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Sloshing in compound cylindrically-conical reservoirs

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Abstract

The issue of vibrations of fluid-filled compound shells is considered. The shell is supposed to be consisted of cylindrical and conical parts. The liquid is ideal and incompressible, and its motion is potential. The fluid pressure acting on the wetted shell surface is obtained from the linearized Bernoulli's equation for a potential flow. The frequencies and vibration modes are defined using reduced boundary element method. Both sloshing effect and elasticity of the shell walls are taking into account. The possibility is discussed of replacing the compound cylindrically-conical shell by cylindrical one of the equal height at sloshing frequency analysis. The validity of hypothesis of spectrum separation for sloshing and elastic modes is testified. The most dangerous frequencies from the point of view of resonance and stability losing are estimated for compound cylindrically-conical shells.

Keywords: Compound cylindrically- conical shells, reduced boundary element method, sloshing, frequencies and modes of vibration.

1. Introduction

In engineering, shells and shell structures are widely used as liquid-containment vessels. Among them there are fuel tanks and containers that are used in different engineering areas such as aerospace industry, oil-gas industry, power machine building, and transport. These reservoirs often operate under excess process loads.

Liquid sloshing is a physical effect that has been observed in many engineering applications. It occurs in rocket tanks, marine and space vehicles as well as in oil and liquefied gas storage tanks, reactors, and pressure vessels. Sloshing is low frequency vibrations of a liquid free surface in partially filled reservoirs. One of the crucial effects connected with seismic responses of fluid-filled containers is the sloshing motion at their free surfaces. When rocket tanks are partially filled with the fuel, substantial liquid masses can move inside the tanks under operative accelerations, generating the fuel slosh. So sloshing control of fuel is a challenging problem. Mission failures have been explained by effects of sloshing that induced instabilities of spacecrafts [1-3].

Compound shell structures are the subject of intensive research during the last decades, and scientific literature devoted to the problems in this area is numerous. A lot of test and engineering problems were proposed and considered in the area. The profound and detailed analysis of research devoted to liquid sloshing in tanks and reservoirs was done by R. A. Ibrahim's in [4,5].

Shells composed of cylindrical and conical parts in interaction with a fluid have not received sufficient attention in scientific literature in spite of wide usage of such thin walled shell structures. They are important in a number of different engineering areas. In aerospace industry such structures are used as fuel tanks in aircrafts and satellites. They are also used in torpedoes, submarines, and for off-shore drilling in ocean engineering. In civil engineering such compound shell structures present containment vessels for elevated water storage tanks [6]. Compound cylindrically-conical shells are also intensively used as fermentation reservoirs [7].

The case of conical shells partially filled with liquids has been successfully investigated by Lakis et al. [8]. The overview of the research on the topic [9-15] demonstrates that the dynamic response of structures containing liquids can be essentially influenced by vibrations of their elastic walls and their interaction with the sloshing liquid. However the great part of research has described the phenomenon of fluid-structure interaction neglecting gravity or elasticity effects. The important results in the area were obtained in [10, 11].

2. Problem statement

In this paper the problem concerned with liquid vibrations in the compound shell of revolution is considered. It is supposed there that the shell may consist of cylindrical and conical parts. Let designate the wetted shell surface by S_1, and the free liquid surface by S_0. Suppose the Cartesian coordinate system $0xyz$ is connected with the compound shell, the liquid free surface S_0 coincides with the plane $z=H$ at the state of rest (Figure 1).

For modeling the fluid domain, a mathematical model has been developed based on the following hypotheses: the fluid is incompressible and inviscid, its motion is irrotational, and only small oscillations need to be considered. In these suppositions a scalar velocity potential $\Phi(x,y,z,t)$ whose gradient represents the fluid velocity can be introduced. The fluid pressure $p = p(x,y,z,t)$ acting on the wetted shell surface is obtained from the linearized Bernoulli's equation for a potential flow

$$p = -\rho_l\left(\frac{\partial \Phi}{\partial t} + gz\right) + p_0,\ \ p_s = -\rho_l gz,\ \ p_d = -\rho_l \frac{\partial \Phi}{\partial t},$$

where g is the gravity acceleration, z is the vertical coordinate of a point inside the liquid, ρ_l is the liquid density, p_s and p_d are static and dynamic components of the fluid pressure, p_0 is the atmospheric pressure.

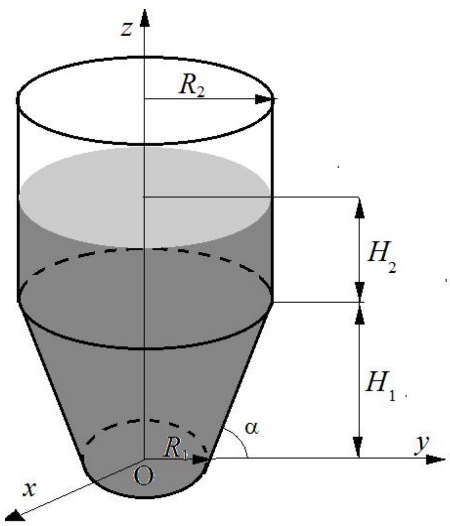

Fig. 1: Compound cylindrically-conical shell

On the liquid free surface the following boundary conditions are given:

$$\left.\frac{\partial \Phi}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial \zeta}{\partial t}; \quad p - p_0 = 0 .$$

The unknown function ζ here describes shape and location of the free surface.

Thus, for the above-mentioned potential Φ we have the following boundary value problem:

$$\nabla^2 \Phi = 0 ; \ \left.\frac{\partial \Phi}{\partial \mathbf{n}}\right|_{S_1} = 0 ; \left.\frac{\partial \Phi}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial \zeta}{\partial t}; \quad p - p_0\big|_{S_0} = 0 ; \ \left.\frac{\partial \Phi}{\partial t} + g\zeta\right|_{S_0} = 0 .$$

The zero eigenvalue obviously exists for above-mentioned problem, but we will exclude it using the next orthogonality condition:

$$\iint_{S_0} \frac{\partial \Phi}{\partial \mathbf{n}} dS_0 = 0 .$$

Here **n** is an external unit normal to the considered surfaces.

Having determined unknown functions Φ and ζ, we obtain the raising height ζ of the free surface and determine the liquid pressure p upon shell walls.

3. The method of solution

Consider the potential Φ as the following series:

$$\Phi = \sum_{k=1}^{M} \dot{d}_k \varphi_k . \qquad (1)$$

For functions φ_k introduced here the next boundary value problems are considered:

$$\nabla^2 \varphi_k = 0 , \ \left.\frac{\partial \varphi_k}{\partial \mathbf{n}}\right|_{S_1} = 0 , \qquad (2)$$

$$\left.\frac{\partial \varphi_k}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial \varsigma}{\partial t}; \ \frac{\partial \varphi_k}{\partial t} + g\zeta = 0 . \qquad (3)$$

Let differentiate the second equation in relationship (3) with respect to t and substitute there the expression for ς_t' from the first one of relations (3). Suppose hereinafter that $\varphi_k(t,x,y,z) = e^{i\chi_k t}\varphi_k(x,y,z)$. Then we obtain the sequence of eigenvalue problems with following conditions on the free surface for each φ_k:

$$\frac{\partial \varphi_k}{\partial \mathbf{n}} = \frac{\chi_k^2}{g}\varphi_k . \qquad (4)$$

Here the own values are

$$\lambda_k = \frac{\chi_k^2}{g}.$$

It follows from Equation (3) that unknown function ζ can be received in the form

$$\zeta = \sum_{k=1}^{M} d_k \frac{\partial \varphi_k}{\partial \mathbf{n}}. \qquad (5)$$

The natural liquid frequencies in the shell are defined from (3), (4) as follows

$$-\omega_k^2 \varphi_k(x,y,H) + \chi_k^2 \varphi_k(x,y,H) = 0 \Rightarrow \omega_k^2 = \chi_k^2; \quad \omega_k = \sqrt{\lambda_k g}.$$

The effective numerical procedure for solution of these eigenvalue problems using boundary element method was proposed in [9].

4. System of the boundary integral equations

To define unknown basic functions φ_k (k=1,2, … M) the direct formulation of boundary element method is in use as it was proposed by C. Brebbia in [14]. Dropping index k we obtaine the principal integral relation as follows:

$$2\pi\varphi(P_0) = \iint_S q \frac{1}{|P - P_0|} dS - \iint_S \varphi \frac{\partial}{\partial \mathbf{n}} \frac{1}{|P - P_0|} dS, \qquad (6)$$

where $S = S_1 \cup S_0$; points P and P_0 belong to surface S.

By $|P - P_0|$ we denote the Cartesian distance between points P and P_0. In doing so, the function φ, defined on the surface S_1, is the pressure on the moistened shell surface, and the function q, defined on the free surface S_0, presents the flux, $q = \frac{\partial \varphi}{\partial \mathbf{n}}$.

Using boundary conditions (4), one can obtain the system of singular integral equations in the form presented in [9, 12]

$$\begin{cases} 2\pi\varphi_1 + \iint_{S_1} \varphi_1 \frac{\partial}{\partial \mathbf{n}}\left(\frac{1}{r}\right) dS_1 - \frac{\chi_k^2}{g} \iint_{S_0} \varphi_0 \frac{1}{r} dS_0 + \iint_{S_0} \varphi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right) dS_0 = 0, \\ -\iint_{S_1} \varphi_1 \frac{\partial}{\partial \mathbf{n}}\left(\frac{1}{r}\right) dS_1 - 2\pi\varphi_0 + \frac{\chi_k^2}{g} \iint_{S_0} \varphi_0 \frac{1}{r} dS_0 = 0. \end{cases} \qquad (7)$$

Here for convenience the potential values are denoted by φ_0 on the free surface, and by φ_1 on the shell walls.

5. Reducing to the system of one-dimensional integral equations

It would be noted that there exist two types of kernels in the integral operators introduced above in (7). Namely they are

$$A(S,\sigma)\psi = \iint_S \psi \frac{\partial}{\partial \mathbf{n}} \frac{1}{|P-P_0|} dS; \quad B(S,\sigma)\psi = \iint_S \psi \frac{1}{|P-P_0|} dS; \quad P_0 \in \sigma. \quad (8)$$

In formulas (8) the surfaces S and σ may be either different or coincident ones. If the surface S coincides with σ then integrals in (8) are singular and so their numerical treatment has to take into account the presence of this singularity and strongly non-uniform distribution of integrands over the boundary element. Noted, that standard quadratures failed in these calculations.

As in [9,12] let replace the Cartesian co-ordinates (x, y, z) with cylindrical co-ordinates (r, θ, z), and integrate with respect to z and θ taking into account that

$$|P-P_0| = \sqrt{r^2 + r_0^2 + (z-z_0)^2 - 2rr_0\cos(\theta-\theta_0)}\,.$$

Here r, θ and r_0, θ_0 are radii-vectors and circumferential coordinates of points P and P_0.

Represent furthermore unknown functions as Fourier series with harmonics

$$\psi(r,z,\theta) = \psi(r,z)\cos m\theta; \quad m = 1,2\,, \qquad (9)$$

where m is a given integer (the number of nodal diameters).

In doing so we obtain the integral operators in the following forms

$$\iint_S \psi \frac{\partial}{\partial \mathbf{n}} \frac{1}{|P-P_0|} dS = \int_\Gamma \psi(P)\Theta(P,P_0)d\Gamma;$$
$$\iint_S \psi \frac{1}{|P-P_0|} dS = \int_\Gamma \psi(P)\Phi(P,P_0)d\Gamma; \; P_0 \in \sigma. \qquad (10)$$

In (10) Γ is a generator of the surface S, and kernels $\Theta(P,P_0)$ and $\Phi(P,P_0)$ are defined by following formulas

$$\Theta(z,z_0) = \frac{4}{\sqrt{a+b}} \frac{1}{2r}\left[\frac{r^2 - r_0^2 + (z_0 - z)^2}{a-b} \mathsf{E}_m(k) - \mathsf{F}_m(k)\right] n_r + \frac{4}{\sqrt{a+b}} \frac{z_0 - z}{a-b} \mathsf{E}_m(k) n_z\,,$$

$$\Phi(P,P_0) = \frac{4}{\sqrt{a+b}} \mathsf{F}_m(k)\,.$$

Here n_r, n_z are components of the unit normal **n** to the considered surface. The following notations are introduced beforehand

$$\mathsf{E}_m(k) = (-1)^m\left(1-4m^2\right)\int_0^{\pi/2}\cos 2m\theta\sqrt{1-k^2\sin^2\theta}\,d\theta\,,\ \mathsf{F}_m(k) = (-1)^m\int_0^{\pi/2}\frac{\cos 2m\theta\, d\theta}{\sqrt{1-k^2\sin^2\theta}}\,, \qquad (11)$$

$$a = r^2 + r_0^2 + (z - z_0)^2, \quad b = 2rr_0\,; \qquad k^2 = \frac{2b}{a+b}\,.$$

Letting $m = 0$ in expressions (11), we obtain the standard elliptic first and second kind integrals.

So, the solution of system (7) becomes independent of the angular coordinate θ, and the three-dimensional problem is reduced to a two-dimensional one in the radial coordinate *r* and the axial coordinate *z*. Using dependence $z = z(r)$, we finally reduce the system of singular integral equations to a one-dimensional one. For numerical solution of this system the procedures described in [9-11] are used.

6. Numerical simulation and discussion

Fluid-filled isotropic cylindrical and cylindrically - conical shells are considered. The cylindrically-conical shell is shown in Figure 1. Here R_1 and R_2 are radii of the cone at its small and large edges, R_2 is also for cylinder radius, α =90°-β, β is a semivertex angle of the cone, and H is the cone height, H_1 and H_2 are heights of conical and cylindrical parts, respectively, $H = H_1 + H_2$. Both shells are refereed to the cylindrical coordinate system (x,θ,z).

For following numerical simulation, semivertex angle α=60°, H_1/R_2=1.0, R_1=0.5m, and H_1 = 1m. For H_2 we consider the following values H_2 = 1, 0.5, 0.25, and 0.1 m.

For both shells we estimate further the sloshing frequencies.

First, we determine the requisite number of boundary elements for the accurate calculations of the natural frequencies. The convergence is established when numbers of boundary elements along the shell wall was equal to 150, along bottom this number was 120 elements, the number of boundary elements along the free surface radius was 120 also.

The sloshing frequencies here are calculated accordingly to Degtyarev *et al.* [9].

Below we demonstrate that sloshing frequencies of rigid conical (C) and cylindrically-conical (CC) shells are very close when $H_1 = H_2$=1m. Comparison of results for *m*=0 (axisymmetric modes) and different *n* is shown in Table 1.

Table 1: Axisymmetric sloshing frequencies, *Hz*

Type of shell and method	*n*						
	1	2	3	4	5	6	7
C, numerical solution	6.131	8.300	9.999	11.445	12.718	13.905	14.98
C, analytical solution	6.130	8.295	9.990	11.432	12.711	13.871	14.94
CC, numerical solution	6.131	8.300	9.999	11.445	12.718	13.905	14.98

Here numerical solution is obtained by using the proposed method. We used for comparison and validation the analytical solution of R. Ibrahim [5] that can be expressed in the following form:

$$\frac{\chi_k^2}{g} = \mu_k \tanh\left(\mu_k \frac{H}{R}\right), \quad k = 1,2,\ldots;\ \varphi_k = J_0\left(\frac{\mu_k}{R} r\right)\cosh\left(\frac{\mu_k}{R} z\right)\cosh^{-1}\left(\frac{\mu_k}{R} H\right).$$

The values μ_k above are roots of the equation

$$\frac{dJ_0(x)}{dx} = 0,$$

where $J_0(x)$ is Bessel function of the first kind, χ_k, φ_k are frequencies and modes of liquid sloshing in the rigid cylindrical shell.

Table 1 shows an insignificant difference between frequencies of cylindrical and cylindrically-conical shells. Results obtained here also testify the proposed numerical method accuracy. It follows from good agreement between numerical and analytical solutions.

Figure 2 demonstrates the first axisymmetric sloshing modes (*n*=1,2,3) of the cylindrical shell.

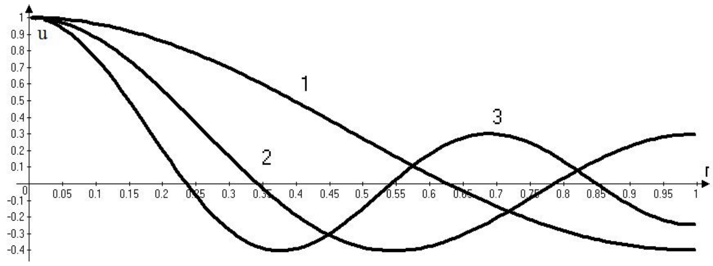

Fig. 2: Axisymmetric sloshing modes *n*=1,2,3 of cylindrical shell

Here and hereinafter numbers 1, 2, 3 in the pictures correspond to 1st , 2nd, and 3rd sloshing modes.

Next, we compare frequencies of non-axisymmertric modes for circumference numbers *m*=1,2,3 and different modes *n* for cylindrical (C) and cylindrically-conical (CC) shells. The results are shown in table 2.

Table 2: Non-axisymmetric sloshing frequencies, *Hz*

m	Type of shell	*n*						
		1	2	3	4	5	6	7
1	C	4.2474	7.2352	9.1573	10.726	12.089	13.312	14.433
	CC	4.2346	7.2352	9.1573	10.726	12.089	13.312	14.433
2	C	5.4733	8.1148	9.8966	11.377	12.678	13.855	14.939
	CC	5.4718	8.1148	9.8966	11.377	12.678	13.855	14.939
3	C	6.4197	8.8719	10.558	11.973	13.226	14.364	15.417
	CC	6.4195	8.8719	10.558	11.973	13.226	14.364	15.417

From these results one can conclude that non-axisymmetric frequencies are similar for both considered shells.

Figure 3 demonstrates first three non-axisymmetric sloshing modes for *m*=1. Solid lines are corresponded to modes of the cylindrical shell, and dot lines are for modes of the cylindrically-conical shell.

Note that the sloshing modes are also practically the same for both shells considered here.

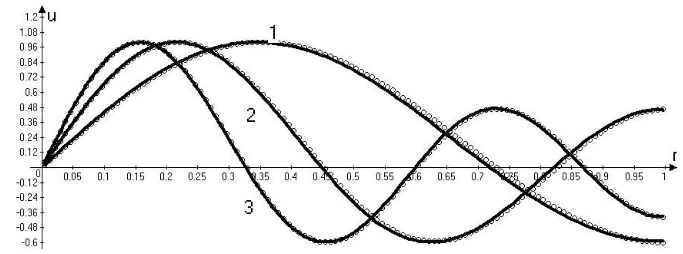

Fig. 3: Non-axisymmetric sloshing modes, *m*=1

The first three sloshing modes for m=2 and different n are shown in Figure 4. Here the sloshing modes for both cylindrical and cylindrically-conical shell practically coincided.

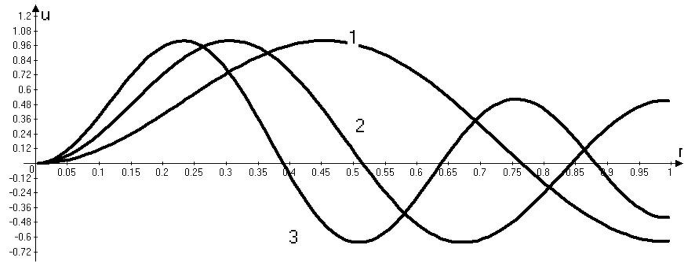

Fig. 4: Non-axisymmetric sloshing modes for *m*=2

At first glance, it seems that it is possible to consider cylindrical shells instead of cylindrically-conical ones with equal heights and upper radii at sloshing frequency analysis. Really, the results demonstrate that frequencies and modes of both shells are differed insignificantly.

But further numerical simulations show essentially different behavior of frequencies changing with decreasing the height H_2 (see Figure 1), i.e. when the cylindrical part is smaller then conical one.

Table 3 below provides the comparison of sloshing frequencies between cylindrical and cylindrically-conical shells at different filling levels H_2 = 1, 0.5, 0.25, and 0.1 m.

Here we consider m=1, because the lowest frequencies correspond namely to this wave number.

Table 3: Non -axisymmetric sloshing frequencies, *Hz, m*=1

H_2	Type of shell	*n*						
		1	2	3	4	5	6	7
1.0	C	4.247	7.235	9.157	10.72	12.08	13.31	14.43
	CC	4.234	7.235	9.157	10.72	12.08	13.31	14.43
0.5	C	4.145	7.230	9.153	10.72	12.08	13.30	14.42
	CC	4.227	7.231	9.153	10.72	12.08	13.30	14.42
0.25	C	4.000	7.214	9.149	10.71	12.08	13.30	14.42
	CC	4.199	7.230	9.151	10.71	12.08	13.30	14.42
0.1	C	3.831	7.153	9.120	10.70	12.07	13.29	14.42
	CC	4.169	7.229	9.150	10.71	12.08	13.30	14.42

From the results obtained one can conclude that with decreasing the height H_2 the difference between sloshing frequencies of cylindrical and cylindrically-conical shells become more considerable. This difference is noticeable even for high *n*. But modes of vibrations are differed slightly. In figure 3 the modes of cylindrical and cylindrically-conical shells are presented. Here solid lines correspond to the cylindrical shell with H_2=1m, and points correspond to the cylindrically-conical shell with H_2=0.1m. The lowest sloshing frequency for both shells corresponds to mode *m*=1. Its space profile for the cylindrical and cylindrically-conical shells shell are shown in Figure 5 and 6, respectively..

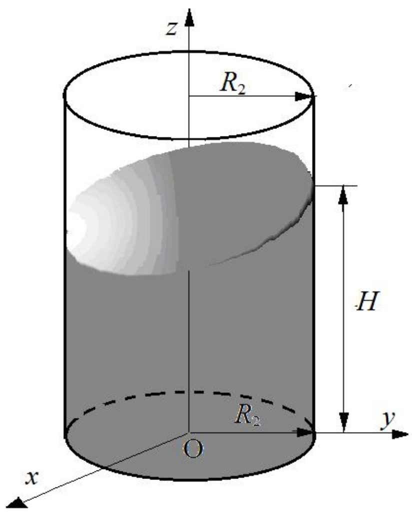

Fig 5: The lowest sloshing mode for cylindrical shell

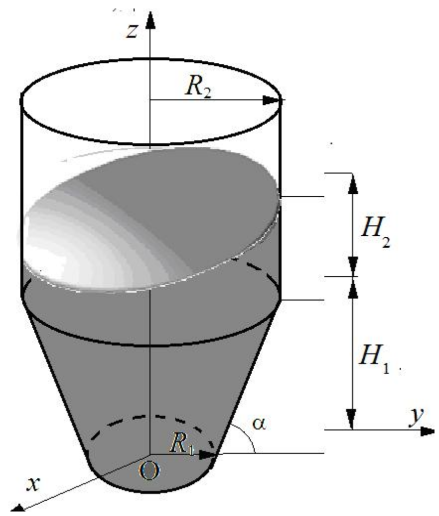

Fig 6: The lowest sloshing mode for cylindrically-conical shell

For both shells the lowest sloshing modes are the same. The modes for other wave numbers *m* demonstrate similar behaviour. Figure 7 shows the sloshing mode space profile for cylindrically-conical shell at *m*=2.

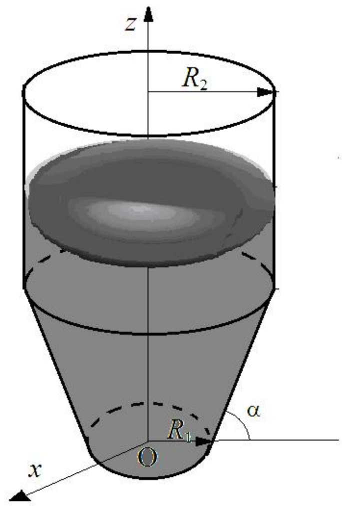

Fig 7: Sloshing mode for cylindrically-conical shell, m=2

To complete the numerical simulation it is necessary to consider vibrations of shells with elastic walls. So the elastic cylindrical and cylindrically-conical shells are under consideration. The method elaborated in [13, 15, 16] is applied for receiving frequencies of both empty and fluid-filled elastic shells. For following numerical simulation, the thickness of the shell and the Poisson's ratio are taken as h/R_2=0.01 and ν =0.3, semivertex angle β is 26.56°, α=63.44°; H_1/R_2=1.0, R_1=0.5m, R_2=1.0m, Young's module $E = 2{,}0 \cdot 10^6$ MPa, ρ_s=7800 kg/m^3, ρ_l=1000 kg/m^3. Here the boundary conditions "clamped – free" for both shells are assumed.

The results of numerical simulation for axisymmetric vibrations of elastic shells are shown in table 4. Here H_2=0.1m.

Table 4: Axisymmetric frequencies of elastic shells, *Hz, m*=0

n	cylindrically-conical shell		cylindrical shell	
	empty	fluid-filled	empty	fluid-filled
1	99.6575	15.9494	24.9144	7.6876
2	387.976	103.952	96.9945	46.7357
3	398.135	398.135	217.307	128.532
4	624.588	294.025	385.780	256.805

From the results obtained here one can conclude that the vibration frequencies of elastic fluid-filled shell are differed drastically from frequencies of empty ones for both cylindrical and cylindrically-conical shells. Noted that third mode of the cylindrically-conical shell is the torsion one, and it does not affect on fluid-structure interaction because an ideal fluid produces only a normal pressure on a moistened body. The frequencies of the cylindrical shell are smaller than those ones of the cylindrically-conical shell. The frequencies ω near 7Hz may be considered as most dangerous for the cylindrical shell. The results of tables 3 and 4 testify it. For example, ω=7.6876 Hz correspond to m=0 and n=1 for the fluid-filled elastic shell; ω=7.1538 Hz correspond to m=1 and n=2 for sloshing in this rigid shell. It will be the reason for the loss of stability.

7. Conclusion

The frequencies and free vibrations modes of the liquid in the rigid cylindrical and cylindrically-conical shells under the gravity force are estimated. The problem is solved using the reduced boundary element method. This method substantially reduces the computer time for the analysis and reveals new qualitative possibilities in modeling the dynamic behavior of shell structures. The difference between sloshing frequencies of cylindrical and cylindrically-conical shells is considerable only for small cylindrical parts in compound shells. The difference become drastic when elastic effects are involved. The lowest frequency corresponds to sloshing non-axisymmetric modes (m=1) for both shells. The supposition about spectrum separation for sloshing and elastic modes is not valid: sloshing frequencies may alternate with those of elastic walls. The assumption about replacing the compound cylindrically-conical shell by the cylindrical shell of equal height is acceptable for sloshing analysis only, and in cases when cylindrical parts are lengthy enough.

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