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The Structure of Groups GL(3,F)

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Abstract

Let M be the **JS**-imprimitive of $GL(3,3)$, that is, $M := GL(1,3)\ wr\ S_3$.

This group has order 48, and is generated by the matrices

$$c := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} a := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, b := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and

In order to obtain a polycyclic presentation for M we introduce the element

$$e := d^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} d := c^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad and$$

Then a polycyclic presentation for M is

$\{a, b, c, d, e \mid \ a^2 = I_3 \ ,$

$b^a = b^2 \ , b^3 = I_3,$

$c^a = d \quad , c^b = d \ , c^2 = I_3 \ ,$

$d^a = c \quad , d^b = e \ , d^c = d, d^2 = I_3,$

$e^a = e \quad , e^b = e \ , e^c = e \ , e^d = e \ , e^2 = I_3\}.$

Note that $\cong C_2 \times S_4$. $M =< cde > \times < a,b,ce,de >$

KeyWords: polycyclic presentation, imprimitive, conjugacy class.

1.Introduction

complete and irredundant list of $GL(3,5)$ – conjugacy class representatives of the irreducible subogroups of M is:

$< a,b,c,d,e >, \quad \cong C_2 \times S_4;$

$< a,b,ce,de >, \quad \cong S_4;$

$< acde,b,ce,de >, \cong S_4;$

$< b,c,d,e >, \quad \cong C_2 \times A_4$

$< b,ce,de >, \quad \cong A_4.$

Now let M be the **JS**-imprimitive of $GL(3,5)$,that is ,

S_3. $M := GL(1,5) \ wr$

This group has order 384 ,and is generated by the matrices

$$c := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad a := \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, b := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and .

In order to obtain a polycyclic presetation for M we introduce the elements

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad d := c^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $e := d^b$.

Then a polycyclic presentation for M is

$\{a,b,c,d,e \mid a^2 = I_3$,

$$b^a = b^2, b^3 = I_3,$$
$$c^a = d \ , c^b = d, c^4 = I_3,$$
$$d^a = c \ , d^b = e, d^c = d, d^4 = I_3,$$
$$e^a = e \ , e^b = c, e^c = e, e^d = e, e^4 = I_3\}$$

Note that

$$M = <cde> \times <a(cde)^2, b, ce^{-1}, de^{-1}>$$
$$= <cde> \times (M \cap SL(3,5)).$$

2-Theorem:

A complete and irredundant list of $GL(3,5)$ - conjugacy class

representatives of the irreducible subgroups of M is:

$$<a(cde)^2, b, ce^{-1}, de^{-1}, (cde)^i>, i = 1,2,4;$$
$$<a(cde)^{2+j}, b, ce^{-1}, de^{-1}>, j = 1,2;$$
$$<b, ce^{-1}, de^{-1}, (cde)^k>, k = 1,2,4;$$
$$<a(cde)^2, b, (ce^{-1})^2, (de^{-1})^2, (cde)^l>, l = 1,2,4;$$
$$<a(cde)^{2+m}, b, (ce^{-1})^2, (de^{-1})^2>, m = 1,2;$$
$$<b, (ce^{-1})^2, (de^{-1})^2, (cde)^n>, n = 1,2,4.$$

Proof: Set B:=$<cde>$ *and* $N := M \cap SL(3,5).$ We use the CAYLEY function lattice to obtain the subgroup lattice of N . If subgroup of N is irreducible，then its projection into the top group must be transitive. Therefore the **2**-subgroups of N are reducible.This leaves just six conjuacy classes of subgroups to consider，

the groups in those classes being isomorphic to one of N ，48.52(this is the number of that group in the tables of Neubuser (1967)，S_4, A_4, S_3 or C_3.

It is easly to show ，then that the only irreducible subgroups of N are N ，**48.52**，S_4 and A_4. The proof now proceeds in the same way as that of

Theorm 4.3.4. Note that none theses groups a normal subgroups of index **4**，and that **48.52** has no subgroups of index **2**.The relevant table is table **1**.

G	$\lvert G \rvert$	$G \cap SL(3,5)$	$\lvert G \cap B \rvert$
$< a(cde)^2, b, ce^{-1}, de^{-1}, (cde)^i >, i = 1,2,4$	$\frac{384}{i}$	N	$\frac{4}{i}$
$< a(cde)^{2+j}, b, ce^{-1}, de^{-1} >, j = 1,2$	$\frac{192}{j}$	48.52	$\frac{2}{j}$
$< b, ce^{-1}, de^{-1}, (cde)^k >, k = 1,2,4$	$\frac{192}{k}$	48.52	$\frac{4}{k}$
$< a(cde)^{2+m}, b, (ce^{-1}), (de^{-1})^2 >, m = 1,2;$	$\frac{48}{m}$	A_4	$\frac{2}{m}$
$< b, (ce^{-1})^2, (de^{-1})^2, (cde)^n >, n = 1,2,4;$	$\frac{48}{m}$	A_4	$\frac{4}{n}$
$< a(cde)^2, b, (ce^{-1})^2, (de^{-1})^2, (cde)^l >, l = 1,2,4$	$\frac{96}{l}$	S_4	$\frac{4}{l}$

Table1:Information on the imprimitive soluble subgroups of $GL(3,5)$

Now for determine the imprimitive soluble subgroups of $GL(5,3)$,let M be the

-imprimitive of $GL(5,3)$,that is, $M := GL(1,3)\ wr\ Hol(C_5)$. JS

By the same methods as in the previous two case we can write down a polycyclic presentation for M ,is:

$$M = \{a,b,c,d,e,f,g \mid a^4 = I_5,$$

$$\begin{aligned}
&b^a = b^2, b^5 = I_5,\\
&c^a = d\ , c^b = d, c^2 = I_5,\\
&d^a = f, d^b = e\ , d^c = d, d^2 = I_5,\\
&e^a = c\ , e^b = f\ , e^c = e\ , e^d = e\ , e^2 = I_5,\\
&f^a = e, f^b = g\ , f^c = f, f^d = f, f^e = f, f^2 = I_5,\\
&g^a = g, g^b = c\ , g^c = g, g^d = g\ , g^e = g, g^f = g\ , g^2 = I_5\}.
\end{aligned}$$

Note that $M =< cdefg > \times < acdefg, b, cd, de, ef, fg >=< cdefg > \times (M \cap SL(5,3))$

3-Theorem D:

A complete and irredundant list of $GL(5,3)$ - conjugacy class representative of the irreducible subgroups of M is:

$$< acdefg, b, cd, ef, fg, (cdefg)^i >, i = 1,2;$$
$$< a, b, cd, de, ef, fg >;$$
$$< a^2, b, cd, de, ef, fg, (cdefg)^j >, j = 1,2;$$
$$< a^2cdefg, b, cd, de, ef, fg >;$$
$$< b, cd, de, ef, fg, (cdefg)^k >, k = 1,2.$$

Proof: Set $B := < cdefg >$ and $N := < a, b, cd, de, ef, fg >$. We use the CAYLEY function lattice to obtain the subgroup lattice of N.

If a subgroup of N is irreducible, then its projection into then its projection into the top group must be transitive. Therefore the **2**-subgroups of N are irreducible. This leaves just six conjugacy classes of subgroups to consider, the groups in those classes being isomorphic to one of N, $\frac{1}{2}N$ (meaning the unique subgroup of N of index **2**), $\frac{1}{4}N$ (meaning the uniqe subgroup of N of index **4**), $Hol(C_5), D_{10}$ or C_5. It is easly to show, then, that the only irreducible subgroups of N are $N, \frac{1}{2}N$ and $\frac{1}{4}N$. The proof now proceeds in the same way as that of **4.3.4** Theorem A. The relevant table is Table **2**.

G	$\|G\|$	$G \cap SL(5,3)$	$\|G \cap B\|$
$< acdefg, b, cd, ef, fg, (cdefg)^i >, i = 1,2$	$\frac{640}{i}$	N	$\frac{2}{i}$
$< a, b, cd, de, ef, fg >,$	320	$\frac{1}{2}N$	1
$< a^2, b, cd, de, ef, fg, (cdefg)^i >, j = 1,2$	$\frac{320}{j}$	$\frac{1}{2}N$	$\frac{2}{j}$
$< a^2cdefg, b, cd, de, ef, fg >$	160	$\frac{1}{4}N$	1

$< b, cd, de, ef, fg, (cdefg)^k >, k = 1,2$	$\frac{160}{k}$	$\frac{1}{4}N$	$\frac{2}{k}$

Table 2 – Information on the primitive soluble subgroups of $GL(5,3)$.

4-A generating set for a JS-primitive of $GL(3, p^k)$.

Let F be the field of p^k elements, where $p^k \equiv 1 (\bmod 3)$. Then since $Sp(2,3)$ is soluble, it follows that there is at just one **JS**-primitive of $GL(3,F)$ whose unique maximal abelian normal subgroup has order p^k -1, namely $M := (C_{p^k-1} \mathrm{Y} E) \ \mathrm{N} \ Sp(2,3)$

Where E is extraspecial of order **27** and exponent **3**.In this section we drive a polycyclic presentation for M, from which we see that $M = C_{p^k-1} \mathrm{Y} (E \succ Sp(2,3))$. This provides a constructive Proof of theorem A in the case $q = 3$. We also need this presentation in the other section when we derive a presentation for one of the **JS-**primitive of $GL(6, p^k)$. We construct a generating set form by the methods described in chapter **2**.Let z be a generator for the scalar group, and define u and v by

and $$v := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon^2 \end{bmatrix}, \quad u := \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

where ε is a primitive cube root of unity in F .Then $\{u, v, z\}$ generate $Fit(M)$. To extented this set to a generating set for M, we first require a generating set for $Sp(2,3)$.The set we use consists of the three matrices

$$a\rho := \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \ b\rho := \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \ and \ c\rho := \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

These matrices satisfy the following relations:

$$(a\rho)^3 = 1$$
$$(b\rho)^{a\rho} = c\rho, (b\rho)^2 = (c\rho)^2$$
$$(c\rho)^{a\rho} = b\rho c\rho, (c\rho)^{b\rho} = (c\rho)^3, (c\rho)^4 = 1.$$

In this case, by the theory in chapter **2** there exist matrices a, b and c of $GL(3,F)$ such that

$$u^a = \lambda_1 uv^2, u^b = \lambda_2 uv, u^c = \lambda_3 u^2 v,$$
$$v^a = \mu_1 v, v^b = \mu_2 uv^2, v^c = \mu_3 uv.$$

Where the λ_i and μ_j are scalars . Setting $\lambda_3 = \mu_3 = \varepsilon^2$ we find that one solution for c is

$$c := (1-\varepsilon)^{-1}\begin{bmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \end{bmatrix}.$$

Then c has determinat **1** and order **4** . Setting $\lambda_2 = 1$ and $\mu_2 = \varepsilon^2$ we find that one solution for b is

$$b := (1-\varepsilon)^{-1}\begin{bmatrix} 1 & \varepsilon & 1 \\ \varepsilon & 1 & 1 \\ \varepsilon & \varepsilon & \varepsilon^2 \end{bmatrix}.$$

Then b has determinat **1**, $b^2 = c^2$ and $c^b = c^3$. Setting $\lambda_1 = \mu_1 = 1$ we find that one solution for a is

$$a := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}.$$

Then a has determinat ε and order 3, and $b^a = c$ and $c^a = bc.$ Then we have that $M = < a, b, c, u, v, z >,$ where

$$\begin{aligned}
&a^3 = I_3 \quad , \\
&b^a = c \quad , b^2 = c^2, \\
&c^a = bc \quad , c^b = c^3 \quad , c^4 = I_3, \\
&u^a = uv^2, u^b = uv \quad , u^c = \varepsilon^2 u^2 v, u^3 = I_3, \\
&v^a = v \quad , v^b = \varepsilon^2 uv^2, v^c = \varepsilon uv \quad , v^u = \varepsilon v \ , v^3 = I_3, \\
&z^a = z \quad , z^b = z \quad , z^c = z \quad , z^u = z \quad , z^v = z \ , z^{p^{k-1}} = I_3.
\end{aligned}$$

This yields a polycyclic presentation for M (after replacing ε by $z^{(p^k-1)/3}$).

From this presentation we see that

$$\begin{aligned} M &=< z > \mathrm{Y} < a,b,c,u,v > \\ &\cong C_{p^k-1} \mathrm{Y} (E \succ Sp(2,3)). \end{aligned}$$

We also have that

$$M \cap SL(2,F) = \begin{cases} < \varepsilon^{-\frac{1}{3}} a,b,c,u,v > & if \quad p^k \equiv 1(\bmod 9), \\ < b,c,u,v > & if \quad p^k \equiv 1(\bmod 9). \end{cases}$$

Finally we investigate the action of field automorphism on M .

$M_1(5,p^k) = GL(1,p^k) \;\; wr \;\; Hol(C_5), p^k \neq 2,$
$M_2(5,p^k) = C_{p^{5k}-1} \succ C_5,$
$M_3(5,p^k) = (C_{p^k-1} \mathrm{Y} \;\; E_{125}) \;\; \mathrm{N} \;\; Sp(2,5) \;\; , p^k \equiv 1(\bmod 5).$

Thus we have the following table for $q = 3,5,7$ and $p^k = 1,...,30.$

$GL(3,1)$	M_1	M_2		$GL(5,1)$	M_1	M_2		$GL(7,1)$	M_1	M_2	
$GL(3,2)$		M_2		$GL(5,2)$		M_2		$GL(7,2)$		M_2	
$GL(3,3)$	M_1	M_2		$GL(5,3)$	M_1	M_2		$GL(7,3)$	M_1	M_2	
$GL(3,4)$	M_1	M_2	M_3	$GL(5,4)$	M_1	M_2		$GL(7,4)$	M_1	M_2	
$GL(3,5)$	M_1	M_2		$GL(5,5)$	M_1	M_2		$GL(7,5)$	M_1	M_2	
$GL(3,6)$	M_1	M_2		$GL(5,6)$	M_1	M_2	M_3	$GL(7,6)$	M_1	M_2	
$GL(3,7)$	M_1	M_2	M_3	$GL(5,7)$	M_1	M_2		$GL(7,7)$	M_1	M_2	
$GL(3,8)$	M_1	M_2		$GL(5,8)$	M_1	M_2		$GL(7,8)$	M_1	M_2	M_3
$GL(3,9)$	M_1	M_2		$GL(5,9)$	M_1	M_2		$GL(7,9)$	M_1	M_2	
$GL(3,10)$	M_1	M_2	M_3	$GL(5,10)$	M_1	M_2		$GL(7,10)$	M_1	M_2	
$GL(3,11)$	M_1	M_2		$GL(5,11)$	M_1	M_2	M_3	$GL(7,11)$	M_1	M_2	
$GL(3,12)$	M_1	M_2		$GL(5,12)$	M_1	M_2		$GL(7,12)$	M_1	M_2	
$GL(3,13)$	M_1	M_2	M_3	$GL(5,13)$	M_1	M_2		$GL(7,13)$	M_1	M_2	
$GL(3,14)$	M_1	M_2		$GL(5,14)$	M_1	M_2		$GL(7,14)$	M_1	M_2	

$GL(3,15)$	M_1	M_2		$GL(5,15)$	M_1	M_2		$GL(7,15)$	M_1	M_2	M_3
$GL(3,16)$	M_1	M_2	M_3	$GL(5,16)$	M_1	M_2	M_3	$GL(7,16)$	M_1	M_2	
$GL(3,17)$	M_1	M_2		$GL(5,17)$	M_1	M_2		$GL(7,17)$	M_1	M_2	
$GL(3,18)$	M_1	M_2		$GL(5,18)$	M_1	M_2		$GL(7,18)$	M_1	M_2	
$GL(3,19)$	M_1	M_2	M_3	$GL(5,19)$	M_1	M_2		$GL(7,19)$	M_1	M_2	
$GL(3,20)$	M_1	M_2		$GL(5,20)$	M_1	M_2		$GL(7,20)$	M_1	M_2	
$GL(3,21)$	M_1	M_2		$GL(5,21)$	M_1	M_2	M_3	$GL(7,21)$	M_1	M_2	
$GL(3,22)$	M_1	M_2	M_3	$GL(5,22)$	M_1	M_2		$GL(7,22)$	M_1	M_2	M_3
$GL(3,23)$	M_1	M_2		$GL(5,23)$	M_1	M_2		$GL(7,23)$	M_1	M_2	
$GL(3,24)$	M_1	M_2		$GL(5,24)$	M_1	M_2		$GL(7,24)$	M_1	M_2	
$GL(3,25)$	M_1	M_2	M_3	$GL(5,25)$	M_1	M_2		$GL(7,25)$	M_1	M_2	
$GL(3,26)$	M_1	M_2		$GL(5,26)$	M_1	M_2	M_3	$GL(7,26)$	M_1	M_2	
$GL(3,27)$	M_1	M_2		$GL(5,27)$	M_1	M_2		$GL(7,27)$	M_1	M_2	
$GL(3,28)$	M_1	M_2	M_3	$GL(5,28)$	M_1	M_2		$GL(7,28)$	M_1	M_2	
$GL(3,29)$	M_1	M_2		$GL(5,29)$	M_1	M_2		$GL(7,29)$	M_1	M_2	M_3
$GL(3,30)$	M_1	M_2		$GL(5,30)$	M_1	M_2		$GL(7,30)$	M_1	M_2	

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