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Musical acoustics in Riemann ζ-function

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Abstract

The first eight terms in the Riemann hypothesis is analysed as harmonics with certain ratios to the ground level. The ratios were chosen from acoustics. To obtain the best fit, terms 5 and 7 was assumed to interact in a beat.

Key words: harmonics, beat, fractal, surr, f, cycle

Introduction

The transportation of sound in a room may occur in a fashion that invoke fractals. In a whispering gallery, the sound at one location appear also at another place, such that acoustics aligns with the geometry of the room. For tones, the ratios, eg octave 2, and quint 3/2, have a certain width, denoted surr in Strömberg (2016), and this is related to fine structure, which may be fractal. In appendix, a definition of surr and the cycle for f is derived and illustrated. The present paper concerns acoustics in the Riemann ζ-function. The first eight terms will be classified as certain harmonics and a beat. The composition into a beat will be discussed in terms of fractals, and the sublevel will give a change in amplitude of one mean frequency, which is close to the ters over the octave.

Model with harmonics

In the present context, we consider a decomposition of the first terms in the Riemann ζ-function into the ratios of acoustic, derived in Strömberg (2016).

Assuming ln(2) as the ground level, we will characterise the other terms and relate them to the acoustic ratios for f, c.f. Appendix. Then we will have a sum with signals invoking all f, except that below ground level, and also 5/2. The factor 5/2 is discussed in [Correia and Laskar], as a possible solution to the spin orbit ratio for Mercury. In musical acoustics it is close to the ters above the octave; 2*2^(4/12)=2.5198

The next term 3, will not be an exact quint, but the difference is almost within surr.

Surr in this case is assumed as a small number, to embrace the 3 values 2^(7/12) =1.498, 3/2, p/2=1.57, such that they equals.

Theorem. Assuming the exact ratio for the quint as $2^{7/12}$, the difference for the terms in the riemann z-function is ln(3)/ln(2)- $2^{7/12}$ =0.087.

Proof. The harmonics in the Riemann z-function are ln of the integers

Exercise. Calculate the difference, assuming that the ratio for the quint is p/2

Solution. ln(3)/ln(2)-p/2 =0.014 .

In conclusion: The quotient between the first and second term are ln(3)/ln(2)=1.58, which is a large quint.

The term 4 is an exact octave to the ground level since ln(4)= 2*ln(2)

The term 6 will give the ratio close to 5/2, as discussed in the introduction. It decomposes as; ln(6)=2.585*ln(2)=1.79

The term 8 will be exactly 3 times the ground level, which is the duodecima

Composition into a Beat

Next we will assume that 5 interacts with 7, in a beat.

Theorem 'Beat'. For the imaginary part of the ζ-function, the terms 5 and 7 will interact to give a beat, reading $(2/5^s)\cos(0.168b)\sin(1.7777b)$ and a term $(1/7^s-1/5^s)\sin(\ln(7)b)$, where s is the real part of argument in ζ(s+ib)

Proof. sin(ln(5)b)+sin(ln(7)b)= 2cos(d)sin(a) where a =(ln(7)+ln(5))(b/2) and d=(ln(7)-ln(5))(b/2)

Figure 1 shows the beat for such a composition, created by the script.

Since the frequency of the oscillation 1.7777b is large compared with the beat 0.168b, it will sound as a change in amplitude of a tone with 5/2 ratio to the ground level.

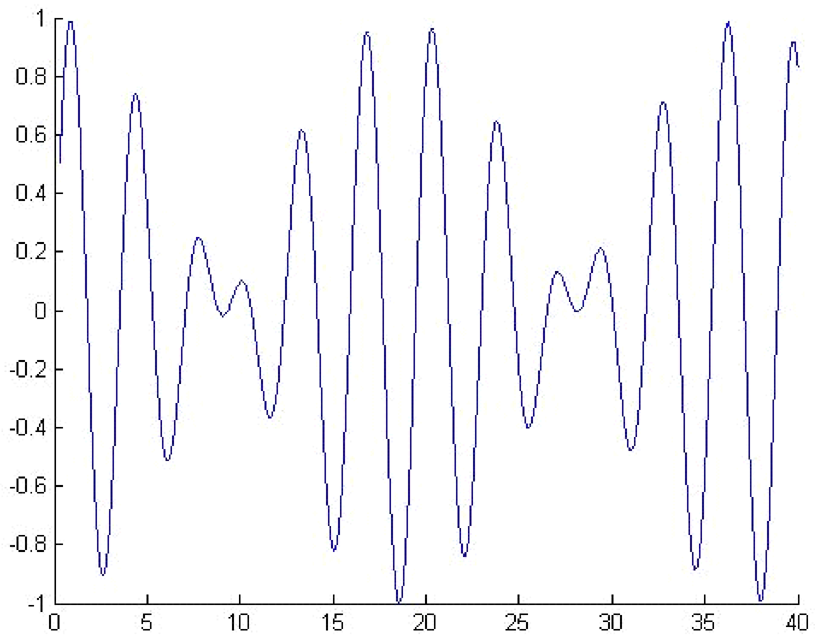


Figure 1. Beat with frequencies given in the Theorem

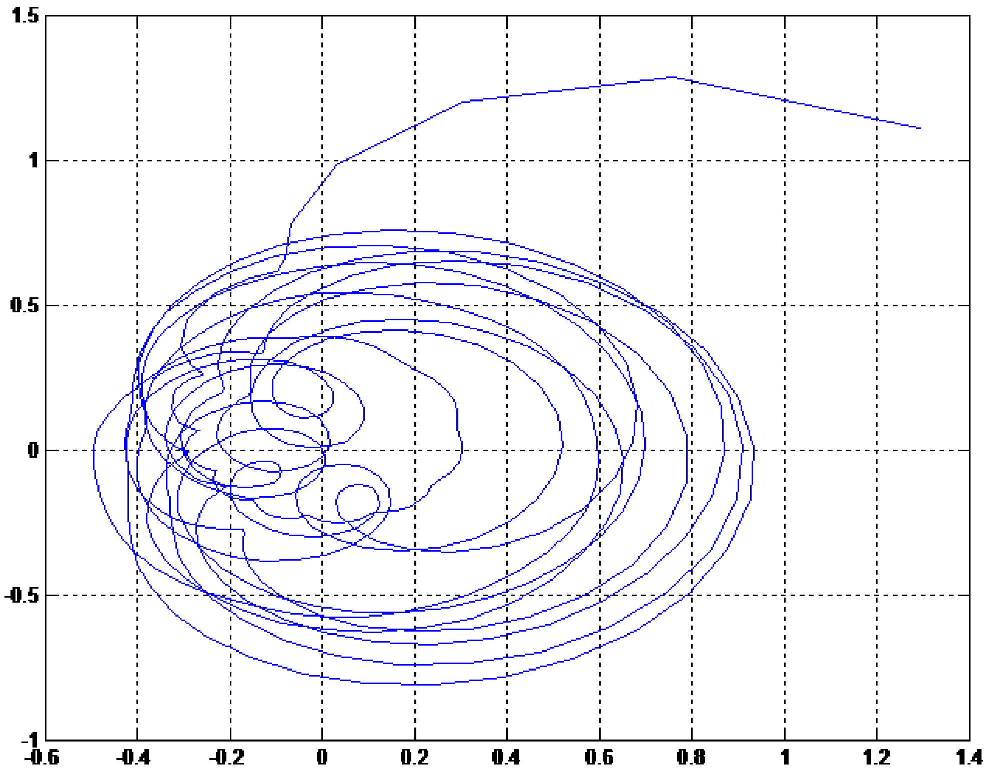
```
clf
hold
b=[0.3:0.01:40]
v=cos(0.1682*b).*sin(1.777*b)
plot(b,v)
```

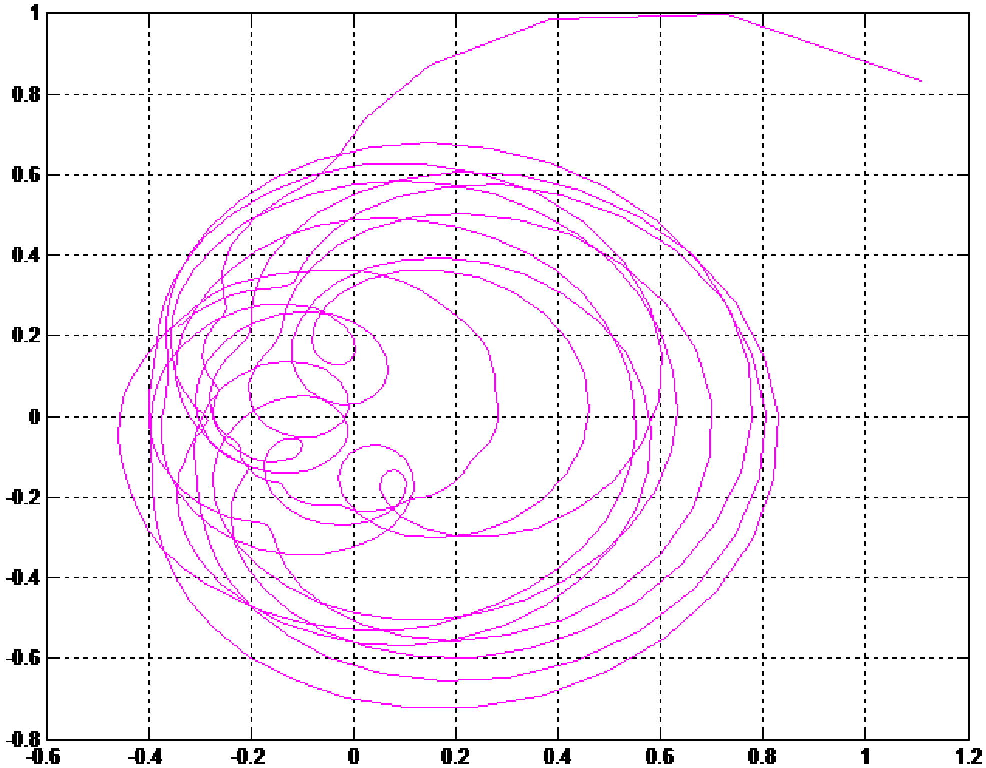
The remaining part, of ln(7) is not small since the amplitude is proportional to $-1/5^s$, but this can be assumed to interact with ln(8) to the duodecima, within surr, since ln(7) is quite close to ln(8). The difference 0.13 can be compared with a surr of p-3=0.14. That will give a negative sign for this term.

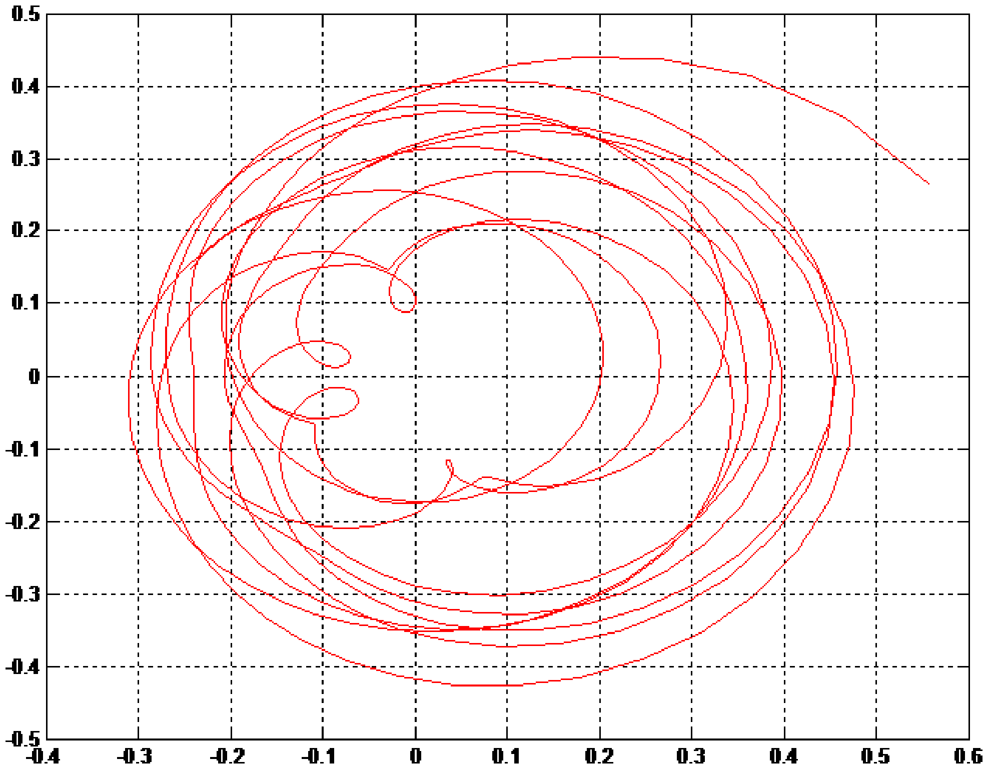
Terms larger than 8 are neglected, since the amplitude get small. (The terms that can be factorized, could be invoked in the signals within the domain for f, giving a very small alteration of the amplitudes.)

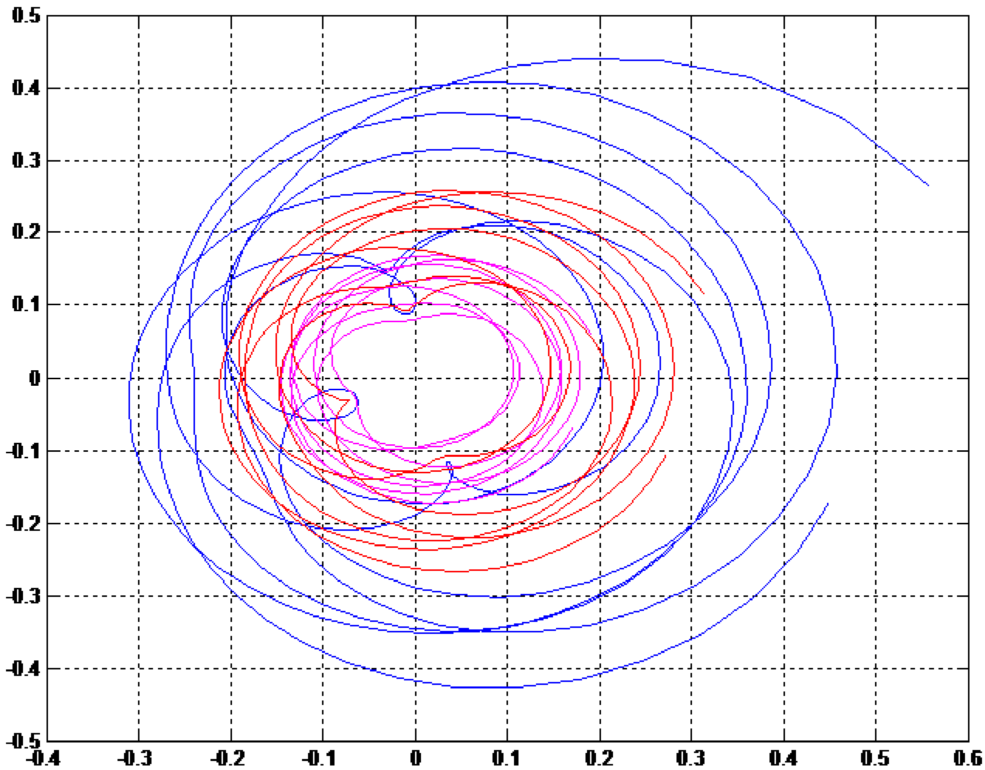
Illustration of the the Riemann ζ-function

Another more abstract illustration of frequency content of the Riemann ζ-function is given in Figure 2-5. The imaginary part is plotted versus the real part for 3 different values of s given by 1.4,1.5,2.




1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6




```
function lena8
  close all
  n=(1:500)';
  sv=[1.4,1.5,2];
  bv=pi*(0.1:0.05:30);
  xm=zeros(max(size(sv)),max(size(bv)));
  xi=zeros(max(size(sv)),max(size(bv)));
  for ls=1:max(size(sv))
    for lb=1:max(size(bv))
      t=exp(-log(n)*sv(ls)).*cos(-log(n)*bv(lb));
      tim=exp(-log(n)*sv(ls)).*sin(log(n)*bv(lb));
      csv=cumsum(t);
      stim=cumsum(tim);
      xm(ls,lb)=csv(500);
      xi(ls,lb)=stim(500);
    end
```

```
end
figure(1)
plot(xm(1,:)-1,xi(1,:),'b')
grid
figure (2)
plot(xm(2,:)-1,xi(2,:),'m')
grid
figure (3)
plot(xm(3,:)-1,xi(3,:),'r')
grid
```

Conclusion

The first terms in the Riemann hypothesis was analysed to be harmonics with certain ratios to the ground level. The ratios were chosen from the cycle for f in Strömberg(2016), and 5/2. To obtain this for the first eight values, 5 and 7 was assumed to interact in a beat. Hereby, the product will appear as one harmonic, (but with slowly varying amplitude) with the ratio 5/2 to the first ln(2).

The composition into a beat may be considered as a fractal, and then the substructure is heard as a change in amplitude of one mean frequency which originally was resolved as two.

Appendix. Surr, values for f, and a cycle

Here, we will discuss the factor f, and its discrete values. A Galois extension of discrete values for f is defined. This and functions on f will result in a cycle.

The factor 3/2 occurs when emanating from acoustics, memory with harmonic, and geometry as $2^{7/12}$, 3/2 and p/2 respectively. This will be used to assume a surrounding for f or angle, as a width.

Definition: a surrounding width abbreviated surr is defined as

surr=max(abs(p /2-3/2) , abs(p /2-$2^{7/12}$))

The surr will be used as a so-called Galois extension of discrete values of f to derive a cycle.

Theorem f-cycle: The set [1, 2, ln2, 3/2, 3] where values within surr, with the operations ('if larger than 2, then ln' and 'if smaller or equal to 2, then double') is a group and a cycle on the rationales.

Proof: See matlab code below in Exercise 1, but assume a surr, such that the values in the set are maintained and a fixpoint iteration does not converge.

Remark 1: The surr is tacitly assumed to be related with dispersion and dissipation in interaction with surroundings e.g. for a sound wave or water waves, or composed dynamical systems.

Remark 2: Theorem *f-cycle* may be denoted *popcycle*, because in order to be a cycle, there is need to 'pop'-up the values with surr.

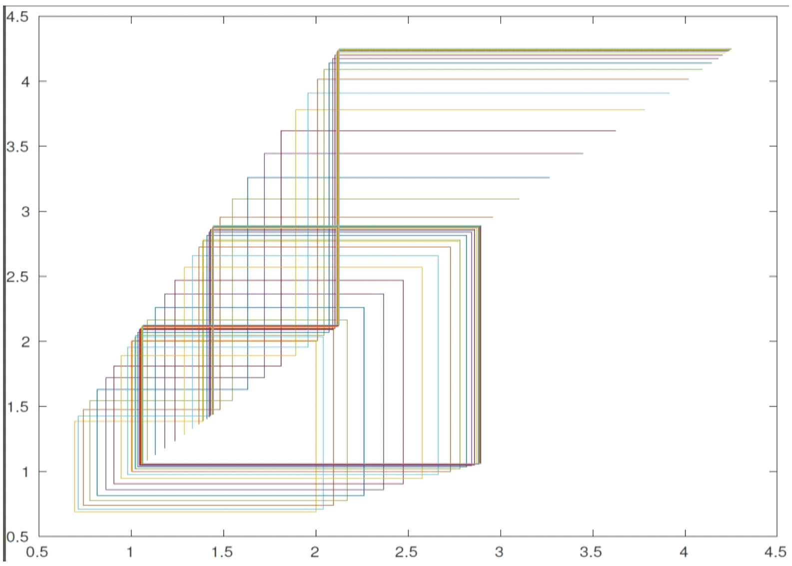
Exercise 1. Without the surrounding; determine the number of cycles (from start value 1), until the value reaches a fix point.

Solution with matlab code provides:

```
x(1)=1

for i=2:30

x(i)=log(2*2*log(2*x(i-1)));

end
```

x= 1.0000 1.0198 1.0476 1.0848 1.1308 1.1830 1.2369 1.2873 1.3305 1.36 47 1.3904 1.4088 1.4215 1.4302 1.4360 1.4398 1.4424 1.4440 1.4451 1.4458 1.4463 1.4466 1.4468 1.4469 1.4470 1.4470 1.4471 1.4471 1.4471 1.4471

The discrete values and the converegence to 1.4471 is illustrated in a map




```
figure(1)
x(1)=1;
for i=1:30
  plot(x(i),x(i) '-');
  hold on
  plot(x(i),2*x(i) '-' );
  plot(2*x(i),2*x(i) '-');
  plot(2*x(i),log(2*x(i)) '-');
  plot(log(2*x(i)),log(2*x(i)) '-');
  plot(log(2*x(i)), 2*log(2*x(i)) '-');
  plot(2*log(2*x(i)), 2*log(2*x(i)) '-');
  plot(2*log(2*x(i)), 2*2*log(2*x(i)) '-');
  plot(2*2*log(2*x(i)), 2*2*log(2*x(i)) '-');
  x(i+1)=log(2*2*log(2*x(i)));
end
```

References

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