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Change of neutron energy at reflection from and transmission trough a matter layer

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Abstract

The experimentally found small heating of the neutrons after transmission through thin foils can be explained by violation of energy conservation because of finite energy width of the neutron wave packet. The part of the wave packet subcritically transmitted through a foil has energy distribution higher than that of the incident one, and the part totally reflected has energy a little bit lower than the incident one. It is proven here with de Broglie's wave packet function. Increase and decrease of energy is analytically calculated. In average the energy is conserved.

Key words: de Broglie's wave packet, coherence length, small heating, law of energy conservation violation

1. Introduction

The laws of energy and angular momentum conservation are absolute in classical physics. However in quantum physics the laws of energy and angular momentum conservation can be violated in a single event and satisfied only on average.

Violation of the angular momentum and parity. Let's consider a nucleus in a scalar s-state, which decays into two scalar parts. After decay the state is not symmetric, because it has the axis, along which particles fly away. Of course, the projection of angular momentum on any direction is zero. Nevertheless the state of two particles flying away is not a symmetric one, therefore it is a superposition of states with many different orbital moments L. There are odd and even L in between them. Therefore at such a decay the parity $(-1)^L$ is also violated. Of course, in average, for many events everything is conserved, but it is not so in a single event. The above claim is supported in [1].

In a similar way there is no spin conservation at a decay. If a scalar particle decays into two spin ½ particles, two outgoing particles have opposite spin projection on a direction, which is arbitrary. The general belief of entangled state has no foundation. It can be directly checked [2], but the experimentalists prefer to prove entanglement by demonstration of the Bell's inequality violation. No result obtained till now can be considered satisfactory.

Violation of the energy conservation law.

When an atom is excited by absorption of a photon of energy E, it becomes in an excited state, and this state has a width Γ. Therefore, in deexcitation the atom emits a photon with energy E', which lies in the interval $E-\Gamma < E' < E+\Gamma$, i.e. the total energy in a single event is not conserved.

Neutron energy nonconservation at reflection from a foil.

The energy nonconservation is predicted only for particles described by wave packets. Let's consider a Gaussian wave packet

$$\psi_g(\mathbf{r},t) = C\int d^3p \exp\left(i\mathbf{pr} - i\,p^2t/2\right)\exp\left(-(\mathbf{p}-\mathbf{k})^2/2\Delta^2\right), \qquad (1)$$

which corresponds to a particle propagating with wave vector **k**. After transmission through a foil with critical wave number k_c the wave function (1) is transformed approximately to

$$\psi_g(\mathbf{r},t) = C\int d^3p\,\Theta(p_n > k_c)\exp\left(i\mathbf{pr} - i\,p^2t/2\right)\exp\left(-(\mathbf{p}-\mathbf{k})^2/2\Delta^2\right), \qquad (2)$$

where $\Theta(x)$ is the step function equal to unity, when inequality in its argument is satisfied, and to zero otherwise, p_n is the component of the vector **p** normal to the foil surface. The transmitted wave packet is not gaussian, but can be represented as superposition of gaussians. However it is clearly seen, that the total energy of transmitted particle is higher than that of the incident one. The similar considerations show that the total energy of reflected particle is lower than that of

the incident one. The total energy of transmitted and reflected particles is the same as that of the incident one. Nevertheless at every single event, when we have only a single particle, reflected, or transmitted, we find energy nonconservation.

Of course, if the gaussian is not related to a single particle, but describes a distribution of the neutrons in a beam, then reflection from a foil only separates higher energy transmitted neutrons from the lower energy reflected ones, but we are interested here by a wave packet of a single particle. A gaussian cannot be accepted for description of a single particle, because it spreads in time, whereas the form of a real free particle should not change. The best candidate for the wave packet of a particle is the de Broglie's singular function, which will be used here, and it is necessary to investigate it's properties.

Examination of the wave packet properties of elementary particles seems to be one of the most important fundamental problems of physics today. It is clear that the wave function of the neutron is not a plane wave. It should be a vector of the Hilbert space i.e. a wave packet having certain properties. It has some size in space, called the coherent length, and some width in energy distribution, and the size can change with energy. In [3] it was reasonably noted that unstable particle with a lifetime τ can have the size $L=\sqrt{\hbar\tau/m}$, which for the neutron is 0.75 cm. However, the finite lifetime can lead also to the definition of the packet size proportional to the neutron velocity $L = v\,\tau$. So the greater the speed, the larger is the wave packet, although from a physical point of view, it seems more reasonable to consider [4], that the faster the particle, the more it should look like a point one. In [5] wave packet was introduced to explain the UCN anomaly (abnormally high losses in traps). The packet size was estimated to be $L\approx 10^5\,\lambda_{\rm Be}/2\pi$, where $\lambda_{\rm Be}$ is the minimal wavelength of the neutron which can be stored in beryllium traps. In this case L is the order of several millimeters, which does not contradict the assumption made in the [3].

Later [6,7] the assumption was made that the size of the wave packet depends on the speed and is proportional to the wavelength: $L\approx 10^5\,\lambda/2\pi$. Therefore, the thermal neutron wave packet size is up to about 5 microns. Perhaps it is correct, but how experimentally to measure the size of the wave packet that is the question.

One way is to look for neutron transmission through films when incidence is at a subcritical glancing angle [6, 8, 9]. Some indication of the transmission was obtained, but there is still no certainty. More precise experiments are needed.

We accept here a model of a nonspreading singular de Broglie's wave packet

$$\psi_{\rm dB}(\mathbf{r},\mathbf{k},t)=\sqrt{\frac{q}{2\pi}}\exp\left(i\mathbf{kr}-i\omega t\right)\frac{\exp\left(-q|\mathbf{r}-\mathbf{k}t|\right)}{|\mathbf{r}-\mathbf{k}t|}, \qquad (3)$$

wherein $\omega=(k^2-q^2)/2$, and the spatial size is $l=1/q$. To get rid of various constants, the variable t in (3) includes factor $\hbar/m$, so that t has dimension cm^{-2}. The packet (3) is normalized to unity as $\int d^3r|\psi_{\rm dB}|^2=1$, or as a full flux through any plane, integrated over time.

Fourier expansion of this packet has the form

$$\psi_{\rm dB}(\mathbf{r},\mathbf{k},t)=\sqrt{\frac{q}{2\pi}}\exp\left(i\mathbf{kr}-i\omega t\right)\frac{4\pi}{(2\pi)^3}\int d^3p\,\frac{\exp\left(i\mathbf{p}(\mathbf{r}-\mathbf{k}t)\right)}{p^2+q^2}, \qquad (4)$$

or

$$\psi_{\rm dB}(\mathbf{r},\mathbf{k},t)=\sqrt{\frac{q}{2\pi}}\frac{4\pi}{(2\pi)^3}\int d^3p\,\frac{\exp\left(i\mathbf{pr}-i\,p^2t/2\right)}{(\mathbf{p}-\mathbf{k})^2+q^2}e^{i\left((\mathbf{p}-\mathbf{k})^2+q^2\right)t/2}. \qquad (5)$$

The packet satisfies the equation

$$\left(\frac{i\partial}{\partial t}+\frac{\Delta}{2}\right)\psi_{\rm dB}(\mathbf{r},\mathbf{k},t)=-q\sqrt{2\pi}\exp\left(-i\left(k^2+q^2\right)t/2\right)\delta\left(\mathbf{r}-\mathbf{k}t\right), \qquad (6)$$

which can be checked by direct substitution.

If the incident neutron is described by such a packet, then at subcritical incidence, when the normal component k_n of the neutron wave vector is less than the critical value k_c of the reflecting foil with optical potential $u=k_c^2$, there should be the almost total reflection. However with small probability the neutron can go through the foil by a nontunneling way, because the Fourier expansion (5) contains components with $p_n > k_c$.

2. Experimental search for foils penetration

In the first experiment [10] in 1997 for search of transmission of ultracold neutrons (UCN) through foils it was found that "The penetration of subbarrier neutrons through beryllium foils and coatings has been observed experimentally. The penetration probability for a 56-μm vacuum-tight beryllium foil was found to be $(5\pm1)10^{-7}$ per collision. This value is many orders of magnitude greater than the quantum-mechanical tunneling for this thickness. The observed effect is very likely of the same nature as the anomalous loss of UCNs during storage in traps." However in the next experiment [11] it was found that transmitted neutrons have "approximately twofold increase in the neutron energy". After that many experiments have shown [12-18] that the storage of UCN in different tra[s is accompanied with small increase and decrease of energy. The explanation is given here.

It was supposed earlier [19], that the transmitted neutron has the same energy as the incident one because of the fear to violate the energy conservation law. However, in quantum mechanics, as was shown above, one should admit violation of the energy conservation law.

The energy distribution of the de Broglie's wave packet can be described by the probability density

$$dW_{\mathrm{dB}}(p) = C\frac{d^3p}{\left|(\mathbf{p}-\mathbf{k})^2+q^2\right|^2}. \quad (7)$$

where C is the normalization constant. After transmission and reflection the probability density changes. Let's calculate the average speed of the neutron after subcritical reflection from and transmission through an ideal foil. For simplicity we will consider only normal incidence of a neutron on the foil, which is perpendicular to the x-axis.

3. Average speed of the transmitted neutron at normal subcritical incidence

The part of the wave packet, transmitted through the wall, can be approximately described by the probability density

$$dW_{\mathrm{t}}(p) = C_t\frac{d^3p\,\Theta\left(|p_x|>k_c\right)}{\left|\mathbf{p}_\parallel^2+(p_x-k)^2+q^2\right|^2}, \quad (8)$$

where $\mathbf{p}_{\|}$ is the vector perpendicular to the x-axis, and parallel to the foil surface. This function approximates the transmission coefficient to unity, when $|p_x|>k_c$. Below we consider k_c as a unit speed, i.e. all other wave vector components are measured in units of k_c. Therefore the distribution (8) is represented as

$$dW_t(p) = C_t \frac{d^3 p \Theta(|p_x| > 1)}{\left|\mathbf{p}_{\|}^2 + (p_x - k)^2 + q^2\right|^2} . \qquad (9)$$

For calculation of normalization factor let's change the variables p_x-k = s then

$$1 = \int dW_t(p) = \int C_t \frac{d^2 p_{\|} ds \Theta(|s+k| > 1)}{\left|\mathbf{p}_{\|}^2 + s^2 + q^2\right|^2} = \pi C_t \left(\int_{-\infty}^{-1-k} + \int_{1-k}^{\infty} \right) \frac{ds}{s^2 + q^2} =$$

$$= \frac{\pi C_t}{q} \left(\pi - \arctan\left(\frac{1-k}{q}\right) - \arctan\left(\frac{1+k}{q}\right) \right). \qquad (10)$$

Let's note, that for small q arguments of arctans are large. Therefore

$$\arctan\left(\frac{1-k}{q}\right) + \arctan\left(\frac{1+k}{q}\right) \approx \pi , \qquad (11)$$

and

$$\pi - \arctan\left(\frac{1-k}{q}\right) - \arctan\left(\frac{1+k}{q}\right) = \arctan\left(\frac{2q}{1-k^2-q^2}\right) << 1 . \qquad (12)$$

So for small q and not too close k to unity we have

$$C_t \approx \frac{1-k^2-q^2}{2\pi} . \qquad (13)$$

Now, with probability distribution (7) we can find average momentum of the transmitted neutron $\langle p_x \rangle_t$:

$$\langle p_x \rangle = C_t \int p_x \frac{d^3 p \Theta(|p_x| > 1)}{\left|\mathbf{p}_{\|}^2 + (p_x - k)^2 + q^2\right|^2} = C_t \pi \int ds \frac{(s+k)\Theta(|s+k| > 1)}{s^2 + q^2} = k + \Delta k , \qquad (14)$$

where

$$\Delta k = C_t \pi \int ds \frac{s\Theta(|s+k|>1)}{s^2+q^2} \approx \frac{1-k^2-q^2}{4} \ln \frac{(1+k)^2+q^2}{(1-k)^2+q^2}. \qquad (15)$$

The dependence $\Delta k(k)$ is shown in fig.1. It is seen that transmission through a foil is accompanied with increase of the neutron energy.

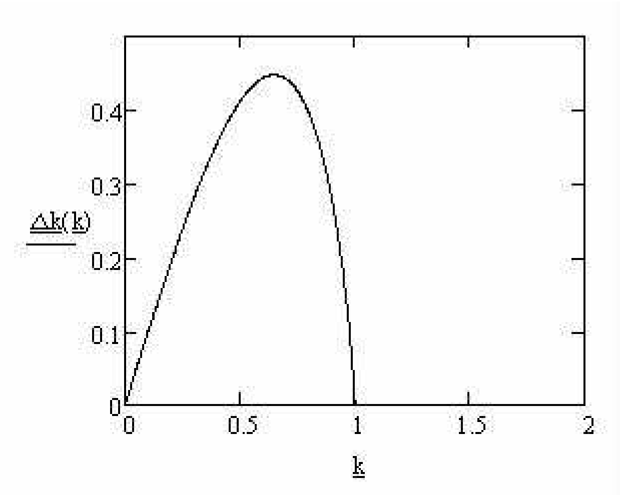

Fig. 1: Increase of the neutron speed after transmission through a foil with critical wave number *kc* = 1 in dependence on the initial speed *k*.

3. Average speed of the reflected neutron at normal subcritical incidence

The part of the wave packet, reflected from the wall, can be approximately described by the probability density

$$dW_r(p) = C_r \frac{d^3 p \Theta(|p_x|<1)}{\left|\mathbf{p}_\parallel^2 + (p_x-k)^2 + q^2\right|^2}. \qquad (16)$$

To calculate the normalization factor we again change the variables p_x-$k = s$, then

$$1 = \int dW_r(p) = \int C_r \frac{d^2 p_\parallel ds \Theta(|s+k|<1)}{\left|\mathbf{p}_\parallel^2 + s^2 + q^2\right|^2} = \pi C_r \int_{-1-k}^{1-k} \frac{ds}{s^2+q^2} =$$

$$= \frac{\pi C_r}{q}\left(\arctan\left(\frac{1-k}{q}\right) + \arctan\left(\frac{1+k}{q}\right)\right). \qquad (17)$$

For small q we can use approximation (11). Therefore

$$C_r \approx \frac{q}{\pi^2}. \qquad (18)$$

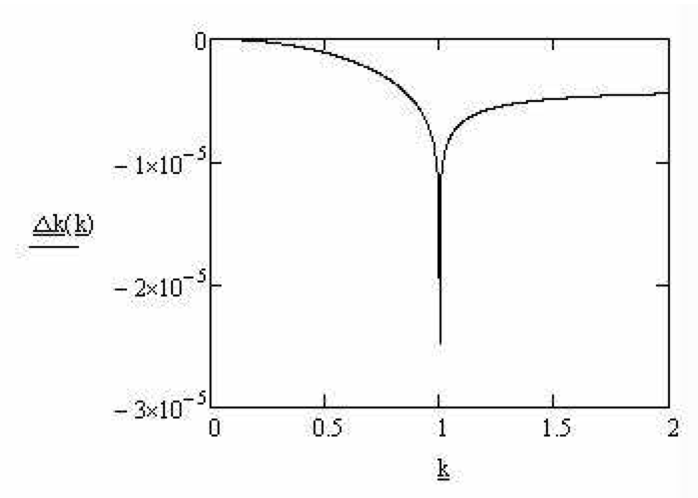

Fig. 2: Small decrease of the neutron speed after reflection through a foil with critical wave number k_c = 1 in dependence on the initial speed *k*.

Now, with probability distribution (16) we can find average momentum of the reflected neutron$\langle p_x\rangle_t$:

$$\langle p_x\rangle = C_r\int p_x\,\frac{d^3p\Theta(|p_x|<1)}{\left|\mathbf{p}_\parallel^2+(p_x-k)^2+q^2\right|^2} = C_r\pi\int ds\,\frac{(s+k)\Theta(|s+k|<1)}{s^2+q^2} = k+\Delta k\,, \qquad (19)$$

where

$$\Delta k = C_r\pi\int_{-1-k}^{-1+k} ds\,\frac{s}{s^2+q^2} \approx \frac{C_r\pi}{2}\ln\frac{(1-k)^2+q^2}{(1+k)^2+q^2} = \frac{1}{2\pi}\ln\frac{(1-k)^2+q^2}{(1+k)^2+q^2}\,. \qquad (20)$$

The dependence $\Delta k(k)$ is shown in fig.2. It is seen that reflection from a foil is accompanied with small decrease of the neutron energy.

4. A possible experiment to measure transformation of the neutron spectrum

It is very easy to check how transmission and reflection change the UCN spectrum. Let's imagine a cylindrical Cu vessel with added radial Al plates, as shown in fig.3. If a cylinder has a height *H*, and it is filled by UCN with energies lower than *H/*2, then after some storage time they can reach the top of the cylinder and jump out of it. The speed with which the neutron spectrum increases in such a trap, depends on number of radial plates, which can be easily verified. In fact the similar experiment with sample foil on the bottom of the vessel was already done [11-12], and it has shown that the larger is the sample the faster is the UCN small heating.

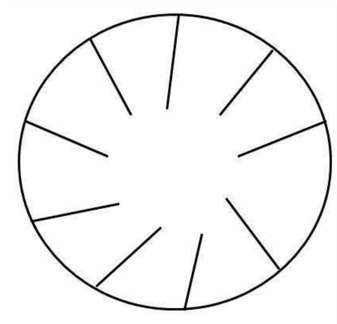

Fig. 3: ***Horizontal section of a UCN cylindrical trap for measurement of spectrum transformation.***

Conclusion

In this paper we have shown, that nontunneling transmission of the ultracold neutrons through a foil with potential u, is accompanied with increase of its energy by amount of order u, and reflection of these neutrons from the same foil is accompanied by decrease of energy by the amount of order 10^{-5}u. Since probability of transmission is of the order 10^{-5}, and probability of reflection is near unity, then total quantum probability of energy change is zero. Nevertheless every event for a single particle is accompanied with violation of energy conservation law, and we proposed an experiment to watch such violation.

In many experiments it was shown that the small heating happens in vessels even without samples. It can be explained by incoherent scattering. Penetration inside matter is provided by components of the wave packet with energy higher than the optical potential u. After incoherent scattering they can go back and represent reflected wave packet with higher energy.

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