SCIREA Journal of Physics

http://www.scirea.org/journal/Physics

December 14, 2016

Volume 1, Issue1, October 2016

Influence of Space-Time Curvature on the Light Propagation.

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Abstract

This article is devoted to the problem of light propagation in a space-time which curvature is due not to massive sources but to the electromagnetic field of the wave itself. Some methodological questions are discussed, such as an isotropy of metric, implementation of the Kalutza-Klein model, topology of space-time, etc.

Keywords: spherical electromagnetic wave, Maxwell-Einstein equations, metric tensor, topology of space-time.

1. Introduction.

This paper deals with the problem of the propagation of light in the space-time whose curvature is determined not by material sources but by the light wave itself. Because the main results obtained in [1-6], are known, the main attention will be focused on some issues of methodological character, which in those studies have not been discussed sufficiently.

As is known, the investigation of the problem requires finding solutions of the system of the Maxwell-Einstein (ME) equations:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi K}{c^4} T_{ik}; F^{ik}_{,k} + \Gamma^{l}_{kl} F^{ik} = 0 \qquad (1)$$

Here R – is a trace of the Ricci tensor R^i_k: $R = R^i_i$, g_{ik} – is a metric tensor; T_{ik} and F^{ik}– are tensors of energy-momentum and electromagnetic one; Γ^i_{kl} – are a Kristoffel symbols; c – is the light speed in vacuum, K – is the gravitation constant; indices *i, k, l* have values 0, 1, 2, 3; on repeated indices assumes summation; comma means usual, not covariant derivative [7]. We have investigated solutions of (1), corresponding to the presence of spherical electromagnetic waves (SEW) at the spatial infinity.

The tensor T^{ik} has the form [7]

$$T^{ik} = \frac{1}{4\pi}\left(-F^{il} F^{k}_{l} + \frac{1}{4} g^{ik} F_{lm} F^{lm} \right) \qquad (2)$$

In solving the system (1) we made a number of assumptions that require a detailed justification.

1. It has been supposed that the metric corresponding to the desired solution (solutions) of (1) is spherically symmetric.

The fact that this assumption requires justification is clear from the very beginning - it is not clear why the solutions of the Einstein's equations (the components of the metric tensor) do not depend on the angular coordinates, if the source of the gravitational field – the energy-momentum tensor of EMW (2) depends on them.

2. In solving the Einstein's equations the constant in the expression for the metric, is determined from the Maxwell's equations, the same way as similar constant in the course of solving the Schwarzschild problem is determined using the Newton's equation [7].

Indeed, the usage of the Newton's equations does not cause questions, because they are the limiting case of the Einstein's equations for the flat space-time, while the use for the same purpose of the Maxwell's equations requires justification.

3. It is assumed that the solutions of the ME equations, which look at spatial infinity like the converging and diverging electromagnetic waves correspondingly belong to the different vacuums of the theory and can not be transformed classically into one another.

This thesis is in fact related to the problem of focusing spherical EMW. Its solution proposed in [1-6] requires a revision of views on the topology of the space.

Without justification of these assumptions the solution of the problem looks unconvincing and raises objections of professionals [1].

2. The spherically symmetric metric.

First of all, we recall that in the article [1] consideration was carried out in the coordinate system in which the projection of the moment momentum of EMW on the axis *OZ* is zero. This eliminates the dependence of the field on the azimuthal angle of EMW ~ $e^{im\varphi}$, due to $m = 0$. Such states are described in [8]. By suitable transformation to the relevant coordinate system the general case of a spherical wave corresponding to $m \neq 0$ may be led to the considered one. This uses the opportunity provided by the general relativity, to solve a problem in the frame of reference, which is most suitable to do so. This method reduces the number of the needed Christoffel symbols.

One can offer an alternative method of reasoning. Consider the symmetry of the generalized Maxwell equations (and Maxwell-Einstein), i.e. which take into account, along with electric charges and currents also magnetic, relatively dual transformation $\vec{E}' + i\vec{H}' = (\vec{E} + i\vec{H})e^{i\alpha}$. Here $\vec{E}, \vec{H}$ - are the electric and magnetic fields of the EMW before transformation and $\vec{E}', \vec{H}'$ - after this one, α – is the dual phase. One can eliminate the dependence on the azimuthal angle in the expressions for the fields, putting $\alpha = -m\varphi$. As was shown in the article [9], the requirement of conservation of the form of the Maxwell's equations[2] with respect to dual transformations leads to the equation for α: $\alpha_t + \vec{V}\nabla\alpha = 0$, where $\vec{V} = \vec{S}/W, \vec{S} = \vec{E} \times \vec{H}, W = (\vec{E}^2 + \vec{H}^2)/2$ (in Heaviside-Lorentz system). For the case of the spherical electromagnetic wave this equation is satisfied for the above selection $\alpha = -m\varphi$,

[1] E. Weinberg, Editor Physical Review D (private communication)
[2] I.e. containing only an electric or only a magnetic charges.

that easily can be shown, for example, for the EMW which is excited by the elementary electric dipole [10].

We now turn directly to the question of the metric. In the article [1] metric is determined by the character of the interval

$$ds^2 = g_{00}c^2dt^2 + g_{11}dr^2 - r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2)$$
$$g_{00} = e^{\nu}, g_{11} = e^{\lambda} \qquad (3)$$

$\nu = \nu(t, r, \theta)$, $\lambda = \lambda(t, r, \theta)$; $x^0 = ct$, t –time; $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ – spherical coordinates. To separate the variables we imposed the additional condition: $\lambda = \alpha(r, t) + \beta(\theta)$, $\nu = -\alpha(r, t) + \beta(\theta)$. According to [1], the angular part of the field of the electromagnetic wave is determined by the known spherical harmonics, which are characterized by the set of values: *j, l, m*, that determine the total and the orbital angular momentum and the projection of the first on the axis OZ, provided $\beta = 0$. Thereafter, the left side of the Einstein equations depends on r and θ additively, whereas the right side - multiplicatively, what leads to a contradiction, which can be eliminated by the averaging on θ.

To substantiate this averaging procedure we consider the observer which is placed in the focus of a converging spherical electromagnetic wave. Illumination that he observes, without taking into account the effects of the general relativity will in accordance with the nature of the spherical functions have peaks corresponding to their extrema, separated by angular intervals $\Delta\theta = \pi/l$ for the spherical function with the orbital number l. This picture takes place if one neglects the effect of the light beam deflection in curved space-time. Accounting for this effect [5], will result in an overlap of the adjacent peaks of illumination for $l \geq 3$ [3]. So, for large l, the observer actually «sees" a uniformly illuminated spherical surface corresponding to the front of the converging EMW[4]. Thus, the procedure of averaging over the angle θ gets the physical justification. Here is the final form of the metric obtained in the article [1]:

$$-g_{11} = g_{00}^{-1} = e^{\alpha}, \alpha = \frac{r_c}{r}, r_c = \frac{l(l+1)c}{\omega} \qquad (4)$$

ω – is a frequency of EMW. As stated in the article [2], this result can be obtained by using only the Maxwell equations written in a curved space-time, because gravitational constant in the expression (4) is absent. To do this, one can use the equality of the energy-momentum

[3] If we assume that the deviation should be taken into account twice, i.e. for the neighboring peaks, then the condition is different: $l \geq 2$ (Y.N. Zayko, not published).

[4] The exceptions are a waves with small l = 1,2 [5] (or, according to updated data l = 1)

tensor components $T_{22} = T_{33}$, which is the consequence of the isotropy of space. This once again emphasizes the similarities of electromagnetic and gravitational phenomena. The curvature of the metric in this approximation is called in the article [2] as "diffractional" due to the analogy between the deviation of the light beam in the metric (4) and the effect of self-diffraction in nonlinear optics.

What are the alternatives to this procedure? The simplest and the most correct way - use accurate metric for the solving the problem. Unfortunately, today such a metric is unknown.

Let's say a few words about the asymptotic nature of the solution of (4). It is derived from Maxwell's equations for the radial part $F_{01}=\Psi(r,t)\ P_l(\theta)$, where $P_l(\theta)$ – is a Legendre polynomial of the order l [1]:

$$\frac{\partial}{\partial r}\left[e^{-\alpha}\frac{\partial f}{\partial r}\right]-e^{\alpha}\frac{\partial^2 f}{c^2\partial t^2}-\frac{l(l+1)}{r^2}f=0, f=r^2\Psi$$
$$e^{\alpha}=\pm\frac{\partial f}{\partial r}\left(\frac{\partial f}{c\partial t}\right)^{-1} \tag{5}$$

In the article [1] this equation was solved in the approximation of smallness of the member ~ α', occurring when placing $e^{-\alpha}$ outside the sign of the derivative with respect to r in the first term in the equation (5) (the prime denotes the derivative with respect to r), which leads to the condition $r > c/\omega$ and which is certainly satisfied when $r > r_c$.

3. Determination of the constant in the expression for the metric.

In the article [2] a specification of the (4) was obtained using the full system of the ME equations, which, given the assumptions made after averaging over the angle θ have the form

$$-e^{-\alpha}\left(\frac{1}{r^2}-\frac{\alpha'}{r}\right)+\frac{1}{r^2}=\frac{r_s^2}{r^4}$$
$$\frac{1}{2}e^{-\alpha}\left[\alpha''-(\alpha')^2+\frac{2\alpha'}{r}\right]=-\frac{r_s^2}{r^4}, r_s^2=\frac{K}{2c^4}|G|^2 \tag{6}$$

G – is the amplitude of the spherical electromagnetic wave, which doesn't depend on r. The equations (6) lead to the expression for the metric

$$e^{-\alpha}=g_{00}=1+t(C+t), t=\frac{r_s}{r} \tag{7}$$

The constant C should be determined. If we choose $C = -r_c/r_s$, then (7) takes the form:

$$e^{-\alpha} = g_{00} = -g_{11}^{-1} = 1 - \frac{r_c}{r} + \left(\frac{r_s}{r}\right)^2 \tag{8}$$

This asymptotically coincides for the $r > r_c >> r_s$ with the previously found asymptotic value $exp(-r_c/r) \approx 1 - r_c/r$. What are the grounds for the use of the asymptotic behavior of (4) to determine the constant C?

Generally speaking, in frame of the 3 + 1 theory, one can't substatiate the above procedure strictly. In order to do it one must suggest that gravitational and electromagnetic fields together constitute a kind of unified field. In our opinion, the most natural choice consists in the use of 4 + 1-dimensional Kaluza-Klein theory [11, 12].

4. The topology of the real space-time.

In recent years, increased interest in the study of topology of real space in connection with the proof in 2003 by G. Perelman the Poincare conjecture [13] which was formulated by him in 1904. It essence consists in that if an arbitrary one-dimensional closed curve in the n - dimensional manifold can be shrunk to a point, then this manifold is homeomorphic to the n - dimensional sphere (has a topology of a sphere).

Numerous popular expositions (see, for example, [14]) on the subject, made attempts to apply this mathematical result to the real space and "prove" the fact which is considered obvious, namely, that our universe resembles a sphere, and not, for example, a torus. At the same time, in a manner typical of mathematicians, the question of the physical implementation of such evidence is not considered[5].

In the report [4] various physical ways to check this result were discussed, and it was shown that they all are untenable - in all cases the methods implemented with the help of physical objects (light rays) cannot be used to achieve target for reasons not associated with the topology.

The report [4] investigated the question of topology by using another homotopy mapping – a sphere into a sphere. According to the articles [1-6], front of the converging spherical EMW can not be shrunk to a point. This question is closely connected with the question of focusing

[5] Rope, which is appeared in [14], is no more than a metaphor.

the spherical EMW. As stated in [1-6], the converging EMW can not classically be transformed into the diverging EMW. This has less to do with the singularity of solutions of the ME equations, how with the possibility of capture of some rays in the metric (7) [2]. The conversion process goes quantum mechanically through an intermediate state – an instanton due to tunneling between the solutions of the original ME equations, which are realized near different electromagnetic vacuums.

In other words, the curvature of space-time associated with the spherical EMW leads to a restructuring of the initial sole electromagnetic vacuum. Both converging and diverging EMW exist in the different vacuums, transition between which is carried out with the help of the instanton. Let us note indirect character of the evidence for this claim - if there is an instanton, then there must be different vacuums which it binds. An immediate proof of this assertion today is absent. We can, however, raise the question of changing the nature of the electromagnetic vacuum near the event horizon for the metric (8).

Consider the classical action for the electromagnetic field [7]

$$S = -\frac{1}{16\pi c}\int F_{ik}F^{ik}\sqrt{-g}\,dx^0dx^1dx^2dx^3 \tag{9}$$

where g – is the determinant of the quantities g_{ik}. In view of the above, (9) can be written as (P_l^1 - associated Legendre polynomial)

$$S = -\frac{1}{8\pi c}\int r^2 dr dx^0 d\Omega\left\{\left[e^{\alpha}\left(\frac{\partial f}{\partial x^0}\right)^2 - e^{-\alpha}\left(\frac{\partial f}{\partial \rho}\right)^2\right]\frac{\left[P_l^1(\theta)\right]^2}{\left[l(l+1)\right]^2} - \frac{f^2}{r^4}\left[P_l(\theta)\right]^2\right\} \tag{10}$$

$$d\Omega = \sin\theta d\theta d\varphi$$

Performing integrating on the angular coordinates, rewrite (10) in the form

$$S = -\frac{1}{2(l+1)c}\int d\rho dx^0\left\{\left[\left(\frac{\partial f}{\partial x^0}\right)^2 - \left(\frac{\partial f}{\partial \rho}\right)^2\right]\frac{1}{l(l+1)} - V(\rho)\right\} \tag{11}$$

$$d\rho = e^{\alpha}dr, V(\rho) = e^{-\alpha(\rho)}\frac{f^2}{r^2(\rho)}$$

The expression in the curly brackets (up to a factor), is the density of the Lagrangian of the electromagnetic field of the spherical waves with the amplitude $f(x^0, \rho)$, and the density of the potential energy $V(f, r(\rho))$. The behavior of $\partial^2 V/\partial f^2$ can give an information on the stability (or instability) of the electromagnetic vacuum [6]. Taking for $e^{-\alpha}$ the expression (8), where for simplicity we put $r_s = 0$ we conclude that for $r > r_c$ [7]electromagnetic vacuum is stable, and for $r < r_c$ - it isn't, what is the cause of arising of the non-wave solutions of the Maxwell's equations, which have been identified with instantons in the works [1-6][8].

Since all the described phenomena take place at distances of order r_c at which the real gravitational phenomena do not affect yet (they begin to affect at the distances $r_s << r_c$) we can talk not about instantons of the Maxwell-Einstein equations, but of the Maxwell equations only. Formally, the equation for the instanton of Maxwell equations is obtained from the condition $T_{00} = T_{11}$, which is performed for a spherically symmetric metrics [1].

Instanton is the solution of the ME (or Maxwell) equations of non-wave nature. It was described in detail in the article [3]. The study of the gravitational instantons goes back to the work of S. Hawking [16]. Note that most of the works, for example, which are available on the resource [17], on the gravitational instantons are devoted to the classification of the instanton solutions of the Maxwell-Einstein equations in many-dimensional Riemannian manifolds and their applications to the physics of black holes.

Homotopy, which is referred to above relates to the contraction of the front of spherical electromagnetic wave, what may be done only till the distances of the order of the instanton size. Inability to shrink the front further is conditioned by the fact that at shorter distances waves do not exist. Although this fact has no direct relationship to the Poincare theorem, it is also associated with the topology of space. This result indicates that the physical space is not homeomorphic to a sphere, and has a more complex topology.

Another indication of this is a violation by instantons of the ME equations the so-called "weak energy condition" $T_{\alpha\beta}\,\xi^{\alpha}\,\xi^{\beta} > 0$ [3], where $T_{\alpha\beta}$ – is the energy-momentum tensor of an electromagnetic field, and ξ – is any non space-like 4 – vector [18]. As is known, this condition is associated with the requirement of the absence of so-called "wormholes" [19], representing the topological features of space-time.

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[6] Usually vacuum instability is considered in the sense of the birth of pairs of particles [15], what can be taken into account in (9) in the first order in the Planck constant $\hbar$. Here, the term "instability" is understood in a different sense, namely in terms of stability (or instability) of the wave-type solutions of the field equations. It can be shown that the inclusion of the said amendment does not change final conclusions.

[7] In the given approximation ($r_s = 0$) the event horizon is located at r_c.

[8] Although this is some overstatement, these solutions further are called as instantons.

5. Conclusion.

In this article, the author summarizes his works on the propagation of light in the space-time which is warped by electromagnetic field of the wave itself. As outlined, the solution of this problem is due to the adoption of a number of assumptions. Some of them can be justified within the framework of generally accepted theories, while others lead to a changes in our understanding of the structure of the space-time, the nature of propagation of the electromagnetic waves and their relationship to gravity phenomena.

Special mention should be on the results described in Section 3. The theory of Kaluza-Klein, in contrast to the analogous Weyl theory does not contradict the facts. The only objection to it is that it does not fulfill the requirements for all sorts of new theories - it does not predict any new results. If we take into account that the present problem cannot be solved outside the unification of the electromagnetic and the gravitational phenomena, its decision may be regarded as that new result, which is required for the recognition of the theory.

Until recently it was thought that the the system (1) has only static solution - the metric of Reissner-Nordström which corresponds to a massive charged body. The formal coincidence of (8) with that metric, though it looks random, actually is an indication of the limitations of the electrodynamics of Maxwell-Einstein at distances of the order of wave- length. For solving this problem at distances of the order of r_s one needs to use non-Abelian gauge theories [20].

Acknowledgments.

The author expresses his thanks to prof. Smolyanskiy S.A. and prof. Prosorkevich A.V for their interest in this work.

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