign experience [1-13]. Because wastewater may differ in features culturing bacteria in them (the presence of sufficient nutrients, toxicity, etc.), in each case, the suitability of water for growing bacteria must be checked. Prediction of water quality in water, carried out under the existing mathematical, mostly one-dimensional, patterns (for generally accepted standardized indicators) largely to determine the necessary degree of wastewater treatment.

The aim of this work is to develop a mathematical model of spatial process of cleaning waste water from nutrients that takes into account the interaction of bacteria, organic and biologically not oxidative substances in porous environment.

2. Statement of the problem

Consider the process of cleaning liquids of organic contaminants in porous biofilter shaped curvilinear parallelepiped $G_z = ABCDA_*B_*C_*D_*$ (fig.1), limited by smooth orthogonal to each other (along the edges) equipotential surfaces $ABB_*A_* = \{z: f_1(x,y,z)=0\}$ (in particular $f_1(x,y,z) = z - z^*$), $CDD_*C_* = \{z: f_2(x,y,z)=0\}$ (in particular $f_2(x,y,z) = z - z_*$), and surfaces flow $ADD_*A_* = \{z: f_3(x,y,z)=0\}$, $BCC_*B_* = \{z: f_4(x,y,z)=0\}$, $ABCD = \{z: f_5(x,y,z)=0\}$, $A_*B_*C_*D_* = \{z: f_6(x,y,z)=0\}$, by filtering solution with the introduction of a biological product. According to the literature (theoretical and experimental data) [9-13] distinguish the following its parts (components): the decomposition of organic contamination by bacteria, the growth and death of bacteria, the making bacteria surfactants, the transition to organic pollution of not oxidative biological agents, the transfer of bacteria given their sorption and desorption, the transfer of biologically not oxidative material with liquid because of its sorption and desorption and diffusion.

To describe the dynamics of bacterial populations including cell death of microorganisms, the equation of Mono was used [9-13]:

$$\frac{\partial B}{\partial t} = \frac{\mu_{\max} U}{U + K_S} B - \lambda B, \tag{1}$$

where $B(x, y, z, t)$ – the concentration of bacteria in the mid-point $(x, y, z,)$ of the loading area at a time t, $\mu_{\max}$ – maximum speed of the growth of bacteria, $U(x, y, z, t)$ – the concentration of pollution (substrate) in the liquid, K_S – constant of affinity substrate for the microorganism, λ – the rate of cell death.

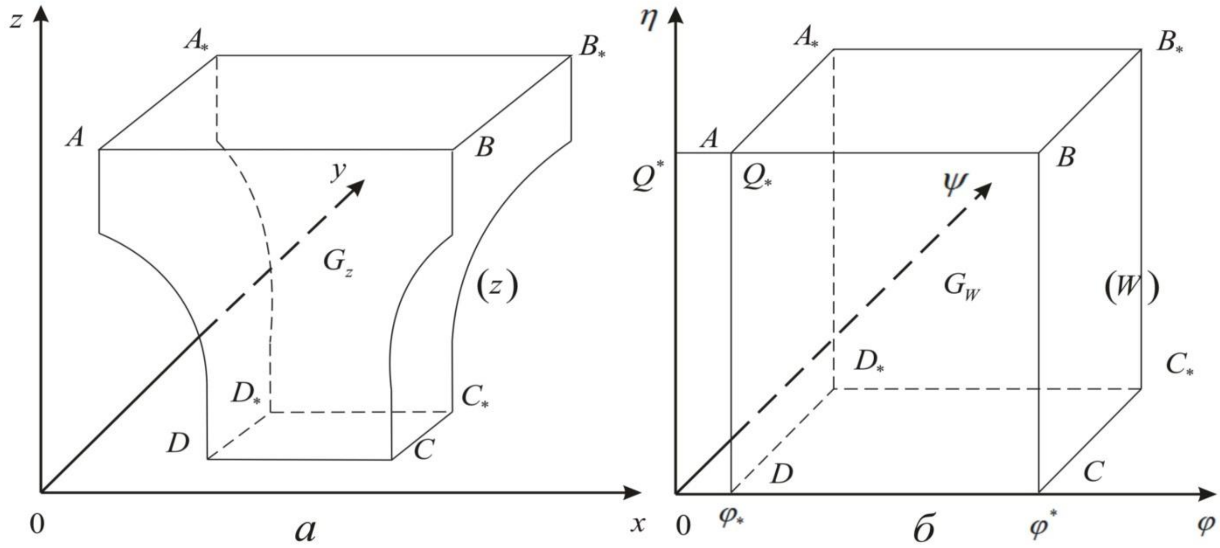

Fig. 1. Physical spatial region G_z (biofilter) (a) and the corresponding integrated capacity G_w (b)

Given that bacteria move together with the contaminated substance in a porous medium, similarly to [6], we arrive at the following equation for growth, dying and carry bacteria:

$$\frac{\partial B}{\partial t} = \frac{\mu_{\max} U}{U + K_S} B - \lambda B - \frac{\vec{v}}{\sigma_e} \cdot \vec{\nabla} B, \tag{2}$$

where $\vec{v}(v_x, v_y, v_z)$ – vector of filtration rate, ($|\vec{v}| > v_* >> \varepsilon > 0$), σ_e – effective porosity, ($\sigma_e = k_a\rho + \sigma$, k_a – rate of adsorption of bacteria, ρ - medium density, σ – porosity), ε – small setting.

Reduction of fluid contamination is due to decomposition by bacteria and filtration of biologically not oxidative substances. The biological decomposition can be described by the equation type Mono and leaching – convection equation, describing the combination of changing of pollution concentration with time:

$$\frac{\partial U}{\partial t} = -\frac{1}{q}\frac{\mu_m U}{U + K_S} B - \vec{v}\cdot\vec{\nabla}U - \frac{\vec{v}}{\rho}\cdot\vec{\nabla}C + D_U \Delta U, \tag{3}$$

where q – proportionality factor that relates the number of cells that formed when they absorbed substrate, $C(x, y, z, t)$ – concentration of biologically not oxidative substances, $\vec{\nabla}$ – Hamilton operator, $\Delta = \vec{\nabla}\cdot\vec{\nabla}$ – Laplace operator, D_U – diffusivity, $0 < b_U \le 1$, $D_U = b_U\varepsilon$ (Diffuse component of the process is small comparably with the other components).

In most cases, we can assume that the bacteria produce surfactants simultaneously with the decomposition of hydrocarbons, that some of the hydrocarbons are used to create surfactants.

Like the previous one, using the equation of Mono, we write the equation describing the change in concentration of biologically not oxidative matter:

$$\sigma\frac{\partial C}{\partial t} = \frac{1}{\rho q_S}\frac{\mu_m U}{K_S + U} B - \vec{v}\cdot\vec{\nabla}C - \beta C + D_C \Delta C, \tag{4}$$

where q_S – proportionality factor that relates the number of formed of not oxidative organic substances absorbed from the substrate β – factor that characterizes the amount of trapped particulate biologically not oxidative matter by filter, D_C – diffusivity, $0 < b_C \le 1$, $D_C = b_C\varepsilon$.

We believe that, at the initial time biofilter is clean, his side "wall" is impervious; at the entrance to filter known (given) time distribution of all three concentrations and there is a "rapid withdrawal" at the output [9-13].

Given the above, we come to the next problem, describing the complex changes in the concentrations of bacteria, organic and biologically not oxidative substances in porous media $G = G_z \times (0, \infty)$:

$$\begin{cases} \dfrac{\partial B}{\partial t} = \left(\Phi(U) - \lambda\right) B - \dfrac{\vec{v}}{\sigma_e} \cdot \vec{\nabla} B, \\ \dfrac{\partial U}{\partial t} = -s\Phi(U) B - \vec{v} \cdot \vec{\nabla} U - \dfrac{\vec{v}}{\rho} \cdot \vec{\nabla} C + D_U \Delta U, \\ \sigma \dfrac{\partial C}{\partial t} = h\Phi(U) B - \vec{v} \cdot \vec{\nabla} C - \beta C + D_C \Delta C, \end{cases} \tag{5}$$

$B\big|_{ABB_*A_*} = B_*(M,t)$, $U\big|_{ABB_*A_*} = U_*(M,t)$, $\dfrac{\partial U}{\partial \vec{n}}\Big|_{CDD_*C_*} = 0$, $\dfrac{\partial U}{\partial \vec{n}}\Big|_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$

$C\big|_{ABB_*A_*} = 0$, $\dfrac{\partial C}{\partial \vec{n}}\Big|_{CDD_*C_*} = 0$, $\dfrac{\partial C}{\partial \vec{n}}\Big|_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0,$

$$B(x,y,z,0) = 0,\ U(x,y,z,0) = 0,\ C(x,y,z,0) = 0; \tag{6}$$

$$\vec{v} = \kappa \cdot grad\ \varphi,\ div\,\vec{v} = 0, \tag{7}$$

$$\varphi\big|_{ABB_*A_*} = \varphi_*,\ \varphi\big|_{CDD_*C_*} = \varphi^*,\ \frac{\partial \varphi}{\partial \vec{n}}\Big|_{ADD_*A_* \cup BCC_*B_* \cup ABCD \cup A_*B_*C_*D_*} = 0, \tag{8}$$

where $\Phi(U) = \dfrac{\mu_{\max} U}{U + K_S}$, $B_*(M,t)$, $U_*(M,t)$ – sufficiently smooth function, coordinated at the edges of the field G , M – arbitrary point corresponding surface; $s = q_*^{-1}\varepsilon$, $h = \left(\rho q_{S*}\right)^{-1} \varepsilon$ ($q_* = q\varepsilon,\ q_{S*} = q_S \varepsilon$), $\varphi = \varphi(x,y,z)$ – filtration capacity ($0 < \varphi_* \le \varphi \le \varphi^* < \infty$), κ – filtration rate of environment, $\vec{n}$ – external normal to the respective surface [10]. This vanishing of the normal derivatives of desired functions along four surfaces are traditional flow conditions impermeability, and along the exit area CDD_*C_* – the terms "rapid withdrawal" (Verigin conditions).

Suppose that the filtration problem (7) – (8) on the spatial conformal mapping $G_{\mathrm{w}} \mapsto G_{\mathrm{z}}$, where $G_{\mathrm{w}} = \left\{ \mathrm{w} = (\varphi, \psi, \eta) : \varphi_* < \varphi < \varphi^*,\ 0 < \psi < Q_*, 0 < \eta < Q^* \right\}$ – on G_{z} integrated potential; $\psi = \psi(x, y, z)$, $\eta = \eta(x, y, z)$ – functions of flow of a complex conjugate with $\varphi = \varphi(x, y, z)$, namely such that: $grad\varphi = grad\psi \times grad\eta$, is solved [9-13], in particular, a dynamic grid and right speed were built and flow filtration was calculated $Q = Q_* \cdot Q^*$. Then, making a change of variables $x = x(\varphi, \psi, \eta)$, $y = y(\varphi, \psi, \eta)$, $z = z(\varphi, \psi, \eta)$ in system (5) and conditions (6), we arrive at the appropriate convection-diffusion mass transfer problem for the region $G_\omega = G_{\mathrm{w}} \times (0, \infty)$:

$$\begin{cases} \dfrac{\partial b}{\partial t} = \left(\Phi(u) - \lambda\right) b - \dfrac{v^2}{\sigma_e} \dfrac{\partial b}{\partial \varphi}, \\ \dfrac{\partial u}{\partial t} = -\dfrac{\varepsilon}{q_*} \Phi(u) b - v^2 \cdot \dfrac{\partial u}{\partial \varphi} - \dfrac{v^2}{\rho} \cdot \dfrac{\partial c}{\partial \varphi} + \varepsilon b_U \left(v^2 \dfrac{\partial^2 u}{\partial \varphi^2} + b_1 \dfrac{\partial^2 u}{\partial \psi^2} + b_2 \dfrac{\partial^2 u}{\partial \eta^2} + d_1 \dfrac{\partial u}{\partial \psi} + d_2 \dfrac{\partial u}{\partial \eta} \right), \\ \sigma \dfrac{\partial c}{\partial t} = \dfrac{\varepsilon}{\rho q_{S*}} \Phi(u) b - v^2 \cdot \dfrac{\partial c}{\partial \varphi} - \beta c + \varepsilon b_C \left(v^2 \dfrac{\partial^2 c}{\partial \varphi^2} + b_1 \dfrac{\partial^2 c}{\partial \psi^2} + b_2 \dfrac{\partial^2 c}{\partial \eta^2} + d_1 \dfrac{\partial c}{\partial \psi} + d_2 \dfrac{\partial c}{\partial \eta} \right), \end{cases} \tag{9}$$

$b(\varphi_*, \psi, \eta, t) = b_*(\psi, \eta, t)$, $b(\varphi, \psi, \eta, 0) = 0$, $u(\varphi_*, \psi, \eta, t) = u_*(\psi, \eta, t)$, $u_\varphi(\varphi^*, \psi, \eta, t) = 0$,

$$u_\psi(\varphi, 0, \eta, t) = u_\psi(\varphi, Q_*, \eta, t) = u_\eta(\varphi, \psi, 0, t) = u_\eta(\varphi, \psi, Q^*, t) = 0, \tag{10}$$

$u(\varphi, \psi, \eta, 0) = 0$, $c(\varphi_*, \psi, \eta, t) = 0$, $c_\varphi(\varphi^*, \psi, \eta, t) = 0$,

$c_\psi(\varphi, 0, \eta, t) = c_\psi(\varphi, Q_*, \eta, t) = c_\eta(\varphi, \psi, 0, t) = c_\eta(\varphi, \psi, Q^*, t) = 0$, $c(\varphi, \psi, \eta, 0) = 0$,

where $b = b(\varphi, \psi, \eta, t) = B\left(x(\varphi, \psi, \eta), y(\varphi, \psi, \eta), z(\varphi, \psi, \eta), t\right)$,

$u = u(\varphi, \psi, \eta, t) = U\left(x(\varphi, \psi, \eta), y(\varphi, \psi, \eta), z(\varphi, \psi, \eta), t\right)$,

$c = c(\varphi, \psi, \eta, t) = C\left(x(\varphi, \psi, \eta), y(\varphi, \psi, \eta), z(\varphi, \psi, \eta), t\right)$,

$b(\varphi_*, \psi, \eta, t) = b_*(\psi, \eta, t) = B_*\left(x(\varphi_*, \psi, \eta), y(\varphi_*, \psi, \eta), z(\varphi_*, \psi, \eta), t\right)$,

$u(\varphi_*, \psi, \eta, t) = u_*(\psi, \eta, t) = U_*\left(x(\varphi_*, \psi, \eta), y(\varphi_*, \psi, \eta), z(\varphi_*, \psi, \eta), t\right)$,

$b_1 = b_1(\varphi,\psi,\eta) = \vec{\nabla}\psi^2$, $b_2 = b_2(\varphi,\psi,\eta) = \vec{\nabla}\eta^2$, $d_1 = d_1(\varphi,\psi,\eta) = \Delta\psi$,

$d_2 = d_2(\varphi,\psi,\eta) = \Delta\eta$ (see., eg., [7]).

3. Asymptotic of the decision

Solution of the problem (9) – (10) with an accuracy of $O(\varepsilon^2)$ at the assumption of strong coordination initial and boundary conditions (10) along the edges and corner points of the field G_ω (see. eg. 10) in a look of asymptotic series [9-13]:

$$b = b_0 + \varepsilon b_1 + R_b,$$

$$u = u_0 + \varepsilon u_1 + \sum_{i=0}^{1}\varepsilon^i P_i + \sum_{i=0}^{1}\varepsilon^i \breve{P}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\tilde{P}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\tilde{\tilde{P}}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\widehat{P}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\widehat{\widehat{P}}_i + R_u, \quad (11)$$

$$c = c_0 + \varepsilon c_1 + \sum_{i=0}^{1}\varepsilon^i \Pi_i + \sum_{i=0}^{1}\varepsilon^i \breve{\Pi}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\tilde{\Pi}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\tilde{\tilde{\Pi}}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\widehat{\Pi}_i + \sum_{i=0}^{3}\varepsilon^{i/2}\widehat{\widehat{\Pi}}_i + R_c,$$

where $R_b(\varphi,\psi,\eta,t,\varepsilon)$, $R_u(\varphi,\psi,\eta,t,\varepsilon)$, $R_c(\varphi,\psi,\eta,t,\varepsilon)$ – the remaining members, $b_i(\varphi,\psi,\eta,t)$, $u_i(\varphi,\psi,\eta,t)$, $c_i(\varphi,\psi,\eta,t)$ – members of the regular asymptotics ($i = 0,1$); $\Pi_i(\xi,\psi,\eta,t)$, $P_i(\xi,\psi,\eta,t)$ – feature type boundary layer in the neighborhood $\varphi = \varphi^*$ (amendments to the outlet of the filter) ($i = \overline{0,1}$), $\breve{\Pi}_i(\breve{\xi},\psi,\eta,t)$, $\breve{P}_i(\breve{\xi},\psi,\eta,t)$ – layer in the neighborhood $\varphi = \varphi^*$ (amendments to the inlet of the filter) ($i = \overline{0,1}$), and functions, $\tilde{\Pi}_i(\varphi,\tilde{\psi},\eta,t)$, $\tilde{\tilde{\Pi}}_i(\varphi,\tilde{\tilde{\psi}},\eta,t)$, $\widehat{\Pi}_i(\varphi,\psi,\tilde{\eta},t)$, $\widehat{\widehat{\Pi}}_i(\varphi,\psi,\tilde{\tilde{\eta}},t)$ and $\tilde{P}_i(\varphi,\tilde{\psi},\eta,t)$, $\tilde{\tilde{P}}_i(\varphi,\tilde{\tilde{\psi}},\eta,t)$, $\widehat{P}_i(\varphi,\psi,\tilde{\eta},t)$, $\widehat{\widehat{P}}_i(\varphi,\psi,\tilde{\tilde{\eta}},t)$ ($i = \overline{0,3}$) – in the neighborhood $\psi = 0$, $\psi = Q_*$, $\eta = 0$, $\eta = Q^*$ (Amendment to the side walls of the filter), respectively; $\xi = \dfrac{\varphi^* - \varphi}{\varepsilon}$, $\breve{\xi} = \dfrac{\varphi - \varphi_*}{\varepsilon}$, $\tilde{\psi} = \dfrac{\psi}{\sqrt{\varepsilon}}$, $\tilde{\tilde{\psi}} = \dfrac{Q_* - \psi}{\sqrt{\varepsilon}}$, $\tilde{\eta} = \dfrac{\eta}{\sqrt{\varepsilon}}$, $\tilde{\tilde{\eta}} = \dfrac{Q^* - \eta}{\sqrt{\varepsilon}}$ – "Stretching" the relevant variables. As a result of the substitution (11) at (9) – (10) and standardized procedure "equalizing" coefficients of the same steps ε, we get the following tasks to find $b_i(\varphi,\psi,\eta,t)$, $u_i(\varphi,\psi,\eta,t)$, $c_i(\varphi,\psi,\eta,t)$ $(i=0,1)$:

$$\begin{cases} \dfrac{\partial b_0}{\partial t} = \left(\Phi\left(u_0\right)-\lambda\right) b_0 - \dfrac{v^2}{\sigma_e}\dfrac{\partial b_0}{\partial \varphi}, \\ \dfrac{\partial u_0}{\partial t} + v^2 \dfrac{\partial u_0}{\partial \varphi} = -\dfrac{v^2}{\rho}\dfrac{\partial c_0}{\partial \varphi}, \\ \sigma \dfrac{\partial c_0}{\partial t} + v^2 \dfrac{\partial c_0}{\partial \varphi} + \beta c_0 = 0, \\ b_0(\varphi,\psi,\eta,0)=0,\, b_0(\varphi_*,\psi,\eta,t) = b_*(\psi,\eta,t), \\ u_0(\varphi,\psi,\eta,0)=0,\, u_0(\varphi_*,\psi,\eta,t) = u_*(\psi,\eta,t), \\ c_0(\varphi,\psi,\eta,0)=0,\, c_0(\varphi_*,\psi,\eta,t) = 0, \end{cases}$$

$$\begin{cases} \dfrac{\partial b_1}{\partial t} = \left(\Phi\left(u_1\right)-\lambda\right) b_1 - \dfrac{v^2}{\sigma_e}\dfrac{\partial b_1}{\partial \varphi}, \\ \dfrac{\partial u_1}{\partial t} + v^2 \dfrac{\partial u_1}{\partial \varphi} = -\dfrac{v^2\sigma}{\rho}\dfrac{\partial c_1}{\partial \varphi} - \dfrac{1}{q_*}\Phi\left(u_0\right) b_0 + b_U\left(v^2\dfrac{\partial^2 u_0}{\partial \varphi^2} + b_1\dfrac{\partial^2 u_0}{\partial \psi^2} + b_2\dfrac{\partial^2 u_0}{\partial \eta^2} + d_1\dfrac{\partial u_0}{\partial \psi} + d_2\dfrac{\partial u_0}{\partial \eta}\right), \\ \sigma \dfrac{\partial c_1}{\partial t} + v^2 \dfrac{\partial c_1}{\partial \varphi} + \beta c_1 = \dfrac{1}{\rho q_{S*}}\Phi\left(u_0\right) b_0 + b_C\left(v^2\dfrac{\partial^2 c_0}{\partial \varphi^2} + b_1\dfrac{\partial^2 c_0}{\partial \psi^2} + b_2\dfrac{\partial^2 c_0}{\partial \eta^2} + d_1\dfrac{\partial c_0}{\partial \psi} + d_2\dfrac{\partial c_0}{\partial \eta}\right), \\ b_1(\varphi,\psi,\eta,0)=0,\, b_1(\varphi_*,\psi,\eta,t) = 0, \\ u_1(\varphi,\psi,\eta,0)=0,\, u_1(\varphi_*,\psi,\eta,t) = 0, \\ c_1(\varphi,\psi,\eta,0)=0,\, c_1(\varphi_*,\psi,\eta,t) = 0, \end{cases}$$

As a result of their solving we obtained:

$$c_0\left(\varphi,\psi,\eta,t\right)=0,$$

$$u_0\left(\varphi,\psi,\eta,t\right) = \begin{bmatrix} \displaystyle\int_0^t g_0\left(\varphi - v^2\left(t-\tilde{t}\right),\psi,\eta,\tilde{t}\right)d\tilde{t},\; t \le \dfrac{\varphi}{v^2}, \\ \dfrac{1}{v^2}\displaystyle\int_0^{\varphi} g_0\left(\tilde{\varphi},\psi,\eta,\dfrac{1}{v^2}\left(\tilde{\varphi}-\varphi+v^2 t\right)\right)d\tilde{\varphi} + u_*\left(\psi,\eta,t-\dfrac{\varphi}{v^2}\right),\; t > \dfrac{\varphi}{v^2}, \end{bmatrix}$$

$$b_0\left(\varphi,\psi,\eta,t\right) = \begin{bmatrix} b_*\left(t-\dfrac{\sigma_e\varphi}{v^2}\right)\cdot e^{\frac{\left(\lambda-\Phi\left(u_0\right)\right)\sigma_e\varphi}{v^2}},\; t \ge \dfrac{\sigma_e\varphi}{v^2}, \\ 0,\; t < \dfrac{\sigma_e\varphi}{v^2}, \end{bmatrix}$$

$$c_1(\varphi,\psi,\eta,t)=\begin{cases}\dfrac{\sigma}{v^2}e^{\frac{\beta\sigma\varphi}{v^2}}\cdot\displaystyle\int_0^{\varphi}e^{-\frac{\beta\sigma\tilde{\varphi}}{v^2}}w_1\left(\tilde{\varphi},\psi,\eta,t+\dfrac{\sigma}{v^2}(\tilde{\varphi}-\varphi)\right)d\tilde{\varphi},\ t\geq\dfrac{\sigma\varphi}{v^2},\\ \dfrac{e^{\beta t}}{\sigma}\cdot\displaystyle\int_0^{t}e^{-\beta\tilde{t}}w_1\left(\dfrac{v^2}{\sigma}(\tilde{t}-t)+\varphi,\psi,\eta,\tilde{t}\right),t<\dfrac{\sigma\varphi}{v^2},\end{cases}$$

$$u_1(\varphi,\psi,\eta,t)=\begin{cases}\displaystyle\int_0^{t}g_1\left(\varphi-v^2(t-\tilde{t}),\psi,\eta,\tilde{t}\right)d\tilde{t},\ t\leq\dfrac{\varphi}{v^2},\\ \dfrac{1}{v^2}\displaystyle\int_0^{\varphi}g_1\left(\tilde{\varphi},\psi,\eta,\dfrac{1}{v^2}(\tilde{\varphi}-\varphi+v^2t)\right)d\tilde{\varphi},\ t>\dfrac{\varphi}{v^2},\end{cases}\qquad b_1(\varphi,\psi,\eta,t)=0,$$

where $g_0(\varphi,\psi,\eta,t)=\dfrac{v^2}{\rho}\dfrac{\partial c_0}{\partial\varphi}$,

$$w_1=\frac{1}{\rho q_S}\Phi(u_0)b_0+b_C\left(v^2\frac{\partial^2c_0}{\partial\varphi^2}+b_1\frac{\partial^2c_0}{\partial\psi^2}+b_2\frac{\partial^2c_0}{\partial\eta^2}+d_1\frac{\partial c_0}{\partial\psi}+d_2\frac{\partial c_0}{\partial\eta}\right),$$

$$g_1=-\frac{v\sigma}{\rho}\frac{\partial c_1}{\partial x}-\frac{1}{q}\Phi(u_0)b_0+b_U\left(v^2\frac{\partial^2u_0}{\partial\varphi^2}+b_1\frac{\partial^2u_0}{\partial\psi^2}+b_2\frac{\partial^2u_0}{\partial\eta^2}+d_1\frac{\partial u_0}{\partial\psi}+d_2\frac{\partial u_0}{\partial\eta}\right).$$

Functions $P_i(\xi,\psi,\eta,t)$, $\Pi_i(\xi,\psi,\eta,t)$ $(i=0,1)$ are found by solving such problems [7]:

$$\begin{cases}b_u\dfrac{\partial^2P_0}{\partial\xi^2}+\dfrac{\partial P_0}{\partial\xi}=0,\\ P_0\underset{\xi\to\infty}{\to}0,\quad\dfrac{\partial u_0(\varphi^*,\psi,\eta,t)}{\partial\varphi}+\dfrac{\partial P_0(0,\psi,\eta,t)}{\partial\xi}=0,\end{cases}$$

$$\begin{cases}b_c\dfrac{\partial^2\Pi_0}{\partial\xi^2}+\dfrac{\partial\Pi_0}{\partial\xi}=0,\\ \Pi_0\underset{\xi\to\infty}{\to}0,\quad\dfrac{\partial c_0(\varphi^*,\psi,\eta,t)}{\partial\varphi}+\dfrac{\partial\Pi_0(0,\psi,\eta,t)}{\partial\xi}=0,\end{cases}$$

$$\begin{cases}b_u\dfrac{\partial^2P_1}{\partial\xi^2}+\dfrac{\partial P_1}{\partial\xi}=\tilde{q}_1,\\ P_1\underset{\xi\to\infty}{\to}0,\ \dfrac{\partial P_1(0,\psi,\eta,t)}{\partial\xi}=0,\end{cases}\quad\begin{cases}b_c\dfrac{\partial^2\Pi_1}{\partial\xi^2}+\dfrac{\partial\Pi_1}{\partial\xi}=\tilde{p}_1,\\ \Pi_1\underset{\xi\to\infty}{\to}0,\ \dfrac{\partial\Pi_1(0,\psi,\eta,t)}{\partial\xi}=0,\end{cases}$$

where $\tilde{q}_1(\xi,\psi,\eta,t) = v^{-2} b_u \left(b_1 \frac{\partial^2 P_0}{\partial \psi^2} + b_2 \frac{\partial^2 P_0}{\partial \eta^2} + d_1 \frac{\partial P_0}{\partial \psi} + d_2 \frac{\partial P_0}{\partial \eta} \right)$,

$$\tilde{p}_1(\xi,\psi,\eta,t) = v^{-2} b_c \left(b_1 \frac{\partial^2 \Pi_0}{\partial \psi^2} + b_2 \frac{\partial^2 \Pi_0}{\partial \eta^2} + d_1 \frac{\partial \Pi_0}{\partial \psi} + d_2 \frac{\partial \Pi_0}{\partial \eta} \right).$$

Tasks for finding functions $\breve{\Pi}_i(\breve{\xi},\psi,\eta,t)$, $\breve{P}_i(\breve{\xi},\psi,\eta,t)$ $(i=\overline{0,1})$ and $\tilde{\Pi}_i(\varphi,\tilde{\psi},\eta,t)$, $\tilde{\tilde{\Pi}}_i(\varphi,\tilde{\tilde{\psi}},\eta,t)$, $\hat{\Pi}_i(\varphi,\psi,\tilde{\eta},t)$, $\hat{\hat{\Pi}}_i(\varphi,\psi,\tilde{\tilde{\eta}},t)$ and $\tilde{P}_i(\varphi,\tilde{\psi},\eta,t)$, $\tilde{\tilde{P}}_i(\varphi,\tilde{\tilde{\psi}},\eta,t)$, $\hat{P}_i(\varphi,\psi,\tilde{\eta},t)$, $\hat{\hat{P}}_i(\varphi,\psi,\tilde{\tilde{\eta}},t)$ $(i=\overline{0,3})$ are constructed similarly. Estimates of the remaining members are held, similar to [9-13].

4. The results of the numerical calculations

Here are the results of calculations by formulas (11) (to simplify calculations assume that there is a one-dimensional filter) at $B_*^*(t)=10$ *kl/ml*, $U_*^*(t)=0.005$ g/g, $v=5\,m/h$, $\beta=36^{-1}$ *m²/c*, $\lambda=0.06\,day^{-1}$, $\sigma_e=5$, $\sigma=0.37$, $\mu_m=2.5\,day^{-1}$, $K_S=0.1$ *g/g*, $b_U=1.25\cdot10^{-4}$ *m²/h*, $b_C=2\cdot10^{-4}$ *m²/h*, $\rho=1.5\,g/cm^3$, $q=2\cdot10^{-9}$ *kl/g*, $q_S=4\cdot10^{11}$ *kl/g*.

Figures 2-4 are simulation results that describes the biological treatment filter thickness, in particular, in Figure 2 shows the distribution of pollution concentration along the filter at different times, in Figure 3 – concentration distribution along bacteria filter, 4 – concentration distribution of biologically not oxidative material along the filter at different times.

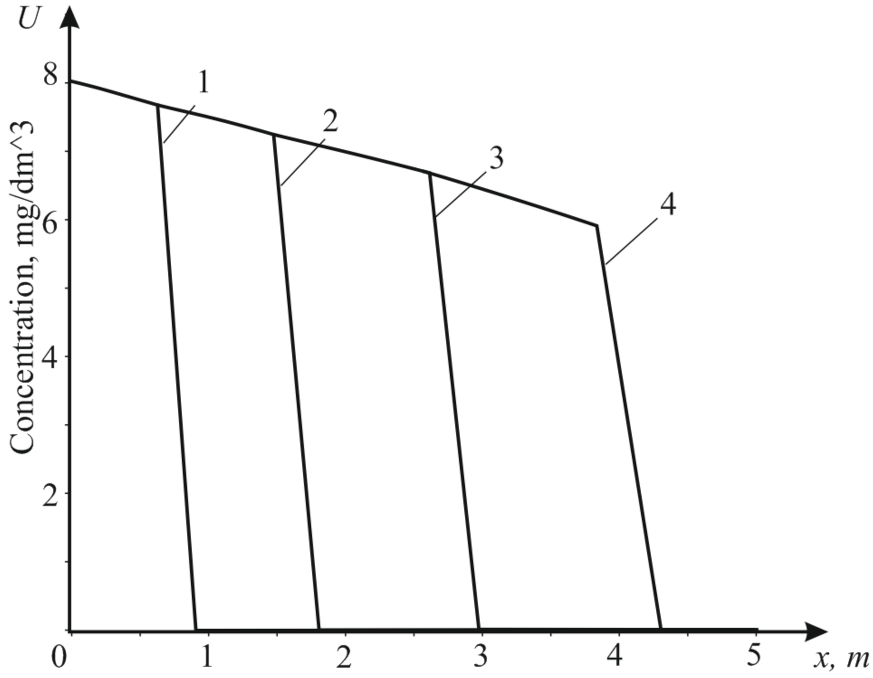

Fig. 2. Distribution of pollution concentration along the filter at a time $t_1 = 10\,h$, $t_2 = 20\,h$, $t_3 = 30\,h$, $t_4 = 40\,h$

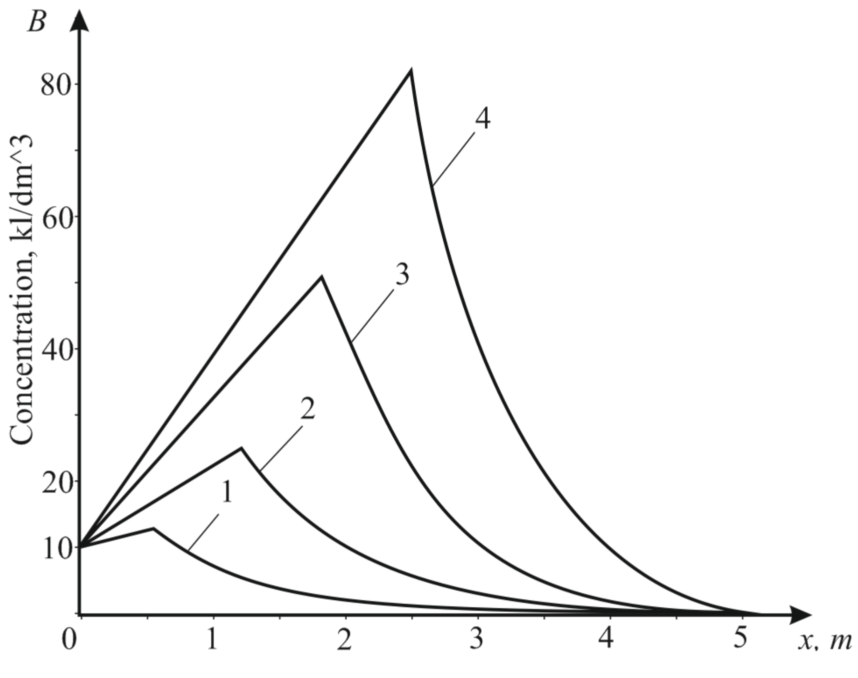

Fig. 3. Distribution of bacteria concentration along the filter at a time $t_1 = 10\,h$, $t_2 = 20\,h$, $t_3 = 30\,h$, $t_4 = 40\,h$

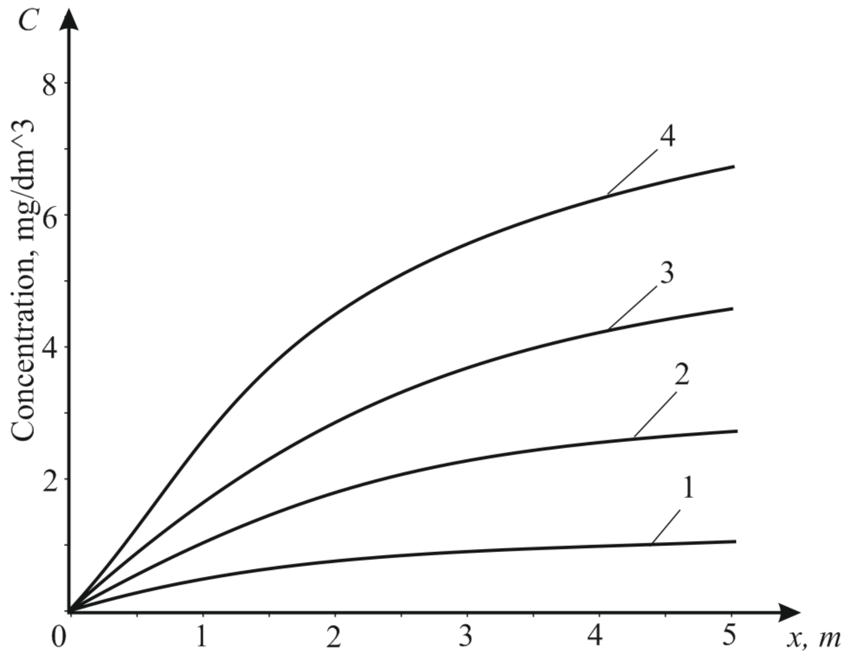

Fig. 4. Average concentrations of biological not oxidative substances along filter at time $t_1 = 10\,h$, $t_2 = 20\,h$, $t_3 = 30\,h$, $t_4 = 40\,h$

5. Conclusions

The dimensional mathematical model describing the basic laws of wastewater treatment in biofilters and appropriate algorithm for solving singularly perturbed problem were developed. The obtained formulas and graphical relationship between variables are effective for theoretical research aimed at optimizing the parameters of the filtering process (time of protective action boot size filter, etc.). Calculations show that after a certain point in time you can stop the flow of bacteria in the biofilter and it will not affect the growth of bacteria in the filter, and the process of filtering, which in turn greatly reduce the cost of the purification process. Thus setting the filter shape plays a significant role in the filter as this may lead to an increase (decrease) filtering options, and to increase the productivity of his work in general.

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