SCIREA Journal of Physics

http://www.scirea.org/journal/Physics

April 9, 2019

Volume 4, Issue 2, April 2019

Propagation of Shock Waves in a Dusty Plasma with Kappa Distributed Electrons and Ions

Louis Ete Akpabio[1,*] and Sekanam Louis Akpabio[2]

[1]Theoretical Physics Group, Department of Physics, University of Uyo, Uyo, Nigeria.

[2]Department of Mechanical Engineering, University of Uyo, Uyo , Nigeria

*email: louisakpabio@uniuyo.edu.ng

ABSTRACT

The Burgers' equation was derived for studying the behaviour of Dust Acoustic (DA) shock wave for a four-component dusty plasma comprising charged dust grains of opposite polarity, super thermally (Kappa) distributed electrons and ions. The numerical solution of Burgers' equation is analyzed. It is established that opposite polarity shock wave potentials exist in such dusty plasma. The properties of the dust acoustic shock wave are studied taking into account the spectral index (K) effect. We have been able to establish the fact that, the polarity of the shock waves amplitude varies from positive to negative at a particular K value.

Keywords: Dusty Plasma; Kappa Distribution; Shock Waves; DA Wave.

1. Introduction

In recent years, much attention has been given to dusty plasmas propagation due to its considerable importance in many aspects of space environments, such as: cometary environments, asteroid zones, planetary rings, interstellar medium and lower ionosphere [1-6].

Noticeable applications of dusty plasmas are also created in laboratory devices [7-9]. More recently, research in dusty plasmas has expanded into a wider range of problems including studies of collective process; this is wave and instabilities [10 – 13]. The presence of charge dust grains does not only affect existing plasmas wave spectra [14-15], it also brings about new novel eigen modes such as Dust Acoustic wave [15, 7], Dust-Ion-Acoustic (DIA) wave [17, 7] etc.

From the first theoretical study by Rao et al [16] on the ultra low frequency DA waves couple with the experimental observations of these waves [7, 12], numerous investigation were made to study the different aspects of the physics of dusty plasmas during the past few years. However, most of the investigations were done on plasmas with dust grains of negative polarity [18 – 20], in this regard; nonlinear solutions of dusty plasmas likewise double layers were studied by some authors [21 – 22]. Whereas, in the space plasmas environments; some plasma systems are found with dust grains of positive polarity [23 – 26]. Processes such as radiative heating that produces thermionic emission, secondary emission of electrons from the surface of the dust grains and irradiation by ultraviolet (UV) light leads to such dust grains with net positive charge [11, 27]. Recently, Mandal and Chatterjee [28] studied dusty plasmas considering the presence of opposite polarity of dust grains in the plasma. They establish the coexistence of opposite polarity shock potentials for electrons and ions. Multi-component dusty plasma containing dust grains of opposite polarity and ions were investigated by Armina et al [29] for the occurrence of shock structure and the formation of shock waves. The study of such shock waves was also presented by Akpabio et al [30] and Akpabio and Ikot [31] recently for dusty plasma. They generalized the work of Paul et al [32] to include the roles of non-thermal ions and electrons. Recently, Sahu [33] established the significant role of the nonplanar geometry effects in adiabatic dusty plasma for the occurrence of shock waves.

It is seen from the above discussions that, almost all the research articles on DA waves are composed of Maxwell – Boltzmann and nonthermal electrons and ions distribution. However, employing Maxwell distribution for describing the long-range interactions in unmagnetized collisionless plasma may be inadequate if the non-equilibrium state exists. Such particle velocity distributions that depart from being Maxwellian are better modeled by the generalized Lorengian or Kappa distribution [34] with functional dependence of the form $f_0(v) \simeq \left[1 + V^2/(k\theta)\right]^{-(k+1)}$. The K value determines the energy spectrum slope of the superthermal electrons forming the tail of the velocity distribution function. Several observations in astrophysical plasmas such as: auroral zone plasma, solar wind, magnetosphere [35 – 43] indicate vividly the occurrence of superthermal or non-maxwellian distribution. For K tending to infinity, the Kappa distribution approaches a Maxwellian distribution $\left[\sim \exp\left(-V^2/\theta^2\right)\right]$. Recently, Kundu et al [44], investigated the nonlinear propagation of waves with opposite dust polarity , where electrons are superthermally distributed by using the standard reductive perturbation method [45] in deriving the Burgers' equation. They presented how the amplitude of shock waves are significantly affected by changes in K. The aim of our study is to determine how electrons and ions superthermality will modify the result of Kundu et al [44].

2. Derivation of Burgers' Equations

We consider the nonlinear propagation of DA waves in four component collisionless, unmagnetized dusty plasma whose constituents are positively and negatively charged dust grains, superthermal electrons and ions. The charge neutrality condition in equilibrium is $n_{i0} + z_2 n_{20} = n_{e0} + z_1 n_{10}$ where n_{i0}, n_{20} (n_{10}) and n_{e0} are the ion, positively (negatively) charged dust and electron respectively. The nonlinear dynamics of DA waves, in such dusty plasma is described by the following set of normalized equations:

$$\partial_T N_1 + \partial_x (N_1 U_1) = 0 \tag{1}$$

$$\partial_T U_1 + U_1 \partial_x U_1 = \partial_x \varphi + \eta_1 \partial_x^2 U_1 \tag{2}$$

$$\partial_T N_2 + \partial_x (N_2 U_2) = 0 \tag{3}$$

$$\partial_T U_2 + U_2 \partial_x U_2 = -\alpha\beta \partial_x \varphi + \eta_2 \partial_x^2 U_2 \tag{4}$$

$$N_1 = (1 + \mu_e - \mu_i) N_2 - \mu_e \left[1 - \frac{\sigma\varphi}{k - \frac{1}{2}}\right]^{-k-\frac{1}{2}} + \mu_i \left[1 + \frac{\varphi}{k - \frac{1}{2}}\right]^{-k+\frac{1}{2}} \tag{5}$$

Where $N_1(N_2)$; is the normalized negative and positive charged dust by their equilibrium value $n_{10}(n_{20}), u_1(u_2)$ is the negative and positive dust fluid velocity normalized by DA speed $C_1 = (z_1 k_B T_i / m_1)^{\frac{1}{2}}$, $\eta_1(\eta_2)$ is the viscosity coefficient of negative (positive) dust fluid normalized by $w_{pd}\lambda_p^2$ and ϕ is the electrostatic wave potential normalized by

$K_B T_i/e$, $\alpha = z_2/z_1$, $\beta = m_1/m_2$, $\mu_e = n_{e0}/z_1 n_{i0}$, $\mu_i = n_{i0}/z_1 n_{i0}$, $\sigma = T_i/T_e$, where $z_1(z_2)$ is the number of electrons (protons) residing on a negative (positive) dust particles, $m_1(m_2)$ is the mass of negative (positive) dust particle and $T_i(T_e)$ is the ion and electron temperature. k_B is the Boltzmann constant. The time and space variables are in units of dust plasma period $w_{pd}^{-1} = \left(m_1/4\pi\ z_1^2 e^2 n_{i0}\right)^{\frac{1}{2}}$ and the Debye length $\lambda_D = \left(z_1 k_B T_i/4\pi z_1^2 e^2 n_{i0}\right)^{\frac{1}{2}}$. Therefore, the normalized number densities of electrons and ions are according expressed as

$$N_e = \mu_e \left[1 - \frac{\sigma\phi}{k - \frac{1}{2}}\right]^{-k-\frac{1}{2}} \tag{6}$$

and

$$N_i = \mu_i \left[1 + \frac{\phi}{k - \frac{1}{2}}\right]^{-k+\frac{1}{2}} \tag{7}$$

respectively; where k is a real parameter measuring deviation from Maxwellian equilibrium (recovered for k infinite). To study dust acoustic shock wave in the dusty plasma using equations (1-5); we employ the standard reductive perturbation technique, by first introducing the stretched variables in in space and time [45] as

$$\xi = \in^{\frac{1}{2}} \left(X - V_0 T\right),\ \ \tau = \in^{\frac{3}{2}} T, \tag{8}$$

Where $\in$ is a smallness parameter $(0 < \in < 1)$ measuring the amplitude of perturbation and V_0 (normalized by C_1) is the phase speed of the perturbation mode. The perturbed quantities are expanded in power series of $\in$ as follows:

$$\left.\begin{aligned} N_1 &= 1 + \in N_1^{(1)} + \in^2\ N_1^{(2)} + \cdots \\ U_1 &= 0 + \in U_1^{(1)} + \in^2\ U_1^{(2)} + \cdots \\ N_2 &= 1 + \in N_2^{(2)} + \in^2\ N_2^{(2)} + \cdots \\ U_2 &= 0 + \in U_2^{(2)} + \in^2\ U_2^{(2)} + \cdots \\ \phi &= 0 + \in \phi^{(1)} + \in^2\ \phi^{(2)} + \cdots \end{aligned}\right\} \tag{9}$$

Now, expressing (1) – (5) in terms of ξ and τ, and substituting equation (9) into the resulting equation, we develop different sets of equations in the form of a power series of $\in$. Considering the lowest order terms in $\in$, we obtain

$$V_0^2 = \frac{(k-1/2)[1+\alpha\beta(1+\mu_e-\mu_i)]}{[\sigma\mu_e(k+1/2)+\mu_i(k-1/2)]} \qquad (11)$$

The linear dispersion relation in the considered dusty plasma is represented by equation (11). The next higher order in $\in$, gives to the following sets of equations:

$$\partial_\tau N_1^{(1)} - V_0\partial_\xi N_1^{(2)} + \partial_\xi U_1^{(2)} + \partial_\xi\left(N_1^{(1)}U_1^{(1)}\right) = 0 \qquad (12)$$

$$\partial_\tau U_1^{(1)} - V_0\partial_\xi U_1^{(2)} + U_1^{(1)}\partial_\xi U_1^{(1)} = \partial_\xi\varphi^{(2)} + \eta_{10}\partial_\xi^2 U_1^{(1)} \qquad (13)$$

$$\partial_\tau N_2^{(1)} - V_0\partial_\xi N_2^{(1)} + \partial_\xi U_2^{(2)} + \partial_\xi\left(N_2^{(1)}U_2^{(1)}\right) = 0 \qquad (14)$$

$$\partial_\tau U_2^{(1)} - V_0\partial_\xi U_2^{(2)} + U_2^{(1)}\partial_\xi U_2^{(1)} = -\alpha\beta\partial_\xi\varphi^{(2)} + \eta_{20}\partial_\xi^2 U_2^{(2)} \qquad (15)$$

$$N_1^{(2)} = (1+\mu_e-\mu_i)N_2^{(2)} - \mu_e\left[\frac{\sigma(k+1/2)\phi^{(2)}}{k-1/2} + \frac{\sigma^2(k+1/2)(k+3/2)}{2(k-1/2)^2}\left(\phi^{(1)}\right)^2\right] -$$

$$\mu_i\left[\varphi^{(2)} - \frac{(k+3/2)}{2(k-1/2)}\left(\varphi^{(1)}\right)^2\right] \qquad (16)$$

where $\eta_1 = \in^{1/2}\ \eta_{10}$ and $\eta_2 = \in^{1/2}\ \eta_{20}$ are assumed. Eliminating $N_1^{(2)}$, $N_2^{(2)}$, $U_1^{(2)}$, $U_2^{(2)}$ and $\phi^{(2)}$ from equation (12) – (16) by combining equations (10) – (11), we finally deduce a nonlinear equation of the form:

$$\partial_\tau\varphi^{(1)} + A\varphi^{(1)}\partial_\xi\varphi^{(1)} = B\partial_\xi^2\varphi^{(1)} \qquad (17)$$

where A and B are nonlinear and dissipation coefficient respectively, given as

$$A = \Big\{ (k-1/2)^2\left[3\alpha^2\beta^2(1+\mu_e-\mu_i)-3\right] -$$

$$V_0^4\ \left[\mu_e\sigma^2(k+1/2)(k+3/2)+\mu_i(k-1/2)(k-3/2)\right]\Big\}$$

$$\times\left\{2V_0(k-1/2)^2\left[1+\alpha\beta(1+\mu_e-\mu_i)\right]\right\}^{-1} \qquad (18)$$

and

$$B = \frac{\eta_{10}+\eta_{20}\alpha\beta(1+\mu_e-\mu_i)}{2\left[1+\alpha\beta(1+\mu_e-\mu_i)\right]}. \qquad (19)$$

Equation (17) is called Burgers' equation which is use in describing the nonlinear propagation of DA shock wave in a multi component dusty plasma system.

3. **Discussions and Numerical Results**

Now we numerically solve equation (17) the Burgers' equation. For this, we first analyze the stationary shock wave solution of equation (17) by transforming the independent variable ξ and τ to $\xi = \eta - U_0\tau$ and $\tau = \tau$ where U_0 is a constant velocity normalized by C_i, them the appropriate boundary condition is imposed as

$$\phi^{(1)} \to 0 \ \text{ and } \ \frac{\partial \phi^{(1)}}{\partial \eta} \to 0 \ \text{ at } \ \eta \to \infty .$$

The stationary solution of Burgers' equation will yield the following expression:

$$\phi^{(1)} = \phi_m^{(1)}\left[1 - \tanh\left(\frac{\eta}{\Delta}\right)\right] \tag{20}$$

where

$$\phi_m = \frac{U_0}{A} \tag{21}$$

$$\Delta = \frac{2B}{U_0} \tag{22}$$

Equation 20 represents a monotonic shock-like soliton whose shock potential profile is positive (negative) when A is positive (negative). It can be seen from equation (21) and (22) that the amplitude (width) increases (decreases) as U_0 increases. The nonlinear coefficient A from the Burgers' equation as obtained by Kundu et al [44] reads.

$$\begin{aligned} A = &\{(k-1/2)^2\left[3\alpha^2\beta^2(1+\mu_e-\mu_i)-3\right]- \\ &V_0^2\left[\sigma^2\mu_e(k+1/2)(k+3/2)-\mu_i(k-1/2)^2\right]\}\times \\ &\{2V_0(k-1/2)^2\left[1+\alpha\beta(1+\mu_e-\mu_i)\right]\}^{-1}, \end{aligned} \tag{23}$$

while that obtained by us and Mandal and Chatterjee [28] are given respectively as

$$\begin{aligned} A = &\left\{(k-1/2)^2\left[3\alpha^2\beta^2(1+\mu_e-\mu_i)-3\right] - \right. \\ &\left. V_0^4\left[\sigma^2\mu_e(k+1/2)(k+3/2)+\mu_i(k-1/2)(k-3/2)\right]\right\}\times \\ &\{2V_0(k-1/2)^2\left[1+\alpha\beta(1+\mu_e-\mu_i)\right]\}^{-1} \end{aligned} \tag{24}$$

and

$$\begin{aligned} A = &\{3\alpha^2\beta^2(1+\mu_e-\mu_i)-3-V_0^4(\sigma^2\mu_e-2\mu_i\beta)\}\times \\ &\{2V_0\left[1+\alpha\beta(1+\mu_e-\mu_i)\right]\}^{-1} \end{aligned} \tag{25}$$

We strongly feel that the proper expression for the coefficient V_0 in the third term of the given nonlinear coefficient A equations (23) – (25) should be V_0^4 and not V_0^2 as presented by Kundu et al [44].

In Fig. 1, we present shock wave potential $\phi^{(1)}$ for different *k* values. Figure 1 shows shock potential profile that is negative and the potential increases with increase in *k*. Figure 2 show how the positive shock potential vary with μ_i. It is obvious that the developed shock potential profiles are almost similar as μ_i varies. Figures 3, 4 and 5 presents negative shock potentials which varies differently with values of β, σ and α respectively. Figure 3, presents the shock potential profile increasing with increase β. In figure 4, the potential profile decreases as σ increases; while, the potential of figure 5 increases as α increases.

Figure 6 indicates the behaviour of the amplitude (ϕ_m) of shock waves with *k* for different values of μ_i. The figure shows that, the amplitude of the wave's increases as μ_i increases. Variations in the amplitude (ϕ_m) of the wave with different values of μ_e are presented in Figure 7. Figure 7 also shows that, the sign of the amplitude changes at a particular value of *k* equal 1.2 for $\mu_e = 0.1$. Generally, the amplitude decreases as μ_e increases. Also, the behaviour of (ϕ_m) is plotted in figure 8 against several values of σ. It is showed that, the amplitude profile increases as σ increases. The changes in shock waves width against different values of μ_e, α and β according to the chosen parameters are plotted in figures 9, 10 and 11 respectively. It is quite clear from figures 9, 10 and 11 that the width of shock waves always increases as the function μ_i increases, while high values of μ_e, α and β gives shorter width of the shock wave respectively.

4. **Conclusion**

Theoretical model to show shock waves propagation (nonlinear) unmagnetized dusty plasma containing mobile positive and negative charged dust, high energy-tail electrons and ions distribution have been presented. We found that the Burgers' equation describes the evolution of shock wave in the unmagnetized dusty plasma system. We have studied the effect of coexistence of super thermal distribution for ions and electrons and the following results have been noticed in this theoretical investigation:

(i) Both negative and positive shock potentials are presented.

(ii) The dust acoustic shock wave amplitude profile are negative and they increase with increasing μ_i and σ, while in the other hand, the amplitude decreases with increase μ_e.

(iii) The dust acoustic shock wave amplitude changes polarity from positive to negative at a particular value of *k=1.2.*

(iv) The of dust acoustic shock waves width decreases with increasing μ_e, α and β.

The presentation of opposite polarity potential in our investigation, affirms the assertion that, the negative (positive) shock potential may trap positively (negatively) dust particles which can attract dust particles of opposite polarity to form larger sized dust or coagulate into extremely large sized neutral dust in cometary environments, upper mesosphere, Jupiter's magnetosphere and even in laboratory experiments. Our present investigations have shown how the basic features of dust acoustic shock waves (nonlinear) are modified by the presence of high energy-tail electrons and ions distribution in dusty plasma. The results of this paper would be useful in understanding nonlinear features of dust acoustic waves propagating in different region of space (viz. cometary environments [46], solar wind, auroral zone, mesosphere, magnetosphere [25, 26, 47, 48]) and the laboratory.

Figures and tables

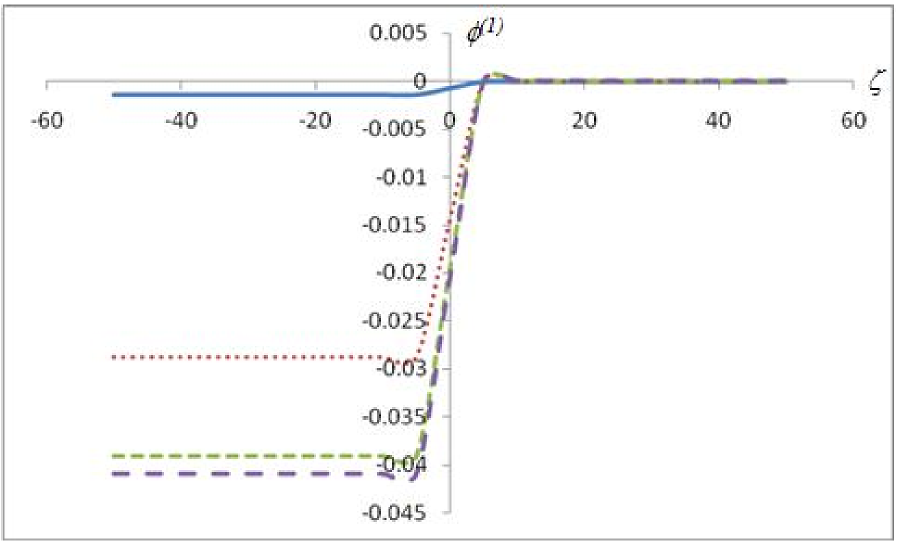


Figure 1. Variation of $\phi^{(1)}$ with ζ, where α = 0.01, β = 500, σ = 0.5, μ_i = 0.5, μ_e = 0.3, η_{10} = 0.1, η_{20} = 0.1, V_o = 10, κ = 0.6 (solid line), κ = 1.0 (dotted line), κ = 1.3 (short dashed line), κ = 1.5 (long dashed line).

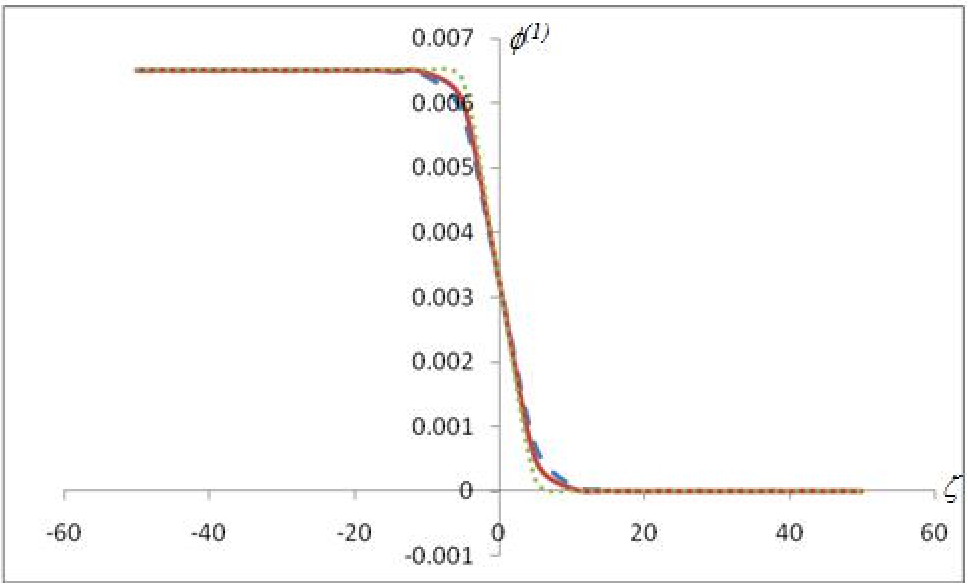


Figure 2. Variation of $\phi^{(1)}$ with ζ , where κ = 1.2 and other values of the given parameters are the same as those in Figure 1, μ_i = 0.2 (dashed line), μ_i = 0.3 (solid line), μ_i = 1.5 (dotted line).

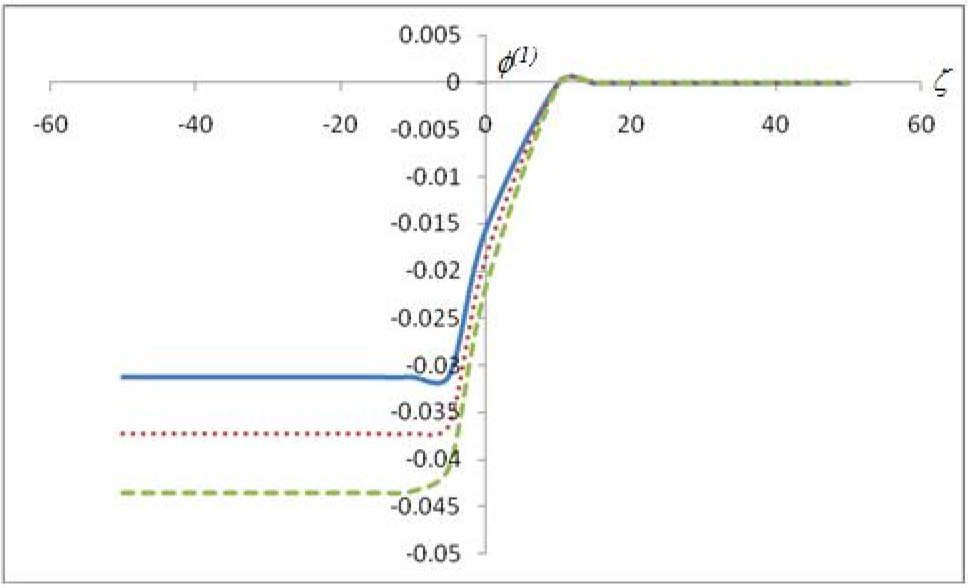


Figure 3. Variation of $\phi^{(1)}$ with ζ , where κ = 1.2, μ_i = 0.5 and other values of the given parameters are the same as those in Figure 1 , β = 400 (solid line), β = 500 (dotted line), β = 600 (dashed line).

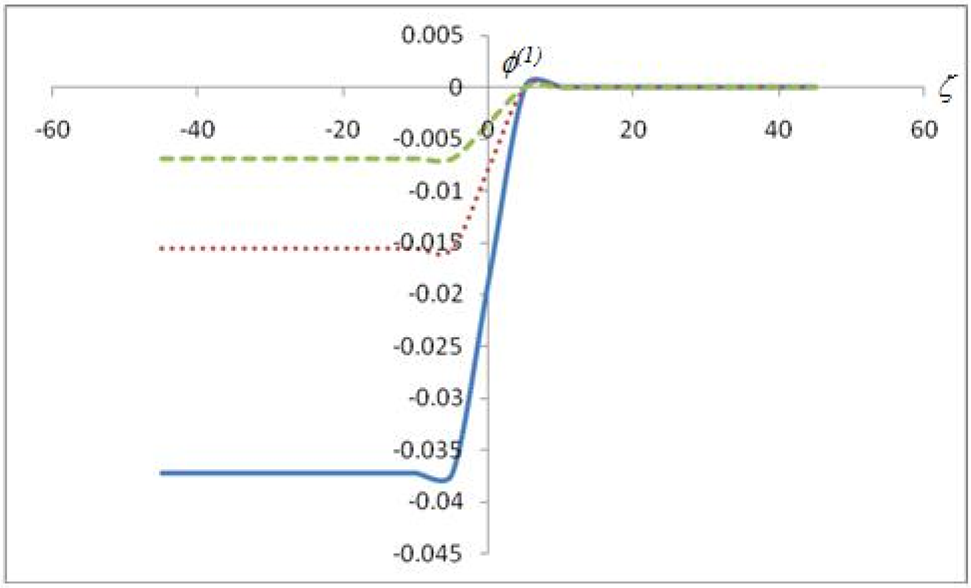


Figure 4. Variation of $\phi^{(1)}$ with ζ, where $\kappa = 1.2$ and other values of the given parameters are the same as those in Figure 1, $\sigma = 0.5$ (solid line), $\sigma = 0.7$ (dotted line), $\sigma = 1$ (dashed line).,

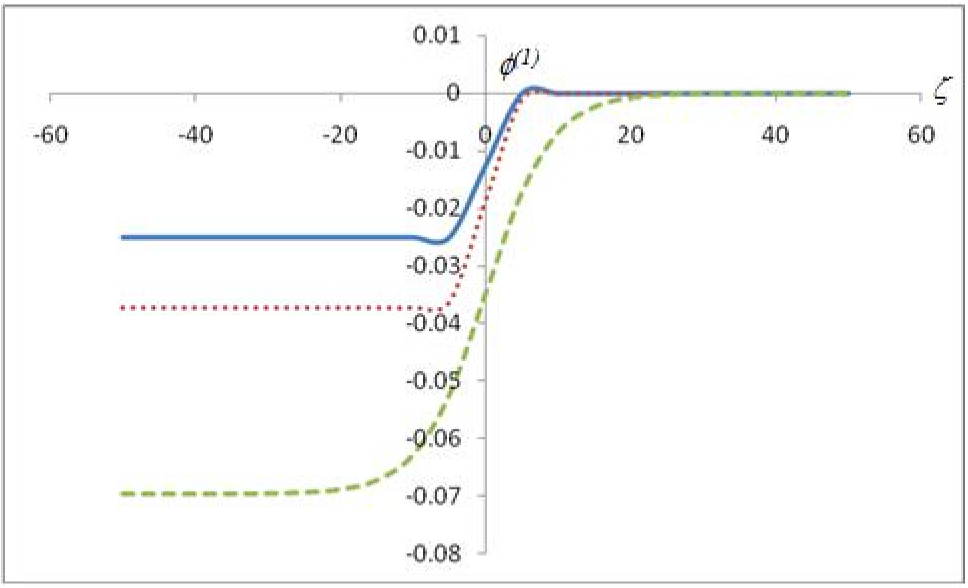
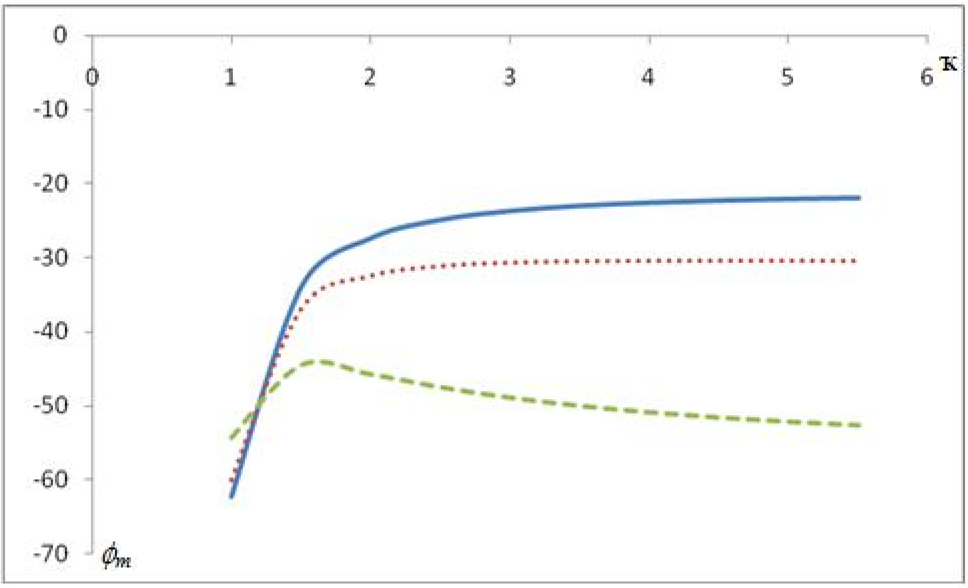
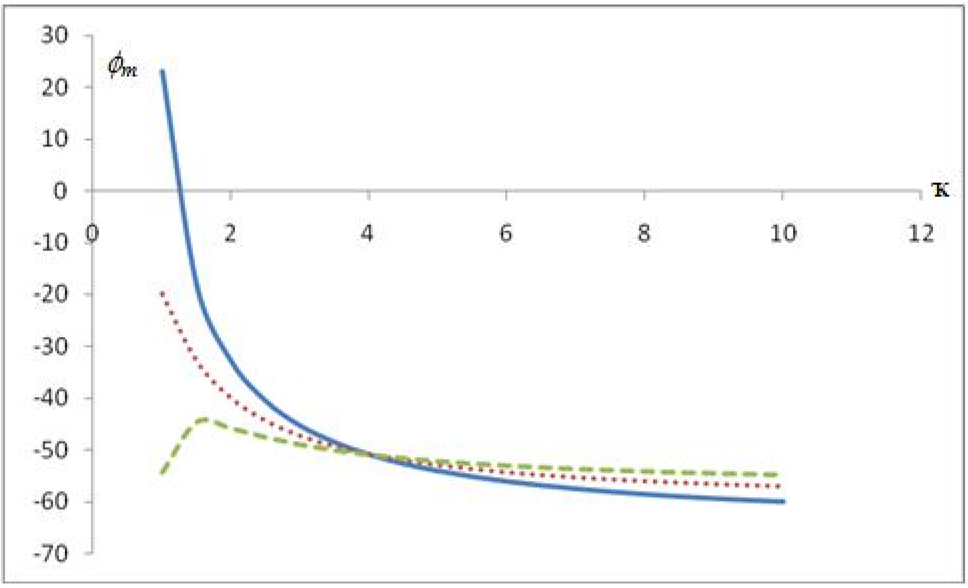


Figure 5. Variation of $\phi^{(1)}$ with ζ, where $\kappa = 1.2$ and other values of the given parameters are the same as those in Figure 1, $\alpha = 0.006$ (solid line), $\alpha = 0.01$ (dotted line), $\alpha = 0.02$ (dashed line).,



Figre 6. Variation of ϕ_m with κ as , μ_i = 0.2 (solid line), μ_i = 0.3 (dotted line) , μ_i = 0.5 (dashed lined).where α =0.01, β = 500 , μ_e = 0.3 , and σ = 0.5.



Figre 7. Variation of ϕ_m with κ as, μ_e = 0.1 (solid line), μ_e = 0.2 (dotted line) , μ_e = 0.3 (dashed lined).where α =0.01, β = 500 , μ_i = 0.5 and σ = 0.5.

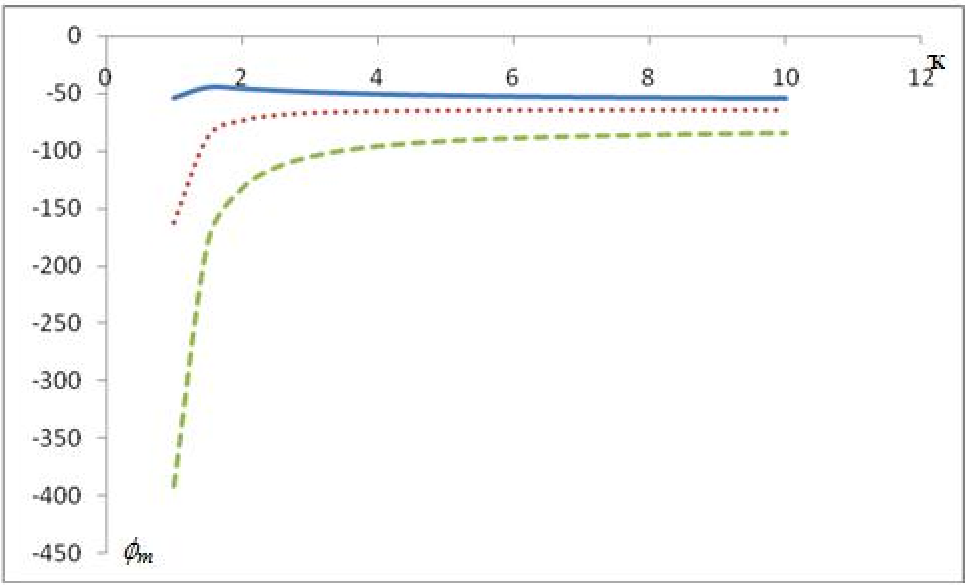


Fig. 8. Variation of ϕ_m with κ as , σ= 0.5 (solid line), σ= 0.7 (dotted line) , σ= 1 (dashed lined).where α =0.01, β = 500 , μ_e = 0.3 , and μ_i = 0.5.

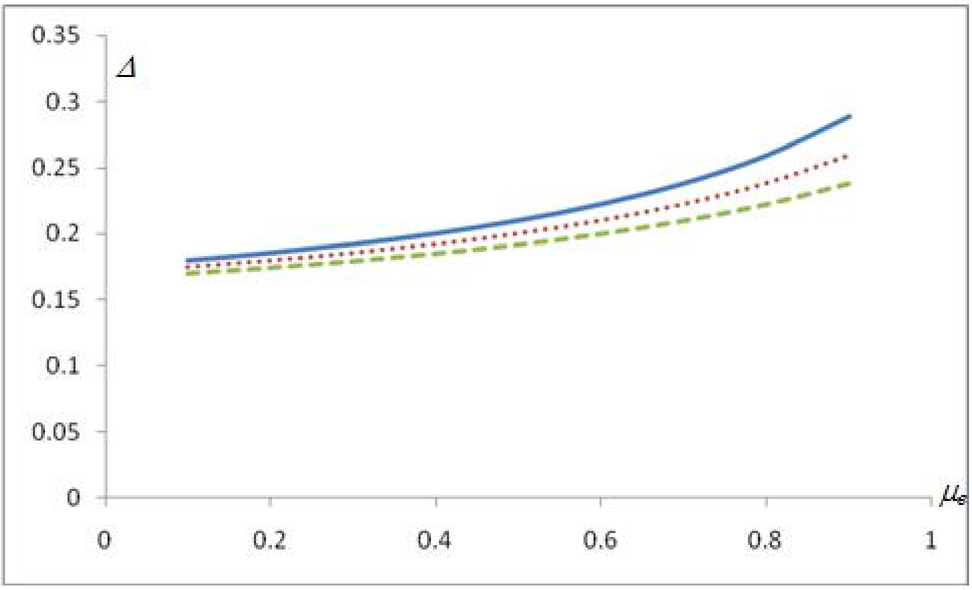


Figure 9. Variation of Δ with μ_i as, μ_e = 0.2 (solid line), μ_e = 0.3 (dotted line), μ_e = 0.4 (dashed line), where η_{10} = 0.5, η_{20} = 0.1, α = 0.01, β = 500.

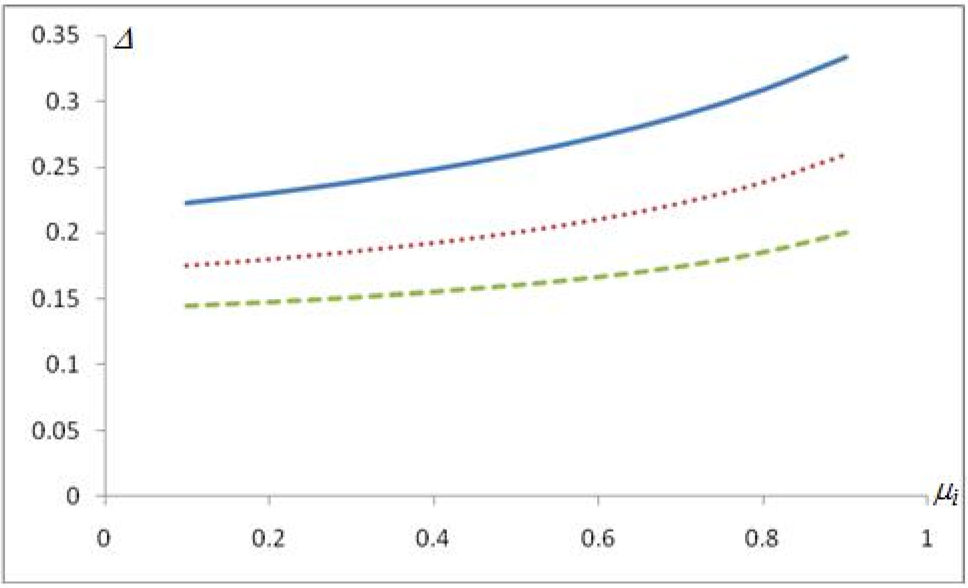


Figure 10. Variation of Δ with μ_i as, α = 0.005 (solid line), α = 0.01 (dotted line), α = 0.02 (dashed line), where η_{10} = 0.5, η_{20} = 0.1, μ_e = 0.3, β = 500.

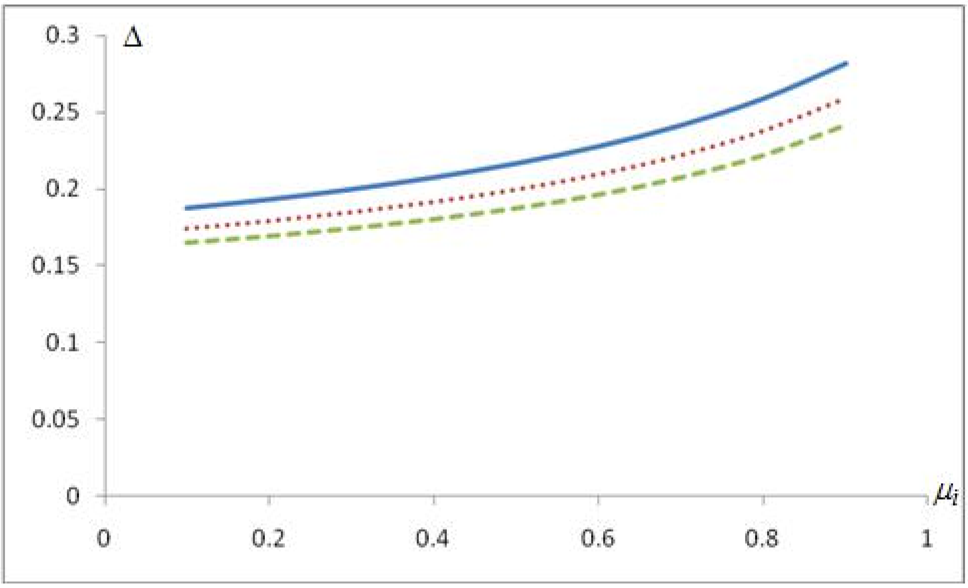


Figure 11. Variation of Δ with μ_i as, β = 400 (solid line), β = 500 (dotted line), β = 600 (dashed line), where η_{10} = 0.5 , η_{20} = 0.1 , α = 0.01 , μ_e = 0.3 .

REFERENCES

[1] Goertz , C. K. *Rev. Geophys*. **1989**, 27, 271- 282.

[2] Mendis D. A. ; Rosenberg, M. C. *Annu. Rev. Astron. Astrophys*. **1994**. 32, 419-427.

[3] Shukla, P. K. *Phys.Plasmas*, **2001**. 8, 1791-1803.

[4] Horanyi, M. ; Mendis, D. A. *J. Geophys. Rev*.**1986**. 91, 355.

[5] F.Verheest, *Waves in Dusty Space Plasmas*; Kluwer Academic, Dordrecht, Netherlands. 2001.

[6] Horanyi, M. Annu. *Rev. Astron. Astrophys*. **1996**. 34, 383.

[7] Barkan, A.;D'Angelo N.;Merlino, R. L. *Planet Space Sci*. **1996**.44, 239.

[8] Merlino, R. L.;Barkan, A.;Thompson, C.;D'Angela, N. *Phys. Plasmas* **1998**. 5, 1067.

[9] Homann, A.;Melzer, A.;Peters, S.;Piel, A. *Phys. Rev. E*. **1997**.56, 713.

[10] Verheest, F. *Planet Space Sci*. 1992.40,1.

[11] Pieper, J. B.;Goree, J. *Phys. Rev. Lett*. **1996**. 77, 3137.

[12] Bliokh, P. V.;Yaroshenko, V. V. *Sov. Astron. (Eng. Transl.)* **1985**. 29, 330.

[13] De Angelis, U.;Formisano, V.;Giordano, M. *J. Plasma Phys*. **1988**. 40, 399-406.

[14] Asbridge, J. R.; Bame,S. J.; Strong, I. B. *J. Geophys. Rev*. **1993**.73, 5777.

[15] Shukla, P. K.; Stenflo, L. *Astrophys. Space Sci*. **1992**. 190(1), 23.

[16] Rao,N. N.;Shukla, P. K.;Yu, M. Y. *Planet. Space Sci*. **1990**.38, 543.

[17] Shukla, P. K.;Silin, V. P. *Physica Scripta*. **1992.** 45, 508-.513.

[18] Amin, M. R.;Morfil, G. E.;Shukla, P. K. *Phys. Rev. E*. **1998**. 58, 6517.

[19] Popel, S. I.;Yu, M.Y. *Contrib. Plasma Phys*. **1995**.35(2), 103.

[20] Ma, J. X,;Lui, J. *Phys. Plasmas*. **1997**. 4, 253.

[21] Bharuthram, R.;Shukla, P. K. *Planet. Space Sci*. **1992**.40, 973-977.

[22] Mamun, A. A.; Cairns, R. A.;Shukla, P. K. *Phys. Plasmas* 1996. **3**,702-704.

[23] Mendis, D. A.;Horanyi, M. *Geophys. Monogr. Ser*. **1991**. 61, 17.

[24] Chow, V. W.;Mendis, D. A.;Rosenberg, M. *J. Geophys. Res*. **1993**. 98, 19065.

[25] Havnes, O.;Troim, J.;Blix, T.;Mortensen, W.;Naeshein, L. I.;Thrane, E.;Tonnesen, T. *J. Geophys. Res*. **1996** 10, 10,839.

[26] Horanyi, M.;Morfill, G.;Gruin, E. *Nature*. **1993.** 363, 144-146.

[27] Shukla, P. K.;Mamun, A. A. *Introduction to Dusty Plasma Physics*; IOP Publishing Ltd. Bristol, U. K., 2002.

[28] Mandal G.;Chatterjee, P. *Z. Naturforsch*. **2010.**65a, 85.

[29] Armina, R.;Mamun, A. A.;Alam, S. M. K. *Astrophys. Space Sci*. **2008**. 315(1), 243.

[30] Akpabio, L. E.;Ikot, A. N.;Udoumuk, A. B. *Elixir Space Sci*. **2012**. 50, 10175-10178.

[31] Akpabio, L. E.; Ikot, A. N. *Research J. Phys*. **2012**. 1-9 DOI: 10. 3923/rjp

[32] Paul, K. S.;Mandal, G.;Mamun, A. A.;Amin, M. R. *IEEE Trans. Plasma Sci*. **2009**. 37, 627-631.

[33] Sahu, B. *Bulg. J. Phys*. **2011**. 38, 175-183.

[34] Vasyliunas, V. M. *J. Geophys. Res.* **1968**. 73, 12839.

[35] Formisano, V.;Moreno, G.;Palmiotto, F. *J. Geophys. Rev*. **1973**. 78, 3714-3730.

[36] Mendis D. A.;Rosenberg, M. *Annu. Rev. Astron. Astro. Phys*. **1994**. 32, 419-463.

[37] Scudder, J. D.;Sittler, E. C.;Bridge, H. S. *J. Geophys. Res.* **1981**. 86, 8157-8179.

[38] Marsch, E.;Muhlhauser, K. H.;Schwenn, R.;Rosenbauer, H.;Phillip, W.;Neubauer, F. M. *J. Geophys. Res*. **1982**. 87, 52-72.

[39] Leubner, M. P. *Phys. Plasmas*, **2004**. 11, 1308-1316.

[40] Lazar, M.;Schlickelser, R.;Poedts, S.;Tantz, R. C. *Mon. Not. R. Astron. Soc*. **2008**. 168, 390.

[41] Lui, A. T. Y.;Krimigis, S. M. *Geophys. Res. Lett*. **1981**. 8, 527-530.

[42] Baluku, T. K.;Hellberg, M. A. *Phys. Plasmas*. **2008**. 15, 1237.

[43] Eslami, P.;Mottaghizadeh, M.;Pakzad, H. R. *Astrophys. Space Sci*. 2011. 333, 263-268.

[44] Kundu, S. K.;Ghosh, D. K.;Chatterjee, P.;Das, B. *Bulg. J. Phys*. **2011**. 38, 409.

[45] Washinai, H.;Tanuiti, T. *Phys. Rev. Lett*. **1966**. 17, 996.

[46] Mendis, D. A.;Rosenberg, M. *Annu. Rev. Astron. Astrophys*. 1994. **32**, 418-463.

[47] El-Awady, E. I.;E1 – Tantawy, S. A.;Moslem, W. M.;Shukla, P. K. *Phys. Lett. A*. **2010**. 374 (31), 3216-3219.

[48] Chuang, S. H.; Hau, L. N. *Phys. Plasmas*. **2009**. 16, 022901.