



Diagrams show Derivatives of Inverse Function in Two Forms

Guojun Fang ^{1,*} and Zhiqing Fu ²

¹ Fuzhou Medical College of Nanchang University, Fuzhou, China;

² Fuzhou Medical College of Nanchang University, Fuzhou, China

Corresponding author: fanggj@163.sufe.edu.cn (Guojun Fang) ; Orcid: 0000-0001-6808-5862

Abstract:

In the calculus class, some students were confused by the expression of Inverse Function in two forms, which causes difficulties in understanding how to calculate the derivative of Inverse Function. It was because they were taught to denote independent variable only by using x in high school. In order to help them understand the problem well, I show the inverse function in two forms with diagrams and the reciprocal relationship between their derivatives.

Keywords: Inverse Function; Derivative; Calculus; Invertible; Diagram.

Subject classification code: 26A06

Introduction

In the calculus class, some students were confused by the expression of Inverse Function in two forms. One is the independent variable denoted by x , the other is the independent denoted by y . The former is mostly taught in high school, so these students have been used with that kind of form, which brings some difficulties in understanding how to calculate the derivative of inverse function. In order to help them deeply learn the problem well, I have constructed two diagrams and showed the reciprocal relationship.

Diagram 1: use y to denote the independent variable

Theorem (Derivative of Inverse Function)[1,2]. Let $y = f(x)$ be a function that is both invertible and differentiable. Let $x = \varphi(y)$ be the inverse function of $y = f(x)$, then,

$$\varphi'(y) = \frac{1}{f'(x)} \text{ or } f'(x) = \frac{1}{\varphi'(y)}$$

Proof:

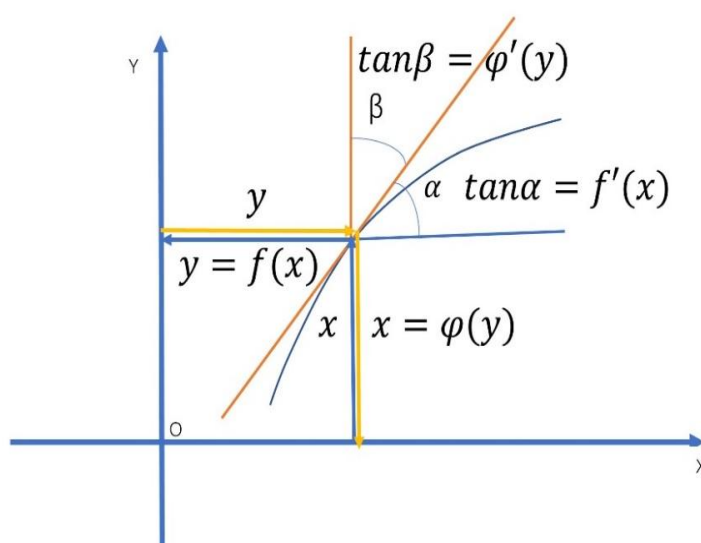
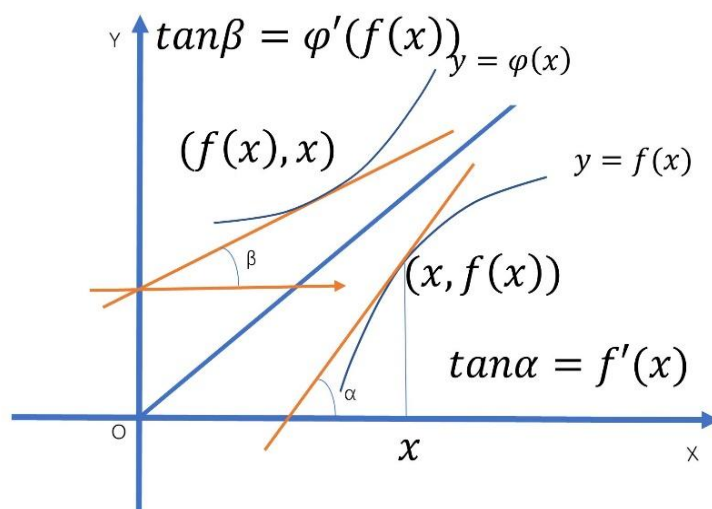


Diagram 2: use x to denote the independent variable

Let $y = \varphi(x)$ be the inverse function of $y = f(x)$, then,

$$f'(x) = \frac{1}{\varphi'(f(x))}$$

Proof:



Acknowledgements

This work was supported by the Science and Technology Project of Educational Department of Jiangxi Province with grant number GJJ2203412 and GJJ180164.

References:

- [1] HUA LOOKENG. An Introduction of Higher Mathematics (Volume 1). Beijing, China: HIGHER EDUCATION PRESS. 816 pp. 2016
- [2] GILBERT STRANG, EDWIN JED HERMAN. Calculus (Volume 1). OpenStax. 865 pp. 2016