SCIREA Journal of Physics

http://www.scirea.org/journal/Physics

August 1, 2017

Volume 2, Issue 3, June 2017

Exclusion of the Magnus Effect as a Mechanism for Shotgun Pellet Dispersion

Jeffrey S. Lee[1,2], Gerald B. Cleaver[1,3]

[1]Early Universe Cosmology and Strings Group, Baylor University, One Bear Place, Waco, TX 76706

[2]Center for Astrophysics, Space Physics, and Engineering Research, Baylor University, One Bear Place, Waco, TX 76706

[3]Department of Physics, Baylor University, One Bear Place, Waco, TX 76706

Abstract

In this paper, it is shown that the Magnus effect cannot be a primary mechanism for the dispersion of shotgun pellets. The one-dimensional motion of objects through air, applied to shotgun pellets traveling a short distance, reveals that throughout the flight, the Reynold's number is approximately constant. The distribution of pellets upon the target is demonstrated to be a phased-shifted Maxwellian distribution in lateral displacement space. The minimum Magnus frequencies required to create a typical pellet distribution pattern are ascertained, and are well in excess of the frequencies that could reasonably be achieved by pellets exiting a smooth bore shotgun.

Key Words: Magnus effect, Shotgun pellets, one-dimensional air resistance, Maxwellian distribution

1. Introduction

The distribution of shotgun pellets has been examined in the literature [1], [2], [3], [4], [5]. By considering the one-dimensional motion in air, it is clear that the variations in velocity and Reynold's number are minimal, and thus, throughout a short flight, both can be taken as being constant. By calculating the Magnus spin frequencies required for the measured lateral deviations in trajectory, this paper eliminates the Magnus effect as a possible cause for the distribution of shotgun pellets in both spatial and velocity spaces.

By examining the distribution of 12 gauge #7½ shotgun pellets at a distance of 7.00 m, a Maxwellian distribution, of the type claimed by Bhattacharyya et al. [5], is verified. The number of shots as a function of their spatial deviation from the principal axis is shown to be a Gaussian, and is in keeping with the results of Nag et al. [2].

2. Air Resistance in One Dimension

The approximate time for a shotgun pellet to travel 7.00 m[i] at a typical velocity of 380 m/s (~ 20 ms) precludes a significant vertical displacement as a result of small off-axis velocities, and thus, the one-dimensional treatment is applicable.

In the absence of gravity,

$$F_D = -\frac{1}{2}\rho C_D A\left(\frac{dz}{dt}\right)^2 = m\frac{d^2z}{dt^2} \tag{1}$$

subject to the boundary conditions:

$$z(0) = 0 \quad \text{and} \quad \left.\frac{dz}{dt}\right|_{t=0} = v_o \tag{2}$$

[i] The target distance for the experiment in Section 5.

Where F_D is the drag force, and z and v_o are the pellet's forward displacement and the initial velocity respectively.

Defining the drag constant as $K = \frac{\rho C_D A}{2m}$, the time and velocity as a function of position are given by eqs. (3) and (4) respectively.

$$t(z) = \frac{e^{Kz} - 1}{K v_o} \tag{3}$$

$$v(z) = \frac{v_o}{e^{Kz}} \tag{4}$$

The corresponding Reynold's Number is

$$\mathrm{Re}(z) = \frac{2\rho v(z) R}{\mu} \tag{5}$$

Where μ is the dynamic viscosity of the dry air, and the characteristic linear dimension is the pellet diameter ($2R$).

Combining eqs. (4) and (5):

$$\mathrm{Re}(z) = \frac{2\rho R v_o}{\mu e^{Kz}} \tag{6}$$

The shotgun pellets can be taken as spheres with a minimal roughness, as revealed by an optical microscopy (Figure 1), with a drag coefficient C_D that can be calculated directly from Morrison's equation [6], given by eq. (7).

Figure 1: A 44× optical microscopy of an unfired 12 gauge #7½ shotgun pellet.

$$C_D = \frac{24}{\text{Re}} + \frac{2.6\left(\frac{\text{Re}}{5}\right)}{1+\left(\frac{\text{Re}}{5}\right)^{1.52}} + \frac{0.411\left(\frac{\text{Re}}{263{,}000}\right)^{-7.94}}{1+\left(\frac{\text{Re}}{263{,}000}\right)^{-8.00}} + \frac{\text{Re}^{0.80}}{461{,}000} \tag{7}$$

Since the deviation of Reynold's Number values is small (10.0%, from Figure 6), the drag coefficient can confidently be taken as an average across the range of Reynold's numbers from the muzzle to the target.

3. Determination of the Minimum Magnus Frequencies

The derivation of the Magnus force acting upon a sphere of radius *R*, mass *m*, moving at speed *v*, and rotating with angular speed ω through air of density ρ is well-established, and the Magnus force F_M is given by:

$$F_M = 4\pi\rho v\omega R^3 \tag{8}$$

In order for the Magnus force to be solely responsible for displacing a pellet from the center of the target (0, 0) to a point with coordinates (r, θ), the Magnus acceleration must be

$$a_M = \frac{4\pi\rho v\omega R^3}{m} + g\sin\theta \tag{9}$$

Where *r* is the radial distance from the target center to the pellet's impact point, *g* is the acceleration due to gravity, θ is the counterclockwise 4-quandrant angle with respect to the positive *x*-axis, and the pellet's spin vector is assumed to be parallel to its lateral displacement vector.

With an approximately constant lateral acceleration resulting from the Magnus force,

$$r = \frac{1}{2}a_M t^2 \approx \left(\frac{2\pi\rho v\omega R^3}{m} - g\sin\theta\right)\left(\frac{z}{v}\right)^2 \tag{10}$$

And thus,

$$f = \frac{m}{4\pi^2 \rho v R^3}\left(\frac{rv^2}{z^2} + g\sin\theta\right) \quad (11)$$

Where $\frac{z}{v}$ is the flight time, and $f = \frac{\omega}{2\pi}$ is the spin frequency of the pellet. Clearly, $r + z \approx z$, since $r << z$.

4. Experimentation

The experimental results were acquired with an un-chocked Remington model 870 12 gauge smooth bore shotgun with a barrel length of 47.00 cm (18.50 inches) and a muzzle diameter of 1.83 cm (0.720 inches).

The ammunition was No. 7½, 2.75 inch (6.99 cm), shotgun cartridges manufactured by Federal Ammunition. The shots were 1.0 ounces (0.0283 kg) with an average of 350 pellets per cartridge and a powder charge of 3 drams (1.77×10^{-3} kg). The nominal shot diameter was 2.41 mm with a mass of 1.25 grains (8.10×10^{-5} kg) and a muzzle velocity of 1250 fps (381.0 m/s). The shots were fired from 7.00 m at standard paper targets with 1.0 inch (2.5 cm) ring widths (shells).

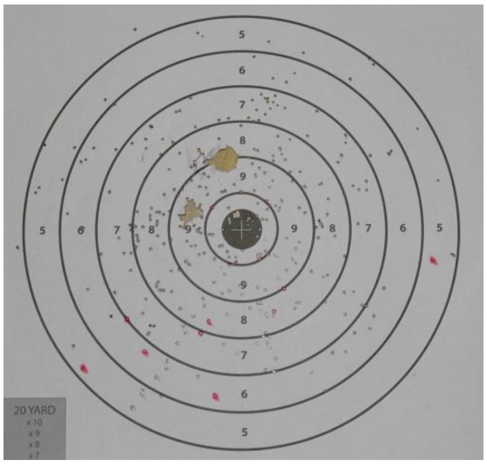

Figure 2: Target showing the spread of 12 gauge #7½ shotgun pellets.

The large hole between the "8" and "9" shells was made by the cartridge wad. The large hole in the "9" shell was formed by a high concentration of pellets. Consequently, data points at these location were not available. Transparent acetate with concentric shells with a width of 0.5 inches was superimposed on the target, providing a total of 15 shells.

The results obtained are shown in

Table 1. The calculated shot density is in good accord with the experimental data and was acquired by applying a phase-shifted Maxwellian distribution in shell radius space (Figure 3). Additionally, the required minimum spin frequencies and corresponding Magnus forces were determined from eqs. (11) and (8) respectively. Since the angular distribution of pellets was not considered, the effect of gravity was neglected, and the spin frequencies apply to only horizontal displacements. Consequently, the required spin frequencies would be greater for upwardly displaced pellets, and less for downwardly displaced pellets.

Table 1: Results for 12 gauge #7½ shotgun pellets incident upon a target from 7.00 m.

Shell #	Shell Radius (cm)	Square of Shell Radius (cm^2)	Shell Area (cm^2)	# of Shots	Shot Density (cm^{-2}) (Experimental)	Shot Density (cm^{-2}) (Calculated)	Spin Frequency (Hz)	Magnus Force (N)	Magnus Force/Weight
0[ii]	0.00	0.00	0.00	0	0	0	0	0	0
1[iii]	1.27	1.61	5.07	13	2.5656	1.9374	89.77	0.0061	7.68
2	2.54	6.45	15.20	24	1.5788	2.0432	179.55	0.0122	15.35
3	3.81	14.52	25.34	31	1.2236	1.9196	269.32	0.0183	23.03
4	5.08	25.81	35.47	49	1.3815	1.6317	359.09	0.0244	30.71
5	6.35	40.32	45.60	45	0.9868	1.2675	448.86	0.0305	38.39
6	7.62	58.06	55.74	38	0.6818	0.9059	538.64	0.0366	46.06
7	8.89	79.03	65.87	29	0.4402	0.5987	628.41	0.0427	53.74
8	10.16	103.23	76.01	36	0.4736	0.3671	718.18	0.0488	61.42
9	11.43	130.64	86.14	17	0.1974	0.2094	807.96	0.0549	69.10
10	12.70	161.29	96.27	12	0.1246	0.1114	897.73	0.0610	76.77
11	13.97	195.16	106.41	8	0.0752	0.0553	987.50	0.0670	84.45
12	15.24	232.26	116.54	12	0.1030	0.0257	1077.27	0.0731	92.13
13	16.51	272.58	126.68	2	0.0158	0.0112	1167.05	0.0792	99.81

[ii] The "0" shell is the central cross.
[iii] The "1" shell is the black bullseye.

14 [iv]	17.78	316.13	136.81	0	0	0.0046	1256.82	0.0853	107.48

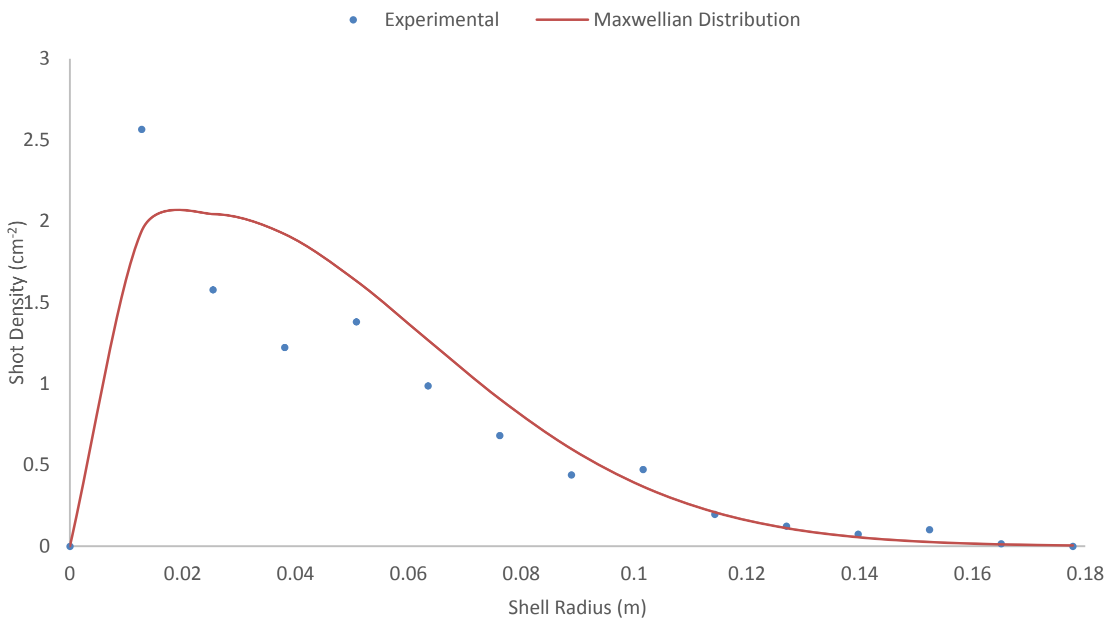

Figure 3: Shot density versus shell radius for 12 gauge #7½ shotgun pellets incident upon a target from 7.00 m.

For the determination of the Reynold's Number, the dynamic viscosity value of 18.37 μPa s was obtained by the interpolation of data from Kadoya et al. [7].

For 12 gauge #7½ pellets fired at a target from a distance of 7.00 m, $\langle C_{\mathrm{D}} \rangle = 0.4111$ [v] from eq. (7), which is combined with eq. (6) and plotted in Figure 4. The range of drag coefficients from Morrison's formula is 0.4098 to 0.4123. The corresponding variation of 1.8% justifies the validity of $\langle C_{\mathrm{D}} \rangle$.

Expectedly, for a range of 7.00 m, the one-dimensional motion establishes approximately linear relationships for v versus z (Figure 5), Re versus z (Figure 6), and f versus z [vi] (Figure 7).

[iv] The "14" shell is bounded on the transparent acetate.
[v] $\langle C_{\mathrm{D}} \rangle$ denotes the average drag coefficient.
[vi] For the Magnus effect to be solely responsible for lateral displacements.

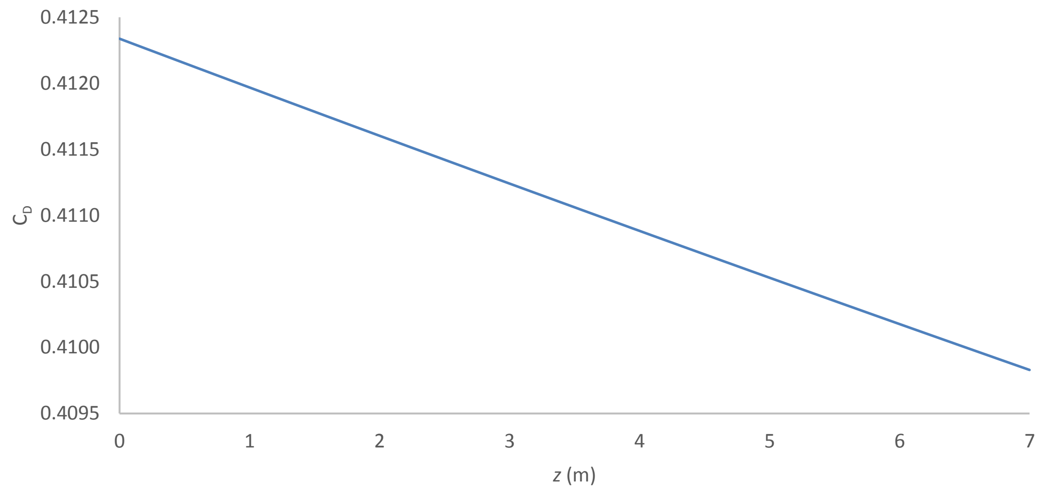

Figure 4: Drag coefficient versus displacement (calculated) for 12 gauge #7½ shotgun pellets.

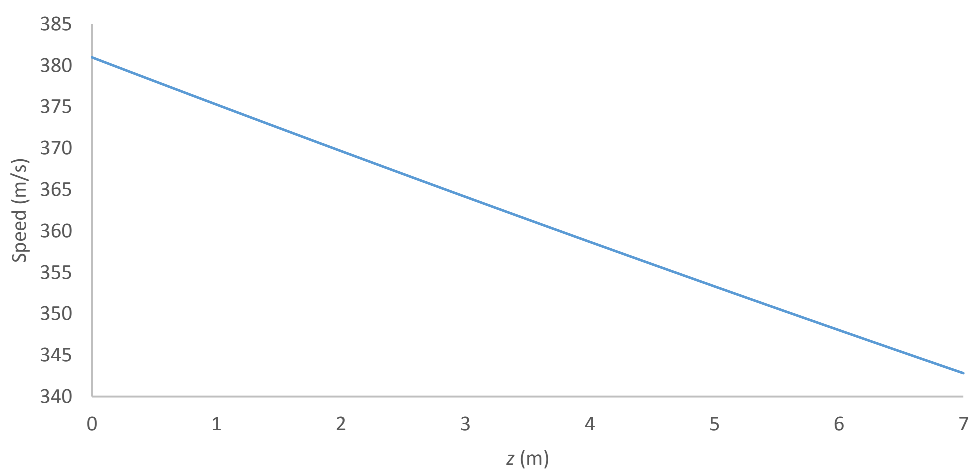

Figure 5: Speed versus distance (calculated) for 12 gauge #7½ shotgun pellets.

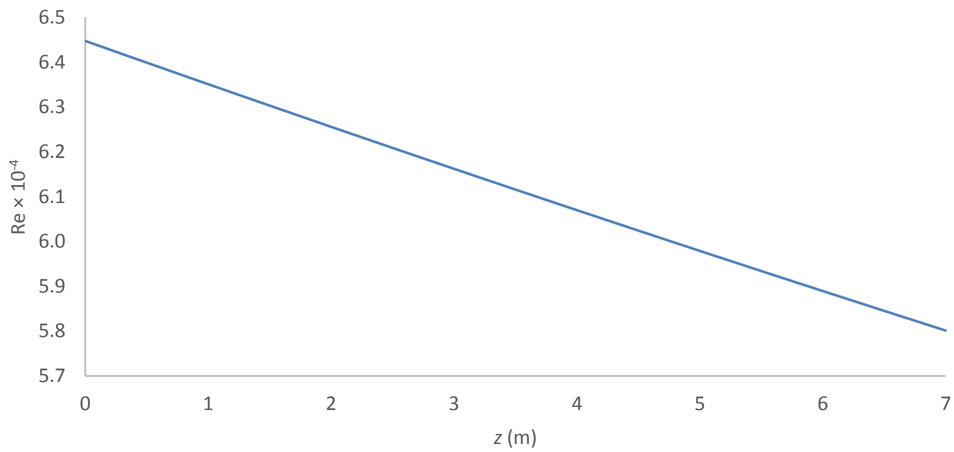

Figure 6: Reynold's Number versus distance (calculated) for 12 gauge #7½ shotgun pellets.

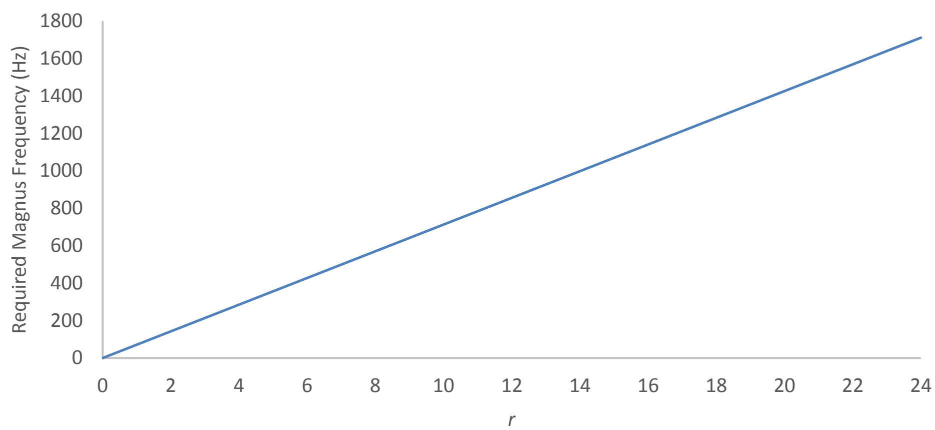

Figure 7: Required Magnus frequency versus lateral radial position for 12 gauge #7½ shotgun pellets incident upon a target from 7.00 m.

When the number of shots in a given target shell is considered as a function of the required minimum frequency, a Gaussian distribution in frequency space expectedly results (Figure 8).

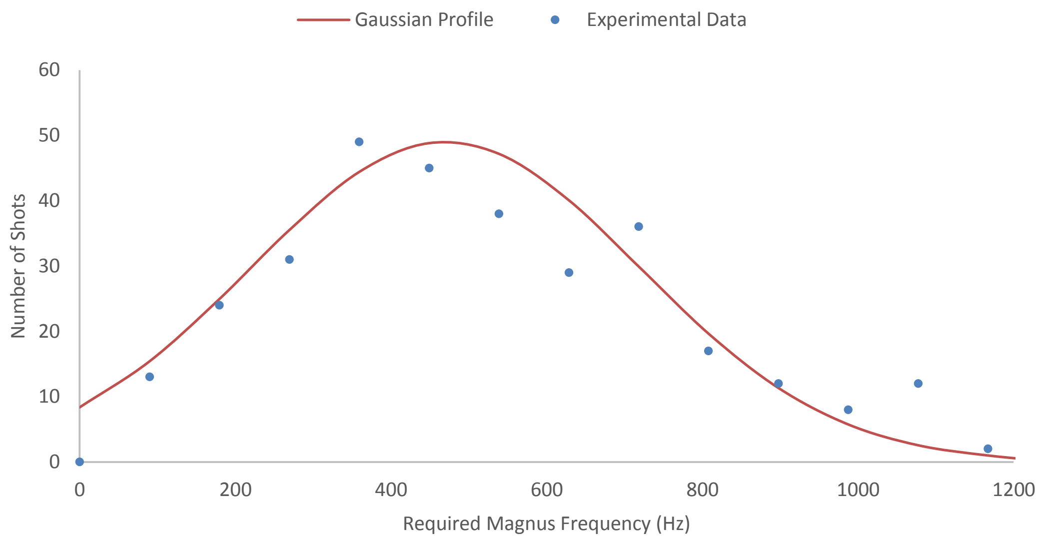

Figure 8: Number of shots versus required Magnus frequency for 12 gauge #7½ shotgun pellets.

Since both the shotgun barrel and pellets are smooth, and many pellets remain in the shotgun wad during travel through the barrel (see Figure 9), there is no physical mechanism capable of initiating pellet rotations approximating the required Magnus frequencies. Equating eq. (8) to the pellet weight shows that a vertically-orientated backspin with a Magnus frequency of 11.7 Hz is required just to offset the gravitational force.

Additionally, the off-axis drag on the pellet was neglected, which if it were to be included, could only increase the required Magnus frequencies. Also, in Figure 7, only lateral displacements were considered. Were the Magnus effect to be non-negligibly responsible for off-axis deviation, greater Magnus frequencies would be required for the pellets impacting above the target center.

This conclusion is further supported by the lack of evidence of high frequency Magnus rotation of the wad as the pellets clear the combustion cloud, as shown in Figure 9.

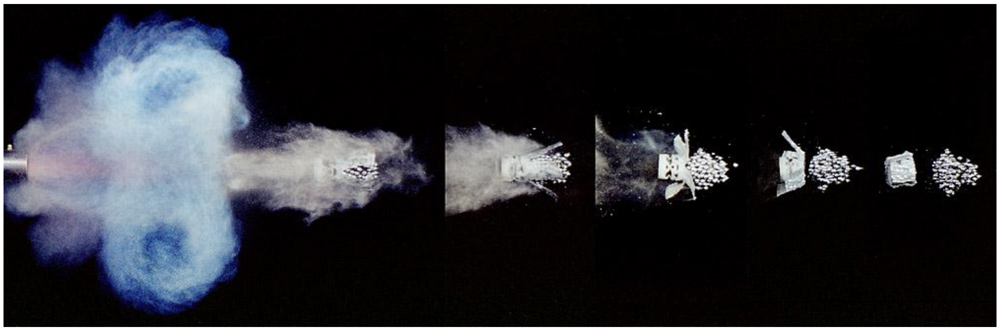

Figure 9: Series of 10^{-6} s exposures of a shotgun firing. Note the separation of the shot and sabot [8].

5. Summary

The approximately kilohertz Magnus frequencies that would be required to explain the lateral distribution of shotgun pellets have been shown to be unachievable, due largely to the lack of a mechanism to induce high frequency pellet rotation. The motion of the pellet in one dimension provides an acceptable description of a non-interacting (with other pellets) pellet's flight across a short range, and the time of flight to the target is insufficient for small random pellet rotations to generate a measurable effect on the pellet distribution. The pellet distribution was shown to be a phase-shifted Maxwellian distribution in shell radius space, and the number of shots was a Gaussian distribution in required-Magnus-frequency space.

6. Acknowledgements

The authors would like to thank Mr. Liam Doherty of Target Sports Canada in Gormley, Ontario for providing the shotgun and range facilities to carry out the experiment, as well as some useful input on the experimental setup.

References

[1] W. Donovan, "Retardation and Condition of Shot Pellets in Free Flight," 1967.

[2] N. Nag and A. Lahiri, "An Evaluation of Distribution of Pellets due to Shotgun Discharge," Forensic Science International, vol. 32, pp. 151-159, 20 May 1986.

[3] B. Kumral, E. Aslan and Y. Buyuk, "Ballistic Examination Findings of Sawed-Off Shotguns Referred to Council of Forensic Medicine, Istanbul, Turkey," Journal of Animal and Veterinary Advances, vol. 12, no. 10, pp. 977-981, 2013.

[4] K. Nag and P. Sinha, "An Investigation into Pellet Dispersion Ballistics," Forensic Science International, vol. 55, pp. 105-130, 1992.

[5] C. Bhattacharyya and P. Sengupta, "Shotgun Pellet Dispersion in a Maxwellian Model," Forensic Science International, vol. 41, no. 3, pp. 205-217, June 1989.

[6] F. A. Morrison, An Introduction to Fluid Mechanics, New York: Cambridge University Press, 2013, p. 625.

[7] K. Kadoya, N. Matsunaga and A. Nagashima, "Viscosity and Thermal Conductivity of Dry Air in the Gaseous Phase," J. Phys. Chem. Ref. Data, vol. 14, no. 4, pp. 947-970, 1985.

[8] A. Davidhazy, "Photographs by Andrew Davidhazy," 7 February 2009. [Online]. Available: https://people.rit.edu/andpph/exhibit-3.html. [Accessed 5 March 2016].