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A proposed new gravitational redshift function to the theoretical model of Morris-Thorne wormholes

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Abstract

This work introduces a new gravitational redshift function to the Morris-Thorne wormhole model that is defined as $\Phi(r) \equiv \frac{r_0}{r} \ln\left(\frac{r}{r_0}\right)$ with spherical symmetry and stabilized by phantom energy.

Keywords : Wormhole, Morris-Thorne, redshift, ghost energy.

Introduction

The possibility incorporating wormholes in the model interstellar travel has been suggested by the equations originally presented by Morris and Thorne [1, 2]. This model expresses how to connect two regions of our universe or different universes through a wormhole.

To date, numerous studies have been published regarding the accelerated expansion of our universe that is caused by the presence of dark matter [3 - 5]. This implies that $\ddot{a} > 0$ in the Friedmann equation:

$$\ddot{a} = -\frac{4\pi}{3}a(\rho + 3p) > 0 \tag{1}$$

On the other hand, it is known that the state equation is given by the relation $p = w\rho$ where p and ρ are the pressure and the energy density, respectively. The range of values for the parameter w is between -1 and -1/3 in the Quintessence model defined by Caldwell, Dave and Steinhardt in 1998 [6]. The particular case of $w = -1$ corresponds to a fluid with a constant energy density associated with the cosmological constant [7]. Finally, the case $w < -1$ is called phantom energy which is precisely the condition to be analyzed in this paper.

Phantom energy

The fundamental characteristic of phantom energy is that the energy density increases as the universe expands at a faster rate than it did with a cosmological constant [8]. Recall that the WMAP satellite has confirmed that approximately seventy percent of the energy of our universe is of the dark type. A study published by Komatsu and co-workers established that the value of w is almost -1 [9]. They obtained w = -1.10 $\pm$0.14 according to WMAP + BAO + Ho (details of the nomenclature and calculation in [9]). The negative value has allowed one to conceptualize the existence of wormholes. However, this type of energy results in a catastrophe known as Big Rip [10]. Gonzalez-Diaz has indicated that Planck-sized wormholes can increase in size quickly until they reach the size of the universe, but they "explode" just before the Big Rip occurs, thus avoiding this catastrophe [10] .

In the next section, we will define a new equation for the gravitational redshift function that could supports intergalactic travel [2]. This function fulfills the condition $w < -1$.

Einstein Field Equations

The first step of our analysis is to consider a static and symmetric line element in spherical coordinates given by [1]:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{\Lambda(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{2}$$

where $\Phi(r)$ is the gravitational redshift function and $e^{\Lambda(r)}$ is given by

$$e^{\Lambda(r)} = \frac{1}{1 - \frac{b(r)}{r}} \tag{3}$$

$b(r)$ is the shape function. The wormhole throat is defined as r_o. According to Morris-Thorne [1], $b(r_o) = r_o$ and $b(r) \to 0$ when $r \to \infty$. Another condition to the wormhole formation is that $\frac{b-b'r}{2b^2} > 0$, which are exactly the condition for a wormhole to exist and can be used to travel through its through, maintaining the weak energy condition, ie., $p + \rho < 0$ [11].

The Einstein's field equation given by

$$G_{uv} = 8\pi T_{uv} \quad (4)$$

where the system of units has been normalized to be G = c = 1

We know that the expression of the impulse energy tensor T_{uv} according to the scientific literature [1,3] , is given by

$$T_{uv} = (p + \rho)u_u u_v - p g_{uv}$$

where $u_u u_v = -1$. Thus Einstein's field equations are employed to resolve the equation (2) taking account the equation (4), is:

$$G_{tt}: \quad \frac{b'}{r^2} = 8\pi\rho \quad (5)$$

$$G_{rr}: \frac{2r(r-b)\Phi' - b}{r^3} = 8\pi p_r \quad (6)$$

$$G_{\theta\theta}: \frac{2r^2(r-b)\left(\Phi'' + \Phi'^2\right) + \Phi' r[(2-b')r - b] - b'r + b}{2r^3} = 8\pi p_t \quad (7)$$

$p_t(r)$ is the pressure measured in the lateral directions (orthogonal to the radial direction). From equations (5) and (6), it follows that

$$b' = 8\pi\rho r^2 \quad (8)$$

$$\Phi' = \frac{8\pi p_r r^3 + b}{2r(r-b)} \quad (9)$$

In addition, the equation of state is $p_r = w\rho$, so we find

$$\Phi' = \frac{w(r) r b'(r) + b(r)}{2r[r - b(r)]} \quad (10)$$

To resolve the last equation is necessary to assume a shape function, $b(r)$. In addition, we can verify with help of equation (10) that $w(r)$ evaluated at r_o must meet the condition $w(r_o) < -1$.

Before defining a redshift function, we assume the following shape function which is defined as:

$$b(r) = r_0 \left(\frac{r}{r_0}\right)^n \tag{11}$$

where n is a constant. It is easy to verify that $b(r_o) = r_o$, $b'(r_o) = n$ and satisfies the equation $b'(r_o) < 1$, so $n < 1$ and greater than zero.

From the equation (10) is easy to obtain the $w(r)$ expression:

$$w(r) \quad = \frac{2r[r - b(r)]\Phi' - b(r)}{rb'(r)}$$

In this point is that we introduce a new redshift function, according to the following equation:

$$\Phi(r) \equiv \frac{r_0}{r} \ln(\frac{r}{r_0}) \tag{12}$$

It is important to verify that this new definition must satisfy the following condition $\omega < -1$, and for this verification, we obtain the ω equation taking account the following equations (10,11,12):

$$\omega = \frac{2r(r-b)\Phi' - b}{rb'} = \frac{\left(r - r_o\left(\frac{r}{r_0}\right)^n\right)\left[2r_0\left(1 - ln\left(\frac{r}{r_0}\right)\right)\right] - rr_0\left(\frac{r}{r_0}\right)^n}{nr_0 r\left(\frac{r}{r_0}\right)^n} \tag{13}$$

At $r = r_o$, we obtain $\omega(r) = -\frac{1}{n}$, and we can verify that $\omega < -1$.

The expression of week energy, $p + \rho$ is

$$p + \rho = \frac{2\left[ln({}^{r}/_{r_0}) - 1\right]\left(\frac{r}{r_o}\right)^n r_0^2 + \left\{(n-1)r\left(\frac{r}{r_o}\right)^n - 2ln({}^{r}/_{r_o}) + 2r\right\} r_o}{8\pi r^4}$$

When $r = r_o$, the last equation is

$$(p + \rho)|_{r=r_o} = \frac{n-1}{8\pi r_o^2}$$

As indicated in the previous section, n must be less than 1. Therefore, the function proposed in equation (12) violates the weak energy condition.

Conclusion

We derive a new gravitational redshift of a wormhole such that it is allow to intergalactic travel between two regions in our universe, assuming phantom energy. To validate this equation such is $\omega < -1$ and also calculate that violates the weak energy condition, $p + \rho < 0$, and only hope that this expression may be useful in the astrophysical community.

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