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Proposal of an Experiment to Investigate Properties of the Neutron Wave-Packet

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Abstract

The concept of the coherence length of the neutron is explored. The generally accepted definition of a Gaussian wave packet based on the method of the beam preparation, and the singular de Broglie's wave packet are considered. Possible ways of measuring the coherence length are discussed.

Key words: wave packet, coherence length, polarization, superposition, interference

1.Introduction

Examination of wave packets is, apparently, one of the most important fundamental problems of physics today. It is evident that a particle wave function is not a plane wave. It should be a vector of the Hilbert space i.e. a wave packet having certain properties. It has some size,called the coherent length, and the size can change with energy. Here the neutron wave properties are discussed, because the neutron seems to be the simplest massive particle. In [1] it was reasonably noted that an unstable particle with a lifetime τ can have the size of the order of $L = \sqrt{\hbar\tau/m}$, which for the neutron is 0.75 cm. However, the finite lifetime can also lead to the definition of the packet size proportional to the particle velocity $L = v\tau$. So the greater the speed, the larger is the wave packet, though from a physical point of view, it seems more reasonable to accept [2] that the faster is the particle, the more close it to a point one. In [3] a wave packet was introduced to explain the UCN anomaly (abnormally high losses in traps). The packet size was estimated to be $L \approx 10^5\, \lambda_{\mathrm{Be}}/2\pi$, where λ_{Be} is the minimal wavelength of the neutron, which can be stored in beryllium traps. In this case L is the order of several millimeters, which does not contradict the assumption made in the [1].

Later [4,5] the assumption was made that the size of the wave packet depends on the speed and is proportional to the neutron wavelength: $L \approx 10^5\, \lambda/2\pi$. Therefore, the thermal neutron wave packet size is up to about 5 microns. Perhaps it is correct, but how experimentally to measure the size of the wave packet that is the question.

One way is to look for neutron transmission through a film when incident at a subcritical glancing angle [4, 6, 7]. Some indication of the transmission was found, but statistics of the experiment was not sufficient to talk about this transmission with certainty. More precise experiments are needed. Here another type of the experiments is discussed. When, because of some coherent process, the neutron wave function of a polarized neutron is split into two diverging in the space oppositely polarized components, one can observe superposition of polarization and find the distance, at which superposition is terminated, i.e. two components diverge and the neutron becomes only in one of the components. To predict theoretically transition of a coherent superposition into incoherent one, it is necessary to accept a model. Gaussian packets are not suitable because they spread out in the space. Gaussian packets are the result of the beam preparation. Their width Δk in the momentum space characterizes the coherent length $lc = 1/\Delta k$ in the coordinate space. This length is well observable in experiments on interference measurements of distances [8-10]. Here will be accepted a more appropriate model: a nonspreading singular de Broglie's wave packet [11]

$$\psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k},t) = \sqrt{\frac{q}{2\pi}}\exp\left(i\mathbf{k}\mathbf{r} - i\omega t\right)\frac{\exp\left(-q\left|\mathbf{r}-\mathbf{k}t\right|\right)}{\left|\mathbf{r}-\mathbf{k}t\right|}, \qquad (1)$$

with the spatial size $l = 1/q$. To get rid of various constants, the variable t in (1) includes factor $\hbar/m$, so that t has dimension cm^{-2}, and the angular frequency is $\omega = (k^2 - q^2)/2$. The packet (1) is normalized to unity by integration over the volume:

$$\int |\Psi_{\rm dB}|^2 d^3 r = 1,$$

or as a full flux through any plane, integrated over time. Fourier expansion of this packet has the form

$$\psi_{\rm dB}(\mathbf{r},\mathbf{k},t) = \sqrt{\frac{q}{2\pi}} \exp(i\mathbf{kr} - i\omega t) \frac{4\pi}{(2\pi)^3} \int d^3 p \frac{\exp(i\mathbf{p}(\mathbf{r}-\mathbf{k}t))}{p^2+q^2}, \qquad (2)$$

and the packet satisfies the equation

$$\left(\frac{i\partial}{\partial t} + \frac{\Delta}{2}\right)\psi_{\rm dB}(\mathbf{r},\mathbf{k},t) = -q\sqrt{2\pi}\exp\left(-i\left(k^2+q^2\right)t/2\right)\delta(\mathbf{r}-\mathbf{k}t). \qquad (3)$$

In the next three sections some possible experiments with the de Broglie wave packet are discussed. Detailed calculations are shifted to appendices.

2. Measurement of packet size after coherent splitting of the neutron wave into 2 parts with equal but not collinear speeds.

Consider a packet (1), which describes a particle with a fixed speed. Imagine a neutron flying at a speed k along x-axis being polarized along y-axis. At some point $x = 0$, as shown in Fig.1, the packet splits into 2 components polarized along and opposite z-axis and propagating at an angle θ to the x-axis. Spinor wave function of the neutron is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\Psi_{\rm dB}(\mathbf{r},\mathbf{k}_+,t)|z\rangle + \Psi_{\rm dB}(\mathbf{r},\mathbf{k}_-,t)|-z\rangle\right), \qquad (4)$$

where $\mathbf{k}_\pm = (k_x, 0, \pm k_z)$, $k_x = k\cos\theta$, $k_z = k\sin\theta \approx k\theta$.

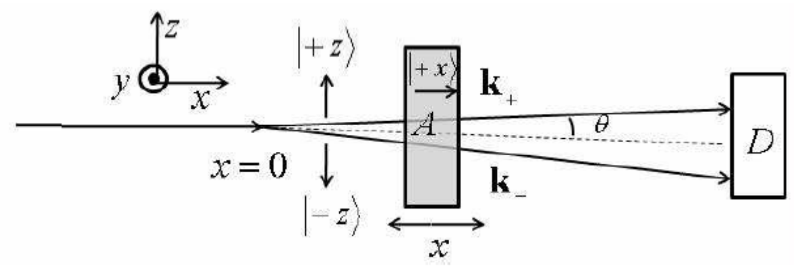

Fig. 1: *The neutron because of a coherent process is split into two oppositely polarized components symmetrically propagating at a small angle θ with respect to the original direction. Superposition of split components passes through an analyzer, which transmits only neutrons polarized along the x-axis. The intensity recorded by the detector after analyzer when shifted along the x-axis, should contain an oscillating component of the type shown in Fig.2.*

Let's put on the way of the split neutron an analyzer transmitting only neutrons, polarized along the *x*-axis (it can be also along the *y*-axis, which is not essential). Since

$$|\pm z\rangle = \left(|x\rangle + |-x\rangle\right)/\sqrt{2}\,,$$

the wavefunction, transmitted by the analyzer is

$$\psi(\mathbf{r},t) = \langle x \| \Psi \rangle = \frac{1}{2}\left(\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{+},t) + \Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{-},t)\right). \qquad (5)$$

Therefore, the total flux through the plane *y*, *z*, located at the point *x* after the analyzer, is

$$I(x) = \frac{1}{4}\left(I_{+} + I_{-} + I_{\pm}\right), \qquad (6)$$

where

$$I_{+,-} = k_x \int dydzdt \left|\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{\pm},t)\right|^2, \qquad (7)$$

$$I_{\pm} = 2k_x \int dydzdt \cos(2k_z z) \left|\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{+},t)\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{-},t)\right|. \qquad (8)$$

The integrals (7), as is easy to show, are equal to unity because of normalization. Of interest is the interference flux $I_{\pm}$. Detailed calculations are shown in the Appendix A. The result is represented by the function

$$f(X) = 2\eta \int_0^1 ds \cos(sX) \frac{\exp\left(-X\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}, \qquad (9)$$

which depends on two dimensionless parameters $X = 2kx\theta^2$ and $\eta = q/k\theta$.

Note that the q can currently be estimated [3] as $10^{-5}k$. If $\theta \approx 10^{-5}$ then $\eta \approx 1$, and the attenuation length is approximately equal to the period of oscillations, so the oscillations after analyzer in fact will not be seen. To observe them, it is desirable to have $\theta \approx 10^{-4}$. Then $\eta \approx 0.1$. With this setting of η the function (9) looks as shown in Fig. 2.

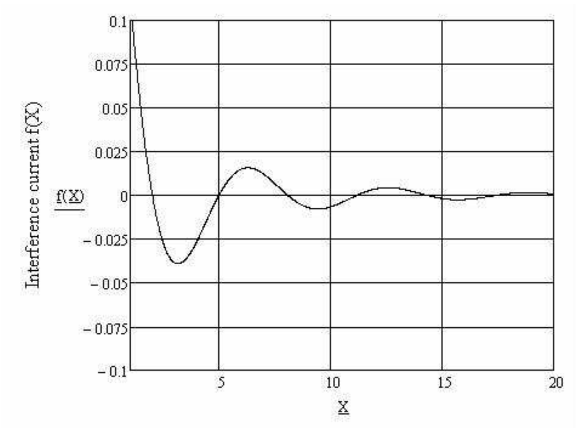

Fig. 2: *The result of a numerical calculation of the functions (9) for η = 0.1.*

The variable X corresponds to displacement x in space equal to

$$x = X\lambda / 4\pi\theta^2 , \qquad (10)$$

where λ is the neutron wavelength. For an experiment the displacement x should be large enough. If the parameter $X = 1$ should correspond to the actual displacement x=1 cm, then for θ=10^{-4} the neutron wavelength should be λ=10Å, which corresponds to the energy of 1 meV. Let's estimate how to get splitting at the angle θ=10^{-4}.

Imagine transmission of a beam through a polarized magnetized prism, as shown in Fig. 3. The neutron beam polarized along the x-axis falls from the left perpendicularly to the vertical face of the prism magnetized along z-axis. Near the exit at the oblique face within the prism the wave vectors of the two components polarized along the z-axis are equal

$$k_{in,1,2} = (\mathbf{n}\cos\phi + \mathbf{t}\sin\phi)\sqrt{k^2 - u_{1,2}} ,$$

where ***n*** and ***t*** are the unit vectors along the normal and along generatrix of the oblique edge, respectively, and $u_{1;2}$ are interaction potentials of the two spin components with prism material and its magnetic induction. After exiting the prism into the empty space without magnetic field the wave vectors become

$$k_{out1,2} = \mathbf{n}\sqrt{(k^2 - u_{1,2})\cos^2\phi + u_{1,2}} + \mathbf{t}\sin\phi\sqrt{k^2 - u_{1,2}} .$$

The square of the difference of two vectors, divided by the square of the sum is equal to θ^2:

$$\theta^2 = \frac{\left(\sqrt{k^2 + u_1\tan^2\phi} - \sqrt{k^2 + u_2\tan^2\phi}\right)^2 + \left(\sqrt{k^2 - u_1} - \sqrt{k^2 - u_2}\right)^2 \tan^2\phi}{\left(\sqrt{k^2 + u_1\tan^2\phi} + \sqrt{k^2 + u_2\tan^2\phi}\right)^2 + \left(\sqrt{k^2 - u_1} + \sqrt{k^2 - u_2}\right)^2 \tan^2\phi}, \qquad (11)$$

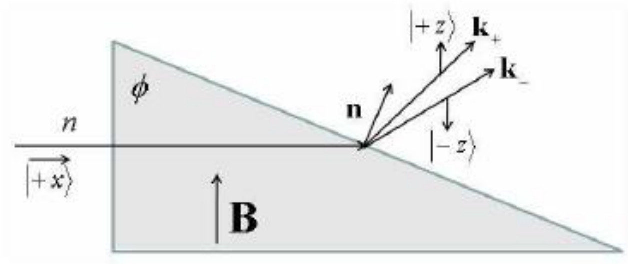

Fig. 3: *Getting slightly split beam of neutrons by transmission through a magnetized prism.*

which for small *u* is reduced to

$$\theta = \frac{\Delta u}{4k^2}\tan\phi\,, \qquad (12)$$

where $\Delta u = u_1 - u_2$. At k^2 corresponding to the energy1 meV the value $\theta = 10^{-4}$ is obtained, if $\Delta u = 10^{-7}$ eV, which corresponds to magnetization 2T, and to tanφ=4, i.e. $\varphi = 75^0$.

The result (9) is obtained for a fixed packet speed $k \approx k_x$. Let us now imagine that in fact the packet speed has a Gaussian distribution

$$w(k) = \frac{1}{\sqrt{\pi}\Delta}\exp\left(-\frac{(k-1)^2}{\Delta^2}\right), \qquad (13)$$

where all the parameters are defined in terms of the average speed k_0. Integration of (9) over this distribution gives

$$F(X) = \int_{-\infty}^{\infty}\frac{2k\eta dk}{\sqrt{\pi}\Delta}\exp\left(-\frac{(k-1)^2}{\Delta^2}\right)\int_0^1 ds\cos(sXk)\frac{\exp\left(-Xk\sqrt{1-s^2+\eta^2}\right)}{\sqrt{1-s^2+\eta^2}}. \qquad (14)$$

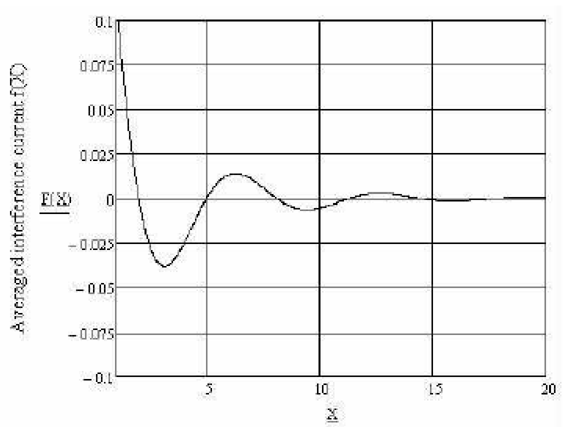

Fig. 4: *The result of the averaging of the function Fig.2, in accordance with (14) for Δ = 0,5.*

The result for the function, shown in Fig.2, is presented in Fig. 4 for Δ = 0,5.

3. Scheme of the packet size measurement by splitting of the neutron into 2 components with different but parallel speeds.

Suppose now that $\mathbf{k}_{\pm} = (k \pm \delta, 0, 0)$, where $\delta/k << 1$. Then (8) takes the form

$$I_{\pm} = 2k\int dydzdt \cos(2\delta x - 2k\delta t)\left|\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{+},t)\Psi_{\mathrm{dB}}(\mathbf{r},\mathbf{k}_{-},t)\right|. \qquad (15)$$

The calculation of this integral, as shown in Appendix B, again leads to the function (9), in which the dimensionless parameters are $X = kx\zeta^2$, and $\eta = q/k\zeta$, where $\zeta = \delta/k$ plays the same role as the parameter θ in (9). The experimental scheme is shown in Fig. 5. If neutrons of energy 10^{-4} eV pass through RF spin flipper with frequency 10 MHz, the velocities of the neutron components polarized up and down become different by the amount

$\delta = k\hbar\omega/k^2 = 10^{-4}/k$

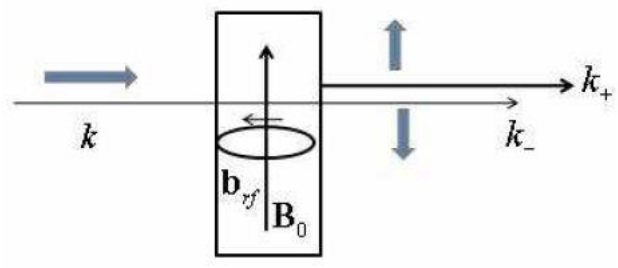

Fig. 5: *Getting a slightly longitudinally split beam of neutrons via RF spin-flipper [11].*

The parameter $\eta \approx 0.1$, and the function (9) has the same form as shown in Fig. 2. Let's see what will be the oscillation period. Since it is determined by the same formula as (10) only with the replacement of $\theta \rightarrow \zeta$, and the energy of 10^{-4}eV corresponds to $\lambda \approx 30$Å, the parameter X = 1 will correspond to $x \approx 3$ cm. Thus, the whole picture, as shown in Fig. 2 can be seen by moving the analyzer to 60 cm.

4. Scheme of the experiment for measuring the packet size by splitting of the neutron into 2 parts with different and non-parallel speeds.

Consider now the case, when the neutron is split into 2 components propagating with different velocities at an angle to each other. Such a beam, for example, can be obtained by reflection of a neutron beam, polarized along the external magnetic field, from the magnetic mirror with magnetization noncollinear to the external field. At the reflection the reflected beam is split into 2 components. One, reflected in the specular direction, retains polarization and the velocity of the incident beam and the other one has the opposite polarization and is reflected in the non-specular direction [12] under larger grazing angle. Choose the x-axis in such a way that the components of the wave vectors are $\mathbf{k}_{1,2} = (k \pm \delta, 0, \pm k_z)$, then the integral (8) becomes

$$I_{\pm} = 2k_x \int dydzdt \cos(2\delta x + 2k_z z - 2k\delta t) \left| \Psi_{\mathrm{dB}}(\mathbf{r}, \mathbf{k}_1, t) \Psi_{\mathrm{dB}}(\mathbf{r}, \mathbf{k}_2, t) \right| . \qquad (16)$$

Obviously, the calculations result in the same function (9), but instead of the parameter θ there will be

$$\xi = \sqrt{\theta^2 + \varsigma^2} = \frac{|\mathbf{k}_+ - \mathbf{k}_-|}{|\mathbf{k}_+ + \mathbf{k}_-|} \approx \frac{\Delta k}{k_0} . \qquad (17)$$

Let's estimate the value of these parameters in the experiment [12]. Since the experiment was performed with thermal neutrons, and external field varied from 33 Oe to 4000 Oe, which corresponds to the magnetic energy from 10^{-10} and 10^{-8}eV, the parameter ξ changed in within 10^{-6}–10^{-8}, and the parameter η in (9) is much greater than unity, then (9) can be approximated as

$$f(X) \approx \int_0^1 ds \cos(sX) \exp\left(-X\eta\right) \approx \exp\left(-2qx\xi\right) = \exp\left(-2 \cdot 10^{-5} kx\xi\right). \qquad (18)$$

Therefore the coherence length, respectively, in this case,

$$L_c = 10^5 \, \lambda / 4\pi\xi \, , \qquad (19)$$

varies in the range 1 --100 m, i.e. in this case practically one deals with a plane wave.

Conclusion

This article shows a possibility of a direct measurement of the size of neutron wave packet, described by the singular de Broglie's wave function. No fundamental difficulties for corresponding experiments are expected.

Appendix

A. Calculation of the integral (8).

We will show here in detail how to calculate the integral (8). We use the Fourier representation (2), write 2cos(2*kzz*) as the sum of two exponents and obtain the integral

$$I_{\pm} = \frac{q(4\pi)^2}{(2\pi)^7}\int \frac{k_x d^3 p d^3 p' dydzdt}{(p^2+q^2)(p'^2+q^2)} \exp(i\mathbf{p}(\mathbf{r}-\mathbf{k}_+ t) + i\mathbf{p}'(\mathbf{r}-\mathbf{k}_- t) \pm 2ik_z z). \qquad \text{(A1)}$$

Integrating over the plane (*y*,*z*) and over time, and then over ***p'***, gives

$$J = \frac{q(4\pi)^2}{(2\pi)^4}\int \frac{d^3 p \exp(-2i(p_z \pm k_z)xk_z/k_x)}{(p^2+q^2)[p_y^2+(p_z\pm 2k_z)^2+(p_x+2(p_z\pm k_z)k_z/k_x)^2+q^2]}. \qquad \text{(A2)}$$

Make the change of variables $p_z \pm k_z \to pz$, and introduce $\theta = kz/kx \approx kz/k$, then the integral is transformed into

$$J = \frac{q}{\pi^2}\int \frac{d^3 p \exp(-2ip_z x\theta)}{[p_y^2+(p_z\mp k_z)^2+p_x^2+q^2][p_y^2+(p_z\pm k_z)^2+(p_x+2p_z\theta)^2+q^2]}. \qquad \text{(A3)}$$

Let us make more change of the variable $p_x + p_z\theta \to p_x$. Then the integral is transformed into

$$J = \frac{q}{\pi^2}\int \frac{d^3 p \exp(-2ip_z x\theta)}{[p_y^2+(p_z\mp k_z)^2+(p_x-p_z\theta)^2+q^2][p_y^2+(p_z\pm k_z)^2+(p_x+p_z\theta)^2+q^2]}. \qquad \text{(A4)}$$

Use the transformation

$$\frac{1}{AB} = \int_0^1 \frac{d\alpha}{(\alpha A+(1-\alpha)B)^2}. \qquad \text{(A5)}$$

$$J = \frac{q}{\pi^2}\int_0^1 d\alpha \int \frac{d^3 p \exp(-2ip_z x\theta)}{[p_y^2+p_z^2(1+\theta^2)+p_x^2+2(p_x p_z\theta \pm k_z p_z)(1-2\alpha)+k_z^2+q^2]^2}. \qquad \text{(A6)}$$

Change variables $s = 2\alpha$-1, and $p_x - sp_z\theta \to p_x$. Then (A6) is

transformed into

$$J = \frac{q}{2\pi^2}\int_{-1}^1 ds \int \frac{d^3 p \exp(-2ip_z x\theta)}{[p_y^2+p_x^2+p_z^2(1+(1-s^2)\theta^2)\mp 2k_z p_z s+k_z^2+q^2]^2}. \qquad \text{(A7)}$$

Integration over $dp_x\, dp_y$ gives

$$J = \frac{q}{2\pi}\int_{-1}^{1} ds \int \frac{dp_z \exp(-2ip_z x\theta)}{[p_z^2(1+(1-s^2)\theta^2) \mp 2k_z p_z s + k_z^2 + q^2]} . \qquad \text{(A8)}$$

For small θ integral can be simplified

$$J = \frac{q}{2\pi}\int_{-1}^{1} ds \int \frac{dp_z \exp(-2ip_z x\theta)}{[p_z^2 \mp 2k_z p_z s + k_z^2 + q^2]} . \qquad \text{(A9)}$$

Change of variables $p_z \mp k_z s \to p_z$, leads to

$$J = \frac{q}{2\pi}\int_{-1}^{1} ds \int \frac{dp_z \exp(\mp 2isk_z x\theta)\exp(-2ip_z x\theta)}{[p_z^2 + k_z^2(1-s^2) + q^2]} . \qquad \text{(A10)}$$

One can now integrate over p_z. The result is

$$I_{\pm} = 2q\int_{-1}^{1} ds \cos(2skx\theta^2)\frac{\exp\left(-2x\theta\sqrt{q^2 + k_z^2(1-s^2)}\right)}{\sqrt{q^2 + k_z^2(1-s^2)}} .$$

(A11)

B. Calculation of the integral (15)

Write 2cos(α) as a sum of two exponents, then get the integral

$$I_{\pm} = \frac{q(4\pi)^2}{(2\pi)^7}\int \frac{k_x d^3p d^3p' dydzdt}{(p^2+q^2)(p'^2+q^2)} \exp(i\mathbf{p}(\mathbf{r}-\mathbf{k}_+ t) + i\mathbf{p}'(\mathbf{r}-\mathbf{k}_- t) \pm 2i\delta x \mp 2ik\delta t) . \qquad \text{(B1)}$$

Integration over time and coordinates leads to three δ-functions which makes it easy to integrate over d^3p', resulting in the expression

$$J = \frac{q}{\pi^2}\int \frac{d^3p \exp(\mp 2ix\delta^2/k)\exp(-2ix\delta p_x/k)}{(p^2+q^2)[p_y^2 + p_z^2 + (p_x \pm 2\delta)^2 + q^2]} . \qquad \text{(B2)}$$

Change of variables $p_x \pm \delta \to p_x$ leads to

$$J = \frac{q}{\pi^2}\int \frac{d^3p \exp(-2ix\delta p_x/k)}{(p_y^2 + p_z^2 + (p_x \mp \delta)^2 + q^2)[p_y^2 + p_z^2 + (p_x \pm \delta)^2 + q^2]} . \qquad \text{(B3)}$$

Transformation (A5) and change of variables 2α-1=s leads to

$$J = \frac{q}{2\pi^2}\int_{-1}^{1} ds \int \frac{d^3p \exp(-2ix\delta p_x/k)}{(p_y^2 + p_x^2 + p_z^2 \mp 2s\delta p_x + q^2)^2} . \qquad \text{(B4)}$$

Change of variable $p_x \mp s\delta \to p_x$ and integration over *dpydpz* gives

$$J = \frac{q}{2\pi}\int_{-1}^{1} ds \int \frac{dp_x \exp(-2i\, sx\delta^2/k)\exp(-2i\, x\delta p_x/k)}{(p_x^2 + \delta^2(1-s^2) + q^2)}. \quad \text{(B5)}$$

After integration over dp_x we finally obtain the function (9), in which $\eta = q/k\zeta$, $\zeta = \delta/k$, and $X = 2xk\zeta^2$.

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