



## Inquiry-Based Science and Mathematics Using Dynamic Modeling

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### Abstract:

Over the past six years, I have integrated student engagement through inquiry- and project-based learning into college mathematics instructions. I attended many workshops about using computation modeling in the classroom supports student learning and reduces student anxiety. Through dynamic modeling students can connect Mathematics skills with real-world questions. At this presentation I will focus on inquiry-based learning enhanced by computational dynamic modeling with content and practice to assist audiences to incorporate modeling and simulation at all levels of students from freshman to senior. The presentation will expose audiences to the basics of computational thinking, including modeling and simulation, with special emphasis on exploring topics that complement each other in math, engineering, and physical, life, and social science.

**Keywords:** dynamic modeling, procedural mathematics, conceptual mathematics, project-based learning.

## **1. Introduction**

Many college students have to take remedial courses to brush up their high school math. Researchers found that math education focusing on procedural math (i.e. knowing the steps to solve a problem) makes it harder for students to remember the math long-term. They argue that "conceptual math sticks with students longer than procedural, and that algebra without real-world connections does not prepare students for college math.

The paper “The Math Class – Paradox” [1] by Professor Jo Boaler, published at the online journal “The Atlantic” on Feb. 4, 2016 pointed out that “mastering the subject has become less about learning and more about performance”. In other words “Passing” is more important than “Learning”. Therefore mathematics instructors concentrate on teaching procedural math to help students earn a satisfactory grade or get a better scores at all standardized tests. After the classes students only have satisfactory grades and don’t get preparation for professional jobs.

My instruction approaches are inquiry based learning using dynamic modeling that connect college mathematics with real-world issues. In such a way students can develop their critical thinking skills and unzipped their potential. What student learn is not only the content from syllabi, they also learn the applications to the real-world problems. Through model building students have fun and less anxiety of memorizing procedures. The ultimate goal of dynamic modeling is that whatever story you give me, I will build a mathematics model for you that models your story.

## **2. Why do I choose Dynamic Modeling?**

Dynamic Modeling is a better way to connect classroom mathematics to real world issues. It is easier to interpret abstract concepts to tangible results. Since there is no correct model, students can build acceptable models according to their background. This fits in differentiated teaching model and reduce student’s anxiety of learning mathematics. Because it is individualized the research topics can be used from freshman to senior. With help of computer software, even freshman can design linear differential equation models. Students will not be scared by Calculus, or Differential Equation classes anymore.

In order to design a model, students need to read the story carefully then identify major variables and auxiliary variables (constants). The next step is to draw a diagram that show relationships between those variables (formulate model). Finally implement formula using modeling apps. If the model is not acceptable then students can modify it until it is acceptable.

### **3. Popular Computer APPS for Dynamic Modeling**

- Microsoft Excel: This is a very popular spreadsheet software. Almost every student know how to use it more or less. In order to guild a dynamic model, students need to have extra skill that can build scroll bars.
- Vensim: Vensim is designed to allow visual model building when we have a quantity (or multiple quantities) that are changing over time for which we can describe the change. Vensim is free for educational use and runs on both Windows and MacOS machines. To draw the graph of model, Vensim is one of the best candidates.
- AgentCubes (<https://www.agentcubesonline.com/>) is a tool for agent modeling that has the advantage of ease of use when building models. You can get all information from the website.
- NetLogo: Netlogo is less user friendly than AgentCubes, and so the learning curve for model building is steeper. AgentCubes uses a “click and drag” technique for building the conditions that govern the model, while Netlogo takes a programming approach. Netlogo has the advantage of being freely available and easy to install on any platform. Depending on the intent of the modeling activity, you may find that giving the students a model to run and modify will meet the learning goals in your course with requiring the student to learn the underlying programming language.
- Matlab Simulink toolbox: Matlab is a very popular high-end computer software. Most college students in STEM fields us it to do their research projects and homework assignments. Certainly the price for complete system is high.

The reason I use Excel as tool to build dynamic models is that every student can access Excel without extra cost and every student has minimal Excel skills. I shall introduce how to create scroll bar for Excel 2013. A scroll bar is used to interactively change the value of a cell and any

cell or graphs that depend on the cell controlled by a scroll bar. This allows you to see how the results in a cell, chart, or graph change as the data changes.

1. First, we need to enable the developer menu if it does not already appear as one of your menu choices. To enable the Developer menu (EXCEL 2013) click File menu then Options, then Customize Ribbon, select Commands not in the Ribbon, check Developer then OK
2. Select Developer, then Insert, then Form Control and choose the Scroll bar option.
3. The mouse will change to a thin crossbar that allows us to draw a rectangle. Drag over the area where you would like your scroll bar to be located and the scroll bar will appear.
4. Once we create scroll bar, we need to tell the scroll bar what cell it will control. To do this, right click on the scroll bar and select Format Control from the resulting menu (last menu in Excel 2013): Format Control dialog Box pop up, we can fill in the information we need than OK.
5. Format Control dialog box:
  - a. Current: value right now
  - b. Minimum value: the lowest setting
  - c. Maximum value: highest setting
  - d. Incremental change: how much the value will change if we click on the scroll bar itself
  - e. Page change: how much the value will change if you click on the scroll bar itself
  - f. Cell link: the cell whose value will be affected by the scroll bar.
6. A scroll bar will not manipulate cells containing decimals or negative numbers. If we need a scroll bar manipulate a value that is negative or a decimal the cell it controls will be a helper cell and then use that helper value in another cell to create the value we need. If scroll bar control cell C1 is 20, and we need a decimal values such as interest rate, we can use different cell C2 and enter  $=c1/100$ . C2 become our real control cell.

#### **4. First Sample Model (Sleeping Beauty)[2]**

Sleeping beauty is a well-known fiction. The main idea is that princess got poisoned by witch became sleeping beauty. There came a prince who kissed sleeping beauty and turned sleeping

beauty back to princess. At this story princess and sleeping are major variables because they had change in quantities. Witch and prince are auxiliary variables. The parameters are rates of poisoned by witch and kissed by prince respectively. The mathematics formula are

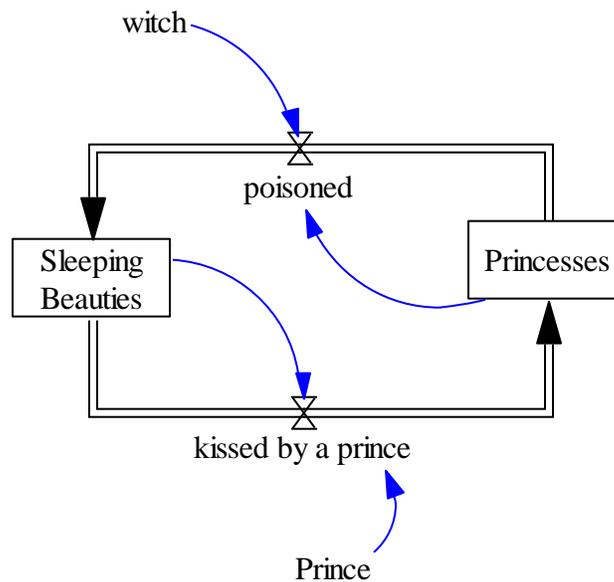
Sleeping Beauties = poisoned princess – kissed by prince;

Princesses = princesses + kissed by prince – poisoned princess;

Poisoned princess = princesses \* poisoned rate, and

Kissed by prince = sleeping beauties \* kissed rate.

The diagram is as following:



How can this fiction related to mathematics? If we replace princess by healthy people in a community, witch by virus, sleeping beauties by infected patients, and prince by medical treatment, it is a simple model of disease control. Certainly students can add more conditions to make the model be better representation of real situations.

## 5. Simple Population Model [2]

This model is a study of population growth based on a growth factor over time. The easiest way to visualize a population growth model is to assume that some proportion of the population

reproduces during each time step and some proportion dies. The difference between the two is the growth factor.

This model simulates a simplified model where the population increases is proportional to the

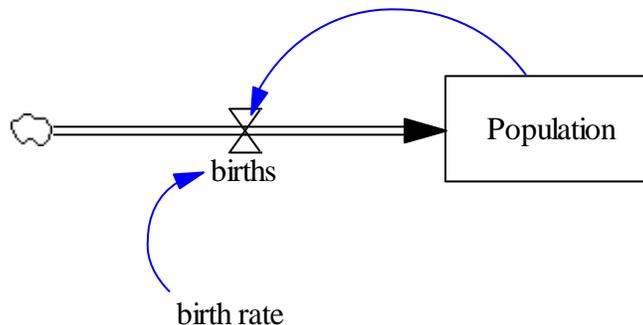
current population with proportional constant  $b$ . The mathematics model is  $\frac{dp}{dt} = b * p$ , where the

instantaneous rate of change of population is  $\frac{dp}{dt}$ . We can use average rate change to approximate

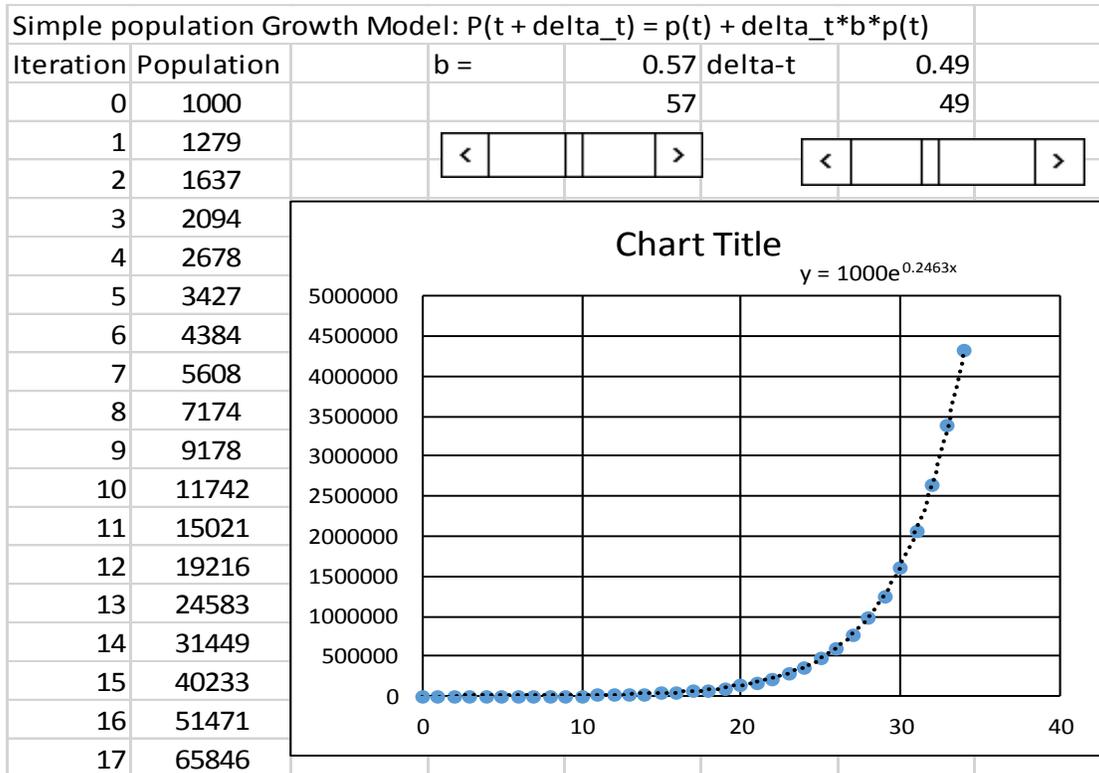
instantaneous rate of change. Therefore we have  $\frac{p(t + \text{delta}_t) - p(t)}{\text{delta}_t} = b * p(t)$ , or  $p(t +$

$\text{delta}_t) = p(t) + \text{delta}_t * b * p(t) \dots (a)$ .

Now we can apply Excel to set up a model for equation a. Let assume initially the population is 1000, or  $p(0) = 1000$ , and proportional constant  $b$  is 0.02. The following Excel worksheet demonstrate this model.



The Excel Solution is as following:



## 6. Lotka-Volterra Predator-Prey model [3]

Suppose foxes and grass eating rabbits interact within the same environment or ecosystem and suppose further that the rabbits eat only grass and the foxes eat rabbits. Let  $x(t)$  and  $y(t)$  denote the fox and rabbit populations, respectively, at time  $t$ . If there is no rabbits, then one might expect that foxes would decline in number according to  $\frac{dx}{dt} = -ax$ ,  $a > 0$ . When rabbits are present in the environment, however, it seems reasonable that the number of interaction between foxes and rabbits per unit times is jointly proportional to their population  $x$  and  $y$ . Adding his condition gives a model of the fox population:  $\frac{dx}{dt} = -ax + bxy$ ,  $b > 0$ . On the other hand, if there no foxes, then the rabbits would, with an addition assumption unlimited grass, grow at the rate that is proportion to the number of rabbits present at time  $t$ :  $\frac{dy}{dt} = dy$ ,  $d > 0$ . But when foxes are present, a model for the rabbit population is decreased by  $cxy$ ,  $c > 0$ . Therefore the rabbit

population at time  $t$  will be  $\frac{dy}{dt} = dy - cxy$ .

Suppose  $\frac{dx}{dt} = -0.16x + 0.08xy$  and  $\frac{dy}{dt} = 4.5y - 0.9xy$ , with initial population  $x(0) = 4$  and  $y(0) = 4$ .

4. This is not a linear model. Certainly we can use numerical method to solve this system. For freshman level students, we can use excel and algebraic approximation to simulate this model.

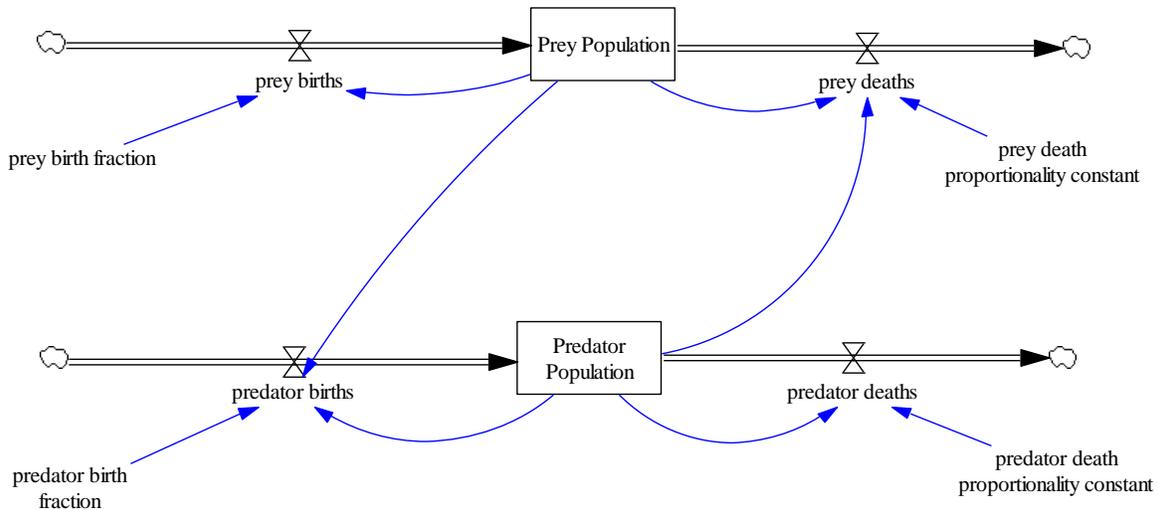
Since  $\frac{dx}{dt} = \lim_{\Delta t \rightarrow 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$ , we can approximate  $\frac{dx}{dt}$  by  $\frac{x(t+\Delta t) - x(t)}{\Delta t}$ . Likewise we

approximate  $\frac{dy}{dt}$  by  $\frac{y(t+\Delta t) - y(t)}{\Delta t}$ . The differential equation model becomes algebraic model for

college freshman. The algebraic model for the example above becomes

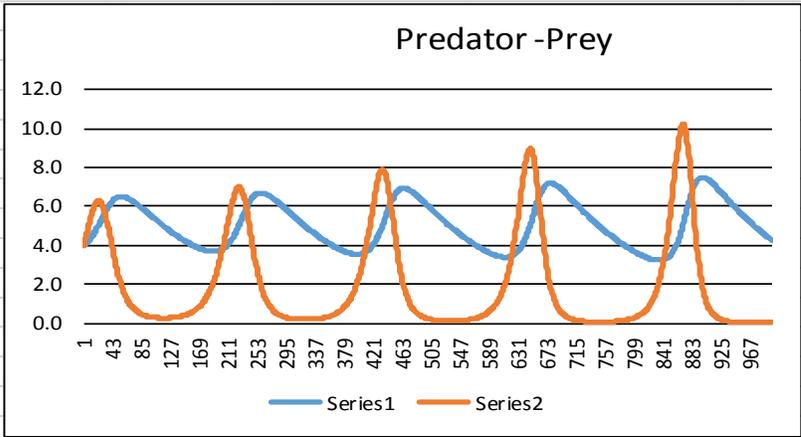
(a)  $\frac{x(t+\Delta t) - x(t)}{\Delta t} = -0.16x + 0.08xy$ , or  $x(t+\Delta t) = x(t) + \Delta t(-0.16x + 0.08xy)$ .

(b)  $\frac{y(t+\Delta t) - y(t)}{\Delta t} = 4.5y - 0.9xy$  or  $y(t+\Delta t) = y(t) + \Delta t(4.5y - 0.9xy)$ .



With help of Excel, we have the following solution:

t	x(t+delta t)	y(t + delta t)	delta t	x(0) =	y(0) =	Iteration n=	25
0	4.0	4.0	0.04	$y(t+\text{delta}t) = y(t) + \text{delta}t * (4.5y(t) - 0.9*x(t)*y(t))$			
1	4.0	4.1	0.04	$x(t+\text{delta}t) = x(t) + \text{delta}t * (-0.16x(t) + 0.08*x(t)*y(t))$			
2	4.1	4.3					
3	4.1	4.4					
4	4.1	4.6					
5	4.1	4.7					
6	4.2	4.9					
7	4.2	5.0					
8	4.3	5.2					
9	4.3	5.3					
10	4.4	5.4					
11	4.4	5.6					
12	4.5	5.7					
13	4.5	5.8					
14	4.6	5.9					
15	4.6	6.0					
16	4.7	6.1					
17	4.7	6.1	0.04				



## 7. Simple SIR Model [5]

The Susceptible, Infected, Recovered model has been used since 1927 to provide estimates of how a disease will effect a population. The SIR model begins with a base population composed of susceptible and infected persons. Rates of change for the susceptible, infected and recovered persons are used to provide estimated numbers of each group at a specific point in time. The SIR model was developed by separate sources and is well supported.

### Assumptions and Limitations:

The SIR model assumes a disease runs its course quickly enough that births and deaths (from other causes) will not affect the population and that the disease itself will not kill the population. The SIR model also assumes recovered persons will not infect others, or that a disease will not mutate and infect the same person multiple times.

### Variables

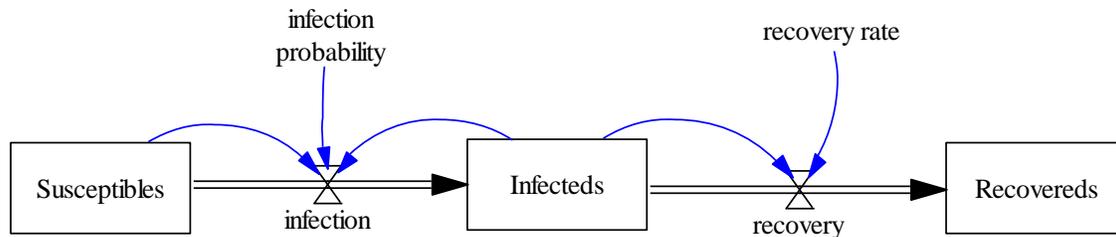
There are three major components of the SIR model which together, represent the population. Initially, there are susceptible and infected person. As the disease runs its course, the susceptible population decreases and the infected population increases. Eventually, recovered individuals begin to emerge as they do, the infected population decreases. The only variables in the basic SIR model are:

P, the total population

S, the number of susceptible individuals

I, the number of infected

R, the number of recovered



It is known that at any point in time, the population is the combination of the susceptible, infected and recovered populations. Therefore:  $P = S + I + R$

In order to establish a differential equation set, a derivative is taken of the population equation.

$$0 = S' + I' + R'$$

The susceptible population changes at a rate dependent on the interaction of the susceptible and infected populations. This rate of susceptible change is modeled by the product of the susceptible, the infected and a coefficient representing a rate of contact between infected and susceptible individuals and the probability that the disease will be transmitted.

$$S' = -a \cdot S \cdot I \quad (a)$$

The infected population increases at the same rate the susceptible population decreases. The infected population also decreases as individuals recover. This recovery is dependent on the number of infected and a coefficient determined by the average duration of the infection.

$$I' = a \cdot S \cdot I - b \cdot I \quad (b)$$

Individuals enter the recovered population at the same rate they leave the infected population.

$$R' = b \cdot I \quad (c)$$

(a), (b) and (c) form a system of linear differential equations. It is very difficult to solve for

general solutions of R, I and S. However, we can use the procedures we did before and get numerical approximations of S, I and R.

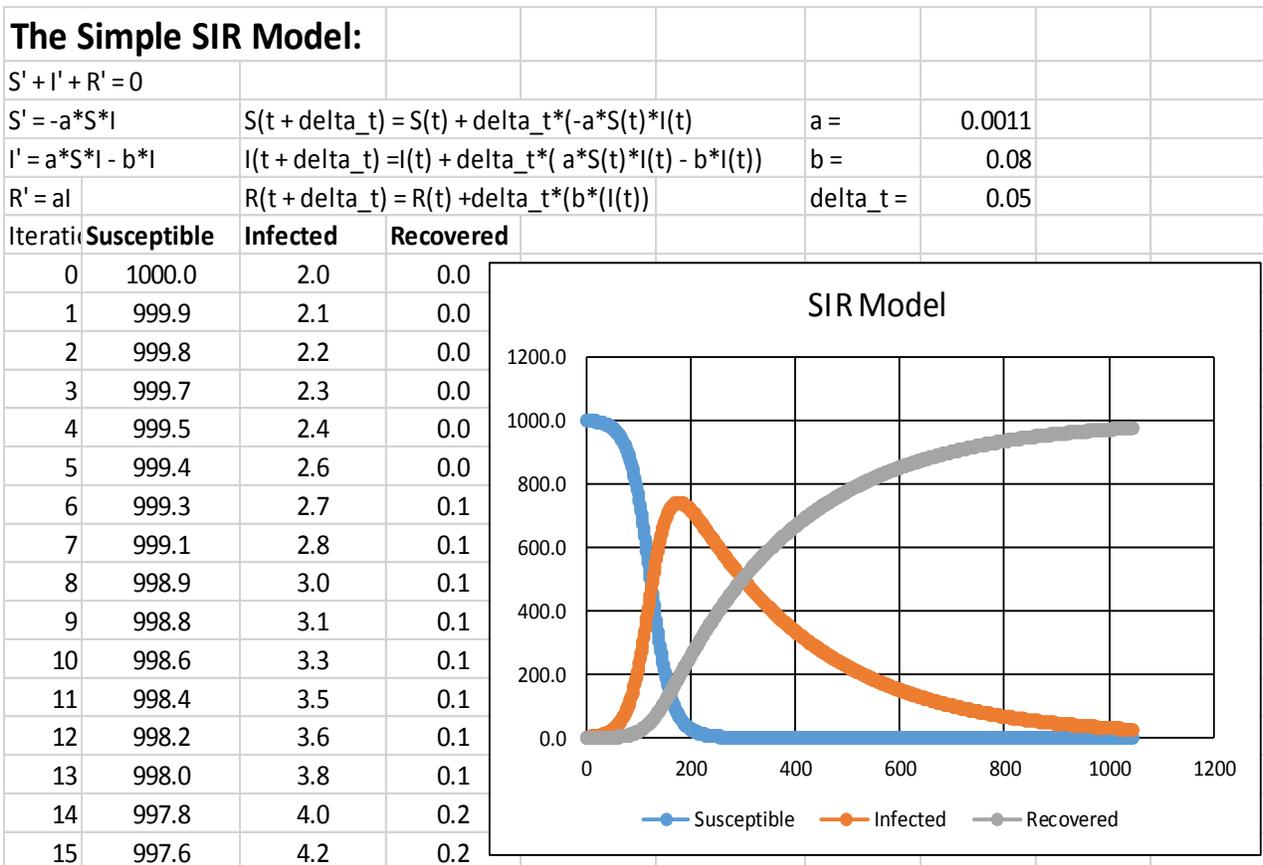
Using average rate of change to approximate derivatives we have the system of equations:

$$\frac{S(t + \Delta t) - S(t)}{\Delta t} = -a * S(t) * I(t)$$

$$\frac{I(t + \Delta t) - I(t)}{\Delta t} = a * S(t) * I(t) - b * I(t)$$

$$\frac{R(t + \Delta t) - R(t)}{\Delta t} = b * I(t)$$

Using Excel we can get the solution as following that is similar to the solution from MATLAB using numerical method to solve this system of differential equations.



Here we did not consider the death rate for all S, I and R. Upper classes students can add conditions such as death from disease, or intervention through quarantine, vaccination and treatment to have better representation of models.

## **8. Conclusion**

Classroom mathematics or procedural mathematics can improve test scores and make politicians, administrators and parents happy. Certainly some of highly motivated students can apply them to professional levels. But average students are not ready to apply them to real life. Most mathematics instructors have some common questions from students or parents such as why do I need to learn Algebra, or Calculus? I will never use algebra or calculus after the classes. Numeracy is the only mathematics that I need to learn. Making connection between classroom mathematics and real-world issues can answer all these questions. Also most mathematics instructors have pressure to pass certain amount of students regardless their achievement. Retention rate, graduation rate or passing rate of standardized tests, will affect teacher's annual evaluation. In addition to procedural mathematics, I assign research projects to every level students. I also use current issues from newspapers, magazines, or television broadcasts and transfer those issues to mathematics models and asked them to improve my sample models. In such a way I can encourage students design their models based on their background. They know there is no correct model. As long as their model is logical, it is acceptable.

That I integrated undergraduate research into curriculum is to promote critical thinking, unzip student's potential, and make connection between classroom mathematics and community issues. Hopefully later students can say "If you give me a story, I will give you a mathematical interpretation with dynamic modeling."

## **References**

- [1] Jo Boaler, "The Math-Class Paradox", The Atlantic, Feb. 4, 2016
- [2] Bob Pannof, Workshop Notes at West Virginia State University, 1-3 August 2016
- [3] Zill, Wright, "Differential Equations with Boundary-valued Problems" 8<sup>th</sup> Edition, 2013 pp 108- 109.

- [4] Jack Andenoro, “The Spread of Infectious Disease”, <http://home2.fvcc.edu/~dhicketh/DiffEqns/Spring2012Projects/M274FinalProjectJackAndenoro/RuftDraft.pdf>, May 11, 2012