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Noncircular orbits and organised chaos exemplified with sine circular map and Arnold's tongues.

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Abstract

Assuming that time is determined by a present frequency, relations are derived based on the angular velocity of a noncircular orbit. Formats relating concepts of noncircular orbits with fix points of the sine circular map, so-called Arnold's tongues, will be obtained.

Keywords: Avd, quantum chaos, frequency ratio, spatial ratio, angular velocity, sine circular map, Arnold's tongues

Introduction

Chaos is a disciplin of mechanics and concerns nonlinear systems which is sensitive to initial conditions, such that second order effects determine the behaviour to some extent. This can show as motions that cannot be described in details with the first order models.

Here, the word organised chaos will be used for classical and general systems where chaos self-organise guided by certain rules. More specifically, we will concern solutions that are related to (the motion in) noncircular orbits. This gives a superimposed harmonic where the L- frequency ω_L is quantised compared with the ground level Strömberg (2014). Quantum levels are given by a ratio, $f=\omega_L/\omega_0$ which typically takes values 2 (octave, tide), 3/2 (quint, Mercury). To describe such systems, a resulting angular velocity is derived Strömberg (2015). This is given by

$$\omega(t)=\omega_0 \exp(-2(r_e/r_0)\sin(f\omega_0 t)) \qquad (1)$$

and may be written approximately as a sum, for the special case when (r_e/r_0) is small. Susequently, (1) will be referred to aas w from Avd.

Sine circular map

The sine circular map [wik], scm, is a model for chaotic systems, and is found to fit with many experiments on dynamical systems subjected to a (repeated) outer loading signal. It is derived from a deterministic system with two independent degrees of freedom, and then assuming some chaos such that they couple. The model relies on dissipation which tacitly invokes interaction and supply. Analysis and experimental results from medical, biological and chemical sciences are found in Ohlen et. Al. (2007), e.g. heart stimuli, and fireflies subjected to flashes.

Modeling resembles that of forced vibrations, and the amplitude will depend on the frequency coupling ratio. Opposite to forced vibrations, the input signal acts only part time. After, both d.o.f are intrinsic, interacts and apparently the system may lock into certain states. Results are illustrated by the famous diagram called Arnold's tongues, Figure 1 and 2.

The diagrams are derived from fix points, at capture into certain frequency ratios due to connection between different levels in the fractal format of iteration formula. This is pictured in several ways, with colours indicating the sublevels, c.f. Figure 1 and 2.

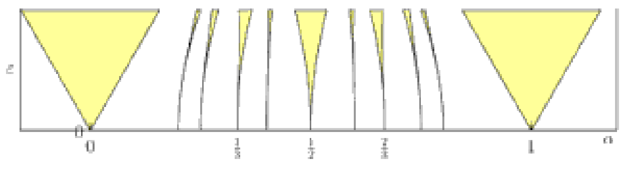

Figure 1. Arnold's tongues. The parameter K as a function of frequency ratio at fix points

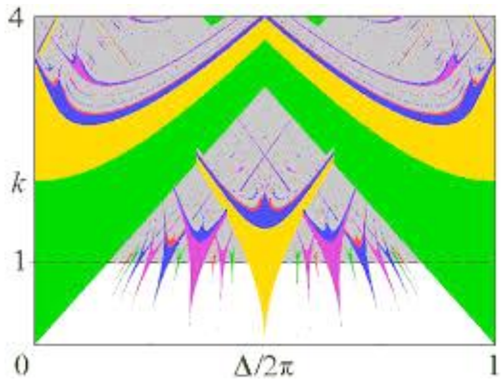

Figure 2. Arnold's tongues as a fractal for mode locking, based on a sine circular map.

The iteration format in scm is formulated in angles, but when compared with signals from experimental data, these may be either mean value of frequencies for a specified time, or a phase. When a system is subjected to a repeated pulse as input, in the first part of the time period, it locks into a constant at the later next half of the period. The sine circular map is an empiric construction that serves to describe the behaviour at the later part.

There are similarities with the angular velocity in Avd equation (1), but this is one d.o.f, time-dependent and determined in its original formulation. Results for updated versions are given in Strömberg (2015), and here we shall derive additional formats in relation with the sine circular map.

Spatial ratios related to quantum of frequencies

Several results may be extracted directly from the definition (1).

This will give 3 different functions depending on how time is created.

i) From (1), we obtain that $\min(\mathrm{abs}(r_e/r_0))=0.5\ln(w/w_0)$.
The function is shown in a graph for frequency ratios embracing those common in acoustics, Figure 3 and 4.

Preliminaries. Assume, similar to the establishment of the sine circular map, that time is determined by the other frequency.
ii) Time from the new frequency. Then, the relation for spatial ratio reads $r_e/r_0=0.5\ln(w/w_0)/(\sin(w_0/w))$.

iii) As in ii), but with an updated format, and time from the first frequency. Relation for spatial ratio $r_e/r_0=0.5\ln(w/w_0)/(\sin(w/w_0))$.

These functions are also show in Figure 3 and 4. The scripts are included in the reference list.

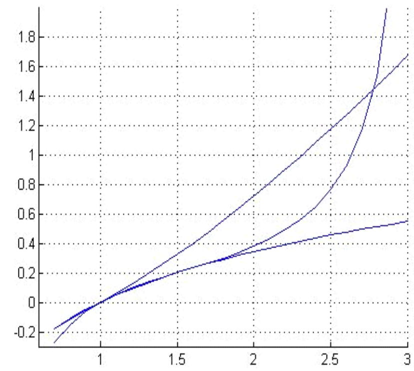

Figure 3. Ratio of eccentricity and radius versus frequency ratio for case i), ii) and iii) from bottom at left, f=1.

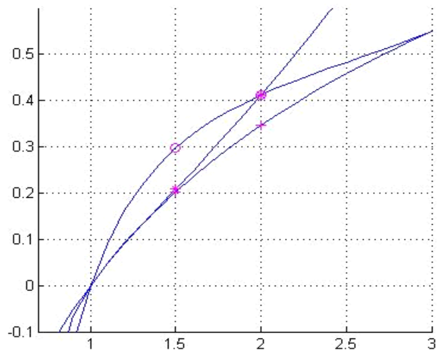

Figure 4. Ratio of eccentricity and radius versus frequency ratio for case i), ii) and iii) from bottom at left, f=2.

Correspondance between scm and w from Avd

The sine circ map is constructed from an iteration equation based on state variables and minimization of d.o.f. together with a functional constitutive assumption. The format and change into a new state depending on the previous is a discrete iteration map for the angle. It differs from the postulations and governing equations in classical continous mechanics. Therefore, the scm cannot be exactly obtained from an integration into angle of the w from Avd. Next, a correspondence between the models will be discussed.

Arnold’s tongues versus spatial ratio.

When the criteria for fix points resulting in Arnold’s tongues, is considered as a function of relative difference in frequency, a correspondence is obtained. Quanta for frequency derived from Avd, are directly proportional to small spatial ratios. Substitution gives that Arnold's tongues can be written as a function of r_e/r_0 instead of frequency ratio.

Conclusion

The format of w in equation (1) provides a possibility to derive relations assuming that time is determined by the other frequency. This method originates from the sine circular map [wik], Ohlen et. al. (2007), and is based on minimisation of states and d.o.f. From that, functional relationships between frequency ratios and spatial ratios were obtained. Results for different definitions of time was derived and showed in diagrams. A correspondence was found by identifying the function K(Δw) in Arnold’s tongues, when Δw is determined from max and min of w in Avd.

```
q=[0.7:0.1:3];
clf
figure(1)
hold on
ra2=0.5*log(q);
ra2=0.5*log(q)./sin(1./q);
ra3=0.5*log(q)./sin(q);
plot(q, ra2)
plot(q, ra3)
plot(q, 0.5*log(q))
grid

q=[0.7:0.1:3];
clf
f=2;
figure(1)
hold on
ra2=0.5*log(q);
```

```
ra2=0.5*log(q)./sin(1./q*f);
ra3=0.5*log(q)./sin(q/f);
plot(q, ra2)
plot(q, ra3)
plot(q, 0.5*log(q))
grid
q=2;
ra2=0.5*log(q);
ra2=0.5*log(q)./sin(1./q*f);
ra3=0.5*log(q)./sin(q/f);
plot(q, ra2, 'm*')
plot(q, ra3, 'mo')
plot(q, 0.5*log(q), 'm+')
q=3/2;
ra2=0.5*log(q);
ra2=0.5*log(q)./sin(1./q*f);
ra3=0.5*log(q)./sin(q/f);
plot(q, ra2, 'm*')
plot(q, ra3, 'mo')
plot(q, 0.5*log(q), 'm+')
```

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