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An extended RLS type algorithm based on a non-linear function of the error

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Abstract

Over the last few years, extended recursive and kernelized algorithms were one of the most promising in terms of tracking signals of state-space models in non-stationary environments. In this work, we intend to propose an EX-RLS (Extended Recursive Least Squares) algorithm based on a non-linear sum function of the error. The simulations were made in the problem by tracking a non-linear Rayleigh fading multipath channel. The results showed that the proposed algorithm exhibits a superior signal tracking capability than the kernelized extended recursive type versions.

Keywords: Recursive filter adaptive, ex-rls algorithm, nonquadratic function, tracking, convergence rate

1. Introduction

The use of the MSE (Mean Squares Error) as the cost function of an adaptive filtering problem is common, and algorithms such as the RLS (Recursive Least Squares) and the LMS (Least Mean Squares) are classic examples of algorithms that seek the minimum of that cost function.

The idea is to develop an algorithm that is capable of tracking the variations of the statistical characteristics of the signals in a non-stationary environment. In this work, we intend to introduce an algorithm, the EX-RNL (Extended recursive Non-Linear), based on an unweighted function of the even powers, and driven by the EX-RNQ (Extended Recursive Non-Quadratic) algorithm developed in the paper [2].

2. The proposed method

We started with the non-linear state-space model

$$x(i+1) = A \cdot x(i) + n(i) \qquad (1)$$

$$d(i) = u^T(i) \cdot x(i) + v(i),$$

In which the state vector $x(i)$, $L \times 1$ is unknown and we need to estimate it; $n(i)$ is the processing noise, $L \times 1$, white with a zero-mean, A is a $L \times L$ matrix of the system; the measurement vector $u(i)$ is known and the measurement noise $v(i)$ has a fixed variance. The initial state $x(1)$ is uncorrelated with $n(i)$ and $v(i)$, $\forall i \geq 1$. The noises $n(i)$ and $v(i)$ are statistically independent, $\forall i \geq 1$. The details of this model can be seen in [3].

Now, we shall consider the non-linear cost function

$$J_n = \sum_{j=1}^{m} \sum_{i=1}^{n} \{\beta^{n-i}[e(i)^{2j}]\}, \qquad (2)$$

In which m and n are positive integers, $0 << \beta < 1$ is the forgetting factor, and $e(i) = d(i) - w(n)$, $1 < i < n$, in which $w(n)$ is the weight vector.

In order to develop the EX-RNL algorithm, we solved the equation

$$\nabla J_n = \sum_{j=1}^{m} \sum_{i=1}^{n} \{\beta^{n-i} 2ju(i)e(i)^{2j}\} = 0, \qquad (3)$$

The development of the binomial $(d(i) - w^T(i)u(i))^{2j-1}$ given in [4], along with the maxim of the inverse matrix [1] and a few mathematical manipulations, led us to the development of the proposed algorithm.

Table 1. Summary of the EX-RNL algorithm

initialize $\mathbf{w}(0) = \mathbf{0}$, $\mathbf{P}(0) = \lambda^{-1}\mathbf{I}$

iterate for $i \geq 1$,

$$s(n) = \left(\sum_{j=1}^{m} j(2j-1)d^{2j-2}(i)\right)^{-1} + \mathbf{u}^T(i)P_{i-1,j}\mathbf{u}(i)$$

$$G_{i,j} = \mathbf{A}P_{i-1,j}\mathbf{u}(i)/s(i)$$

$$\epsilon(i) = \left(\sum_{j=1}^{m} jd^{2j-1}(i)\right)\left(\sum_{j=1}^{m} j(2j-1)d^{2j-2}(i)\right)^{-1} - \mathbf{u}^T(i)\mathbf{w}(i-1)$$

$$\mathbf{w}(i) = \mathbf{A}\mathbf{w}(i-1) + G_{i,j}\epsilon(i)$$

$$P_{i,j} = \mathbf{A}P_{i-1,j}\mathbf{A}^T - G_{i,j}G_{i,j}^T s(i) + \left(\sum_{j=1}^{m} j(2j-1)d^{2j-2}(i)\right)^{-1} q\mathbf{I}$$

3. Experiments

For our experiment, we consider the problem of tracking a non-linear Rayleigh fading multipath channel and compare the performance of the EX-RNL algorithm. Moreover, the performance of the normalized LMS (NLMS), EX-RLS (Extended Recursive Least Squares), KRLS (Kernel Recursive Least Squares), and EX-KRLS (Extended Kernel Recursive Least Squares) algorithms are also included for comparison. To observe the performance of the algorithms, the following parameters were defined: NLMS (regularization factor $\varepsilon = 10^{-3}$ and step size $\eta = 0.25$), EX-RLS ($\alpha = 0.9999999368$, $q = 3.26 \times 10^{-7}$, $\beta = 0.995$, and $\lambda = 10^{-3}$), KRLS (regularization parameter $\lambda = 0.01$ and kernel parameter $a = 1$), EX-KRLS ($A = \alpha I$, $\alpha = 0.999998$, $q = 10^{-4}$, $\beta = 0.995$, $\lambda = 0.01$, and kernel parameter $a = 1$) and EX-RNL ($\alpha = 0.9999999368, \beta = 0.995$, $q = 3.26 \times 10^{-7}$, and $\lambda = 10^{-3}$).

About the problem of tracking a non-linear Rayleigh fading multipath channel, the number of the paths is $M = 5$, the initial maximum Doppler frequency is $f_D = 100Hz$, the sampling rate is $T_s = 0.8\mu s$, and a white Gaussian distributed time series is sent through this channel corrupted with additive white Gaussian noise, with variance $\sigma^2 = 0.001$, output of the Rayleigh channel (x), and saturation non-linearity ($y = \tanh x$).

Furthermore, we generated 2000 symbols for the experiment and performed 200 Monte Carlo simulations. At 1001 symbols, the maximum Doppler frequency was changed to $f_D = 50Hz$, as shown in Figure 2.

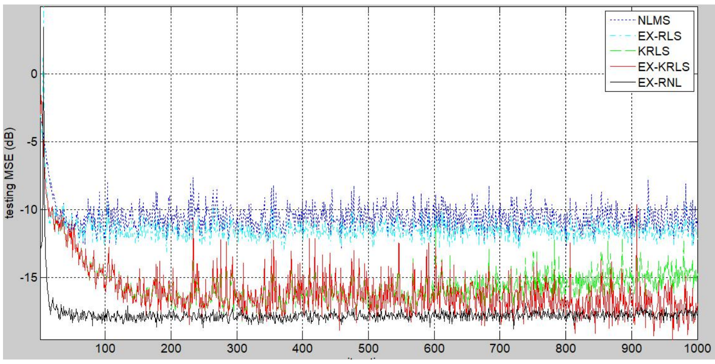

Fig. 1. Ensemble learning curves NLMS, EX-RLS, KRLS, EX-KRLS and EX-RNL algorithm when tracking a Rayleigh fading multipath channel

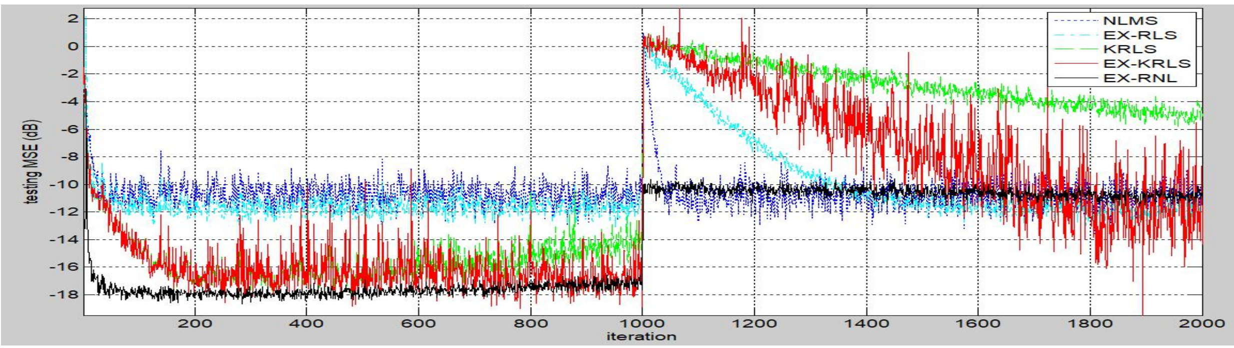

Fig. 2. Learning curves of NLMS, EX-RLS, KRLS, EX-KRLS, and EX-RNL algorithms when tracking a Rayleigh fading multipath channel. At iteration $=1001$, the maximum Doppler frequency was changed to $f_D = 50Hz$.

Table 2. Performance Comparison in Rayleigh channel tracking.

Algorithm	MSE (dB)
NLMS	-10.696 ± 0.87432
EX-RLS	-10.696 ± 0.55673
KRLS	-14.880 ± 0.72817
EX-KRLS	-17.098 ± 1.266
EX-RNL	-17.722 ± 0.26335

4. Conclusions

In this work, we presented an algorithm, in which we updated the weights of an adaptive filter using a cost function based on the sum of the even powers of the error applied to the state-space model. This algorithm is adequate for a non-linear model, with slow fading and a small variation in the state vector. The result is the algorithm EX-RNL, similar to the algorithm EX-RLS, but with a better performance in terms of convergence and misadjustment, compared with the algorithms shown in the experiments, in compliance with Figures 1 and 2 and Table 2.

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