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Electroelastic Actuators for Nano- and Microdisplacement

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Abstract

In the general form for the equation of the electroelasticity the generalized structural-parametric model and the generalized matrix transfer function of the electroelastic actuator with the output parameters displacements are determined from the solutions of the wave equation with using the Laplace transform. The parametric structural schematic diagram and the transfer functions of the electroelastic actuator are obtained. The structural-parametric model of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are constructed. The dynamic and static characteristics of the piezoactuator with the output parameter displacement are obtained.

Keywords: Electroelastic actuator, piezoactuator, structural-parametric model, parametric structural schematic diagram, deformation, transfer functions

1. Introduction and statement of problem

The electroelastic actuator on the piezoeffect and the electrostriction effect is used for precise alignment in the adaptive optics and the nanotechnology [1 − 26]. The piezoactuator uses the inverse piezoeffect and serves for the actuation of mechanisms or the management and converts the electrical signals into the displacement and the force [1, 2, 3, 6]. The piezoactuator is applied for the drives of the scanning tunneling microscopes and the atomic force microscopes [14, 15, 16]. Let us consider the generalized structural-parametric model and the generalized parametric structural schematic diagram of the electroelastic actuator are constructed by solving the wave equation with the Laplace transform for the equation of the electromagnetolasticity, the boundary conditions on loaded working surfaces of the actuator, the strains along the coordinate axes. The transfer functions and the parametric structural schematic diagrams of the piezoactuator are obtained from the generalized structural-parametric model. In [6, 7, 8] was determined the solution of the wave equation of the piezoactuator. The structural-parametric model of the electroelastic actuator was determined in contrast electrical equivalent circuit types Cady and Mason for calculation of piezoelectric transmitter and receiver [9, 10, 11]. In the paper [12] presents the classic analytical two-port lumped-element model (LEM) types Cady and Mason of the piezoelectric composite circular plate with the output pressure. In the paper [13] considers the development of various lumped-element models as practical tools to design and manufacture the actuators with the output velocity. In the [14, 16, 21] were obtained the structural-parametric models, the schematic diagrams for simple piezoactuator and were transformed to the structural-parametric model of the electroelastic actuator. In [8, 18] was used the transfer functions of the piezoactuator for the decision problem absolute stability conditions of system controlling the deformation of the electroelastic actuator. The elastic compliances and the mechanical and adjusting characteristics of the piezoactuator were found in [19] for calculation its transfer functions and the structural-parametric models. The structural-parametric model of the multilayer and compound piezoactuator was determined in [19] with the output displacement. In this paper is solving the problem of building the generalized structural parametric model and the generalized parametric structural schematic diagram of the electroelastic actuator for the equation of the electroelasticity. The difference of this work from the papers [20, 21, 23, 26] is that the construction of the structure-parametric model of the electroelastic actuator is produced immediately in the general form, and not by the method of mathematical induction from the individual examples for the models of the piezoactuators.

2. Structural-parametric model and matrix transfer functions of electroelastic actuator

Let us consider the general structural-parametric model and the parametric structural schematic diagram of the electroelastic actuator with the output parameter displacement. For the electroelastic actuator are presented six stress components T_1, T_2, T_3, T_4, T_5, T_6, where the components T_1 - T_3 are related to extension-compression stresses, T_4 - T_6 to shear stresses. In the electroelastic actuator its deformation corresponds to stressed state.

For polarized piezoceramics PZT the matrix state equations [11, 14] with the electric and elastic variables have the form two equations, where the first equation represents the direct piezoeffect, the second equation describes the inverse piezoeffect

$$\mathbf{D} = \mathbf{dT} + \boldsymbol{\varepsilon}^T \mathbf{E} \tag{1}$$

$$\mathbf{S} = \mathbf{s}^E \mathbf{T} + \mathbf{d}^t \mathbf{E} \tag{2}$$

where $\mathbf{D}$ is the column matrix of electric induction; $\mathbf{S}$ is the column matrix of relative deformations; $\mathbf{T}$ is the column matrix of mechanical stresses; $\mathbf{E}$ is the column matrix of electric field strength; $\mathbf{s}^E$ is the elastic compliance matrix for $E = \text{const}$; $\boldsymbol{\varepsilon}^T$ is the matrix of dielectric constants for $T = \text{const}$; $\mathbf{d}^t$ is the transposed matrix of the piezoelectric modules.

Let us consider the electromagnetoelastic actuator. In general the equation of electromagnetoelasticity [6, 11] has following form

$$S_i = d_{mi} \Psi_m(t) + s_{ij}^{\Psi} T_j(x,t) \tag{3}$$

where $S_i = \partial \xi(x,t) / \partial x$ is the relative displacement along axis i of the cross section of the actuator, $\Psi_m = \{E_m, D_m$ is the control parameter E for the voltage control, D for the current control along axis m, T_j is the mechanical stress along axis j, d_{mi} is the coefficient of electroelasticity, for example, piezomodule, s_{ij}^{Ψ} is the elastic compliance for control parameter $\Psi = \text{const}$.

The piezoactuator (piezoplate) has the following properties: δ is the thickness, h is the height, b is the width, respectively $l = \{\delta, h, b$ the length of the piezoactuator for the longitudinal, transverse and shift piezoeffect. The direction of the polarization axis P, i.e., the direction along which polarization was performed, is usually taken as the direction of axis 3. The equation of the inverse piezoeffect for controlling voltage [6, 11] has the form

$$S_i = \partial\xi(x,t)/\partial x\ ,\ \ \Psi_m(t) = E_m(t) = U(t)/\delta$$

where S_i is the relative displacement of the cross section of the piezoactuator along axis *i*, $\xi(x,t)$ is the displacement of the section of the piezoactuator, d_{mi} is the piezomodule, $E_m(t)$ is the electric field strength along axis m, $U(t)$ is the voltage between the electrodes of actuator, s_{ij}^E is the elastic compliance for $E = \text{const}$, indexes *i*, *j* = 1, 2, … , 6; *m* = 1, 2, 3. The main size $l = \{\ \delta, h, b$ for the piezoactuator, respectively, the thickness, the height, the width for the longitudinal, transverse, shift piezoeffect.

For calculation of the electroelastic actuator is used the wave equation [6, 7, 11, 14] for the wave propagation in a long line with damping but without distortions. After Laplace transform is obtained the linear ordinary second-order differential equation with the parameter *p*, where the original problem for the partial differential equation of hyperbolic type using the Laplace transform is reduced to the simpler problem [6, 14, 21] for the linear ordinary differential equation

$$\frac{d^2\Xi(x,p)}{dx^2} - \gamma^2\Xi(x,p) = 0 \tag{4}$$

with its solution

$$\Xi(x,p) = Ce^{-x\gamma} + Be^{x\gamma} \tag{5}$$

where $\Xi(x,p)$ is the Laplace transform of the displacement of the section of the electroelastic actuator, $\gamma = p/c^{\Psi} + \alpha$ is the propagation coefficient, c^{Ψ} is the sound speed for $\Psi = \text{const}$, α is the damping coefficient.

The constants *C* and *B* of the solution the linear ordinary second-order differential equation [7] are determined from the boundary conditions for the electroelastic actuator

$$\Xi(0,p) = \Xi_1(p) \text{ for } x = 0 \tag{6}$$

$$\Xi(l,p) = \Xi_2(p) \text{ for } x = l$$

whence we obtain the constants in the form

$$C = \frac{\Xi_1 e^{l\gamma} - \Xi_2}{2\text{sh}(l\gamma)}\ ,\ \ B = \frac{\Xi_2 - \Xi_1 e^{-l\gamma}}{2\text{sh}(l\gamma)}$$

Therefore, the solution (5) can be written in following form

$$\Xi(x,p)=\frac{\Xi_1(p)\mathrm{sh}[(l-x)\gamma]+\Xi_2(p)\mathrm{sh}(x\gamma)}{\mathrm{sh}(l\gamma)} \tag{7}$$

The equations for the forces on the faces of the electroelastic actuator

$$T_j(0,p)S_0 = F_1(p)+M_1p^2\Xi_1(p) \text{ for } x=0, \tag{8}$$

$$T_j(l,p))S_0 = -F_2(p)-M_2p^2\Xi_1(p) \text{ for } x=l,$$

where $T_j(0,p)$ and $T_j(l,p)$ are the mechanical stress.

From equations of forces acting on the faces of the electroelastic actuator we obtain the boundary conditions on loaded surfaces

$$T_j(0,p)=\frac{1}{s_{ij}^{\Psi}}\frac{d\Xi(x,p)}{dx}\bigg|_{x=0}-\frac{d_{mi}}{s_{ij}^{\Psi}}\Psi_m(p) \tag{9}$$

$$T_j(l,p)=\frac{1}{s_{ij}^{\Psi}}\frac{d\Xi(x,p)}{dx}\bigg|_{x=h}-\frac{d_{mi}}{s_{ij}^{\Psi}}\Psi_m(p)$$

where S_0 is the cross section area and M_1, M_2 are the displaced mass on the faces of the electroelastic actuator.

From (4), the boundary conditions on loaded surfaces (5), the strains along the axes the system of equations for the generalized structural-parametric model and the generalized parametric structural schematic diagram are determined for Figure 1 of the actuator with the output parameters the Laplace transform for the displacements $\Xi_1(p)$, $\Xi_2(p)$ for the faces of the electroelastic actuator in the form

$$\Xi_1(p)=\left(\frac{1}{M_1p^2}\right)\left\{-F_1(p)+\left(\frac{1}{\chi_{ij}^{\Psi}}\right)\begin{bmatrix}d_{mi}\Psi_m(p)-\\-\left(\frac{\gamma}{\mathrm{sh}(l\gamma)}\right)[\mathrm{ch}(l\gamma)\Xi_1(p)-\Xi_2(p)]\end{bmatrix}\right\}$$

$$\tag{10}$$

$$\Xi_2(p)=\left(\frac{1}{M_2p^2}\right)\left\{-F_2(p)+\left(\frac{1}{\chi_{ij}^{\Psi}}\right)\begin{bmatrix}d_{mi}\Psi_m(p)-\\-\left(\frac{\gamma}{\mathrm{sh}(l\gamma)}\right)[\mathrm{ch}(l\gamma)\Xi_2(p)-\Xi_1(p)]\end{bmatrix}\right\}$$

where $\chi_{ij}^{\Psi}=\frac{s_{ij}^{\Psi}}{S_0}$ $d_{mi}=\begin{cases}d_{33},d_{31},d_{15}\\g_{33},g_{31},g_{15}\end{cases}$, $\Psi_m=\begin{cases}E_3,E_3,E_1\\D_3,D_3,D_1\end{cases}$, $s_{ij}^{\Psi}=\begin{cases}s_{33}^E,s_{11}^E,s_{55}^E\\s_{33}^D,s_{11}^D,s_{55}^D\end{cases}$, $l=\{\delta,h,b$, $c^{\Psi}=\{c^E,c^D$, $\gamma^{\Psi}=\{\gamma^E,\gamma^D$, d_{mi} is the coefficient of the electroelasticity (piezomodule), $F_1(p)$, $F_2(p)$ are the Laplace transform of the forces on the faces. Figure 1 shows the generalized parametric structural schematic diagram of the electroelastic actuator corresponding to the set of equations (10) for the Laplace transform of the displacements of the faces. The generalized transfer functions of the of the electroelastic actuator are the ratio of the Laplace transform of the displacement of the face actuator and the Laplace transform of the corresponding control parameter or the force at zero initial conditions.

From (10) the generalized matrix equation of the transfer functions of electroelastic actuator has the form

$$\begin{pmatrix}\Xi_1(p)\\ \Xi_2(p)\end{pmatrix}=\begin{pmatrix}W_{11}(p) & W_{12}(p) & W_{13}(p)\\ W_{21}(p) & W_{22}(p) & W_{23}(p)\end{pmatrix}\begin{pmatrix}\Psi_m(p)\\ F_1(p)\\ F_2(p)\end{pmatrix} \tag{11}$$

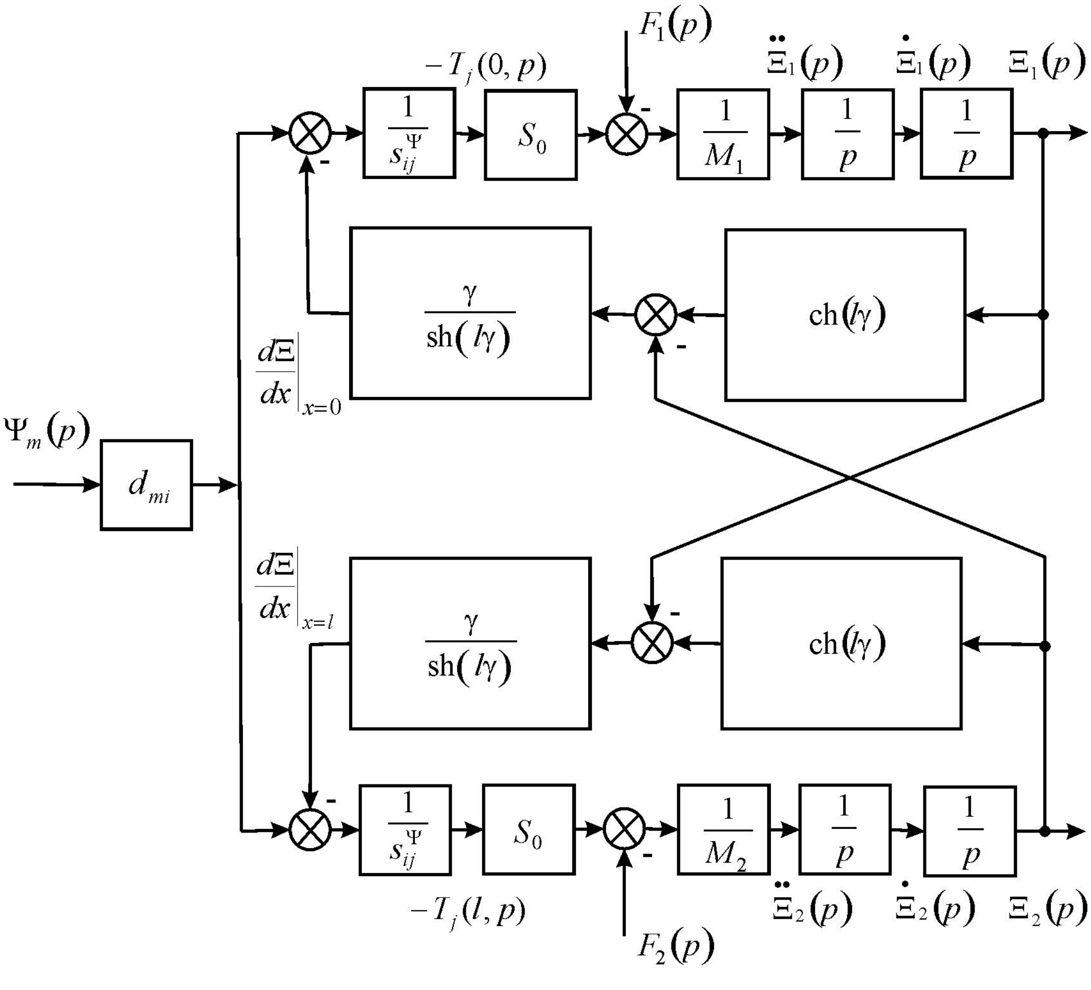

Figure 1. Generalized parametric structural schematic diagram of the of the electroelastic actuator

Let us consider the static displacements of the faces for the electroelastic actuator with the output parameter displacement. For $m \ll M_1$ and $m \ll M_2$ the static displacements of the faces of the piezoactuator for the transverse piezoeffect are obtained from (11) in the form

$$\xi_1(\infty)=\lim_{\substack{p\to 0\\ \alpha\to 0}}\frac{pW_{11}(p)U_0}{\delta p}=\frac{d_{31}hU_0M_2}{\delta(M_1+M_2)} \qquad (12)$$

$$\xi_2(\infty)=\lim_{\substack{p\to 0\\ \alpha\to 0}}\frac{pW_{21}(p)U_0}{\delta p}=\frac{d_{31}hU_0M_1}{\delta(M_1+M_2)} \qquad (13)$$

For the piezoactuator from PZT under the transverse piezoeffect at $m<<M_1$ and $m<<M_2$, $d_{31}=2.5\cdot 10^{-10}$ m/V, $h/\delta=20$, $U=150$ V, $M_1=1$ kg and $M_2=4$ kg, the static displacements of the faces are determined $\xi_1(\infty)=600$ nm, $\xi_2(\infty)=150$ nm, $\xi_1(\infty)+\xi_2(\infty)=750$ nm.

Let us consider the dynamic displacement of the piezoactuator under the transverse piezoeffect with one fixed face. Using equation (11) for unloaded piezoactuator under the transverse piezoelectric effect, we write the expression for the transfer function at $M_1\to\infty$ and $M_2=0$ in the form

$$W_{21}(p)=\Xi_2(p)/E_3(p)=d_{31}/[\gamma\mathrm{cth}(h\gamma)]. \qquad (14)$$

We write the resonance condition for the piezoactuator under the transverse piezoeffect with one fixed face

$$\mathrm{ctg}\left(\omega h/c^E\right)=0 \qquad (15)$$

This means that the piezoactuator is the quarter-wave vibrator with the resonance frequency

$$f_r=c^E/(4h)\ . \qquad (16)$$

Let us consider the resonance condition of the piezoactuator from PZT under the transverse piezoeffect at $c^E==3\cdot 10^3$ m/s and $h=1.8\cdot 10^{-2}$ m, the resonance frequency is determined $f_r=60$ kHz.

For the approximation of the hyperbolic cotangent by two terms of the power series in transfer function (11) the following expressions of the transfer function of the piezoactuator with one fixed face is obtained for the elastic-inertial load at $M_1\to\infty$, $m<<M_2$ under the transverse piezoeffect and control voltage (Figure 2) for the resistance $R=0$ of the voltage source in the form

$$W(p)=\frac{\Xi_2(p)}{U(p)}=\frac{d_{31}\,h/\delta}{\left(1+C_e/C_{11}^E\right)\left(T_t^2p^2+2T_t\xi_tp+1\right)} \qquad (17)$$

$$T_t=\sqrt{M_2/\left(C_e+C_{11}^E\right)},\ \xi_t=\alpha h^2C_{11}^E\Big/\left(3c^E\sqrt{M\left(C_e+C_{11}^E\right)}\right)$$

where $U(p)$ is the Laplace transform of the voltage, T_t is the time constant and ξ_t is the damping coefficient of the piezoactuator.

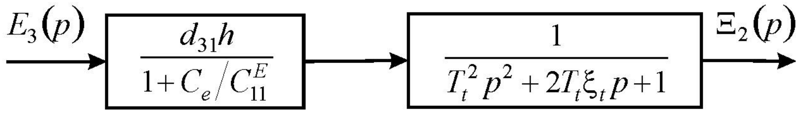

Figure 2. Parametric structural schematic diagram of the voltage-controlled piezoactuator with one fixed face under transverse piezoeffect for elastic-inertial load

Let us consider the transient response of the electroelastic actuator with the output parameter displacement. The expression for the transient response of the voltage-controlled piezoactuator for the elastic-inertial load under the transverse piezoeffect is determined

$$\xi(t) = \xi_m \left[1 - \frac{e^{-\frac{\xi_t t}{T_t}}}{\sqrt{1-\xi_t^2}} \sin(\omega_t t + \varphi_t) \right] \tag{18}$$

$$\xi_m = \frac{d_{31}(h/\delta)U_m}{1+C_e/C_{11}^E}, \quad \omega_t = \frac{\sqrt{1-\xi_t^2}}{T_t}, \quad \varphi_t = \text{arctg}\left(\frac{\sqrt{1-\xi_t^2}}{\xi_t} \right)$$

where ξ_m is the steady-state value of displacement of the piezoactuator, U_m is the amplitude of the voltage. For the voltage-controlled piezoactuator from the piezoceramics PZT under the transverse piezoelectric effect for the elastic-inertial load $M_1 \to \infty$, $m << M_2$ and input voltage with amplitude $U_m = 125$ V at $d_{31} = 2.5 \cdot 10^{-10}$ m/V, $h/\delta = 20$, $M_2 = 4$ kg, $C_{11}^E = 2 \cdot 10^7$ N/m, $C_e = 0.5 \cdot 10^7$ N/m are obtained values $\xi_m = 500$ nm, $T_t = 0.4 \cdot 10^{-3}$ c.

For calculations control systems with the electroelastic actuator the parametric structural schematic diagrams and the transfer functions of the electroelastic actuator are obtained

3. Results and Discussions

The electroelastic actuator solves problem of precise matching, compensation of temperature and gravitational deformations. The structural-parametric model and parametric structural schematic diagrams of the voltage-controlled piezoactuator for the longitudinal, transverse and shift piezoelectric effects are determined from the generalized structural-parametric model of the electroelastic actuator with the replacement of the generalized parameters on the parameters of the piezoactuator.

The transfer functions in matrix form are describe deformations of the electroelastic actuator. The generalized structural-parametric model, the generalized parametric structural schematic diagram and the matrix equation of the electroelastic actuator with the output parameters displacements are obtained from the solutions of the wave equation with the Laplace transform and from its deformations along the coordinate axes. From the generalized matrix equation for the transfer functions of the electroelastic actuator after algebraic transformations are constructed the matrix equations of the piezoactuator for the longitudinal, transverse and shift piezoelectric effects.

4. Conclusions

The generalized structural-parametric model, the generalized parametric structural schematic diagram, the matrix equation of the electroelastic actuator with the output parameters displacements are obtained in the general form.

The structural-parametric model, the matrix equation and the parametric structural schematic diagram of the piezoactuator for the transverse, longitudinal, shift piezoelectric effects are determined from the generalized structural-parametric model of the electroelastic actuator.

From the solution of the wave equation, from the equation of the electroelasticity and the deformations along the coordinate axes the generalized structural-parametric model and the generalized parametric structural schematic diagram of the electroelastic actuator with the output parameters displacements are constructed. The deformations of the electroelastic actuator are described by the matrix equation for the transfer functions of the actuator.

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