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The Validity of Generalized Modal Syllogisms Based on the Syllogism E□M◇O-1

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Abstract

The simple and feasible methods for screening valid generalized modal syllogisms are as follows: (1) First, prove the validity of a generalized syllogism, then add at least one necessary modality (□) or possible modality (◇) to this syllogism, (2) Secondly, according to the basic fact that the conclusion of a modal syllogism is determined by the weakest premise, 12 valid generalized modal syllogisms in the same figure can be obtained by deleting invalid syllogisms; (3) Finally, taking each of these 12 syllogisms as a basic axiom, one can derive generalized modal syllogisms with different figures and forms by means of some definitions, facts, and deductive rules. This paper takes the generalized syllogisms with the general quantifier '*most*' as an example to illustrate the above methods. The results obtained by this deductive method are logically consistent. This study not only has important theoretical value for us to deeply reveal the connections between/among things, but also has important practical significance for knowledge reasoning in artificial intelligence.

Keywords: generalized quantifiers; generalized syllogisms; generalized modal syllogisms; validity

1. Introduction

Syllogistic reasoning is a common form of reasoning in natural language, and is one of the important contents of logic (Łukasiewicz, 1957; Moss, 2008). There are many kinds of syllogisms, such as Aristotelian syllogisms (Hao, 2023), Aristotelian modal syllogisms (Johnson 2004; Malink, 2013; Zhang, 2023), generalized syllogisms (Murinová and Novák, 2012), generalized modal syllogisms (Xu and Zhang, 2023abc), and so on. There are few studies on generalized modal syllogisms, and this article will investigate their validity. There are many generalized quantifiers in natural language (Peters and Westerståhl, 2006), and this paper focuses on the validity of the generalized modal syllogisms that include '*most*' which is a common quantifier in natural language.

There are (24×24×24×4−8×8×8×4−4××12×12×12+4×4×4×4=) 46592 non-trivial generalized modal syllogisms involving the following 24 propositions with 8 quantifiers in this paper. How can we screen out all the valid syllogisms among them? This paper provides some enlightening discussions on the question.

2. Preliminaries

In the following, let *c*, *n* and *z* be lexical variables, and *D* be their domain. The set composed of *c*, *n* and *z* is *C, N*, and *Z,* respectively. Let ϕ, ψ, δ and λ be well-formed formulas (abbreviated as wff). '$|C\cap Z|$' represents the cardinality of the intersection of the set *C* and Z. '$\vdash\phi$' indicates that the wff ϕ is provable, and '$=_{def}$' that the left can be defined by the right. '$\Box$' is a necessary modality, and '$\Diamond$' is a possible one. The others are similar. The operators (such as $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$) in the paper are symbols in set theory (Halmos, 1974) and modal logic (Chagrov and Zakharyaschev, 1997).

A non-trivial generalized modal syllogism needs to satisfy two conditions simultaneously: (1) it must at least contain one necessary modality ($\Box$) or possible modality ($\Diamond$); (2) and must includes at least one of non-trivial generalized quantifiers which are quantifiers outside of Aristotelian ones.

This paper only studies non-trivial generalized modal syllogisms involving the following 8 quantifiers: *all*, *no*, *some*, *not all, most, fewer than half of the, at most half of the, at least half of the*, and the first four quantifiers are called Aristotelian quantifiers. Let Q be any of these 8 quantifiers, $\neg Q$ be its outer negation quantifier and $Q\neg$ be its inner one. Therefore, the generalized modal syllogisms of this paper involve 24 types of propositions as follows: (1) *all(c, z)*, *no(c, z)*, *some(c, z), not all(c, z)*, *most(c, z), fewer than half of the(c, z), at most half of the(c, z), at least half of the(c, z)*, and they are respectively called: Proposition *A*, *E*, *I*, *O*, *M*, *F*, *H*, and *S*; (2)□*all(c, z)*, □*no(c, z)*, □*some(c, z),* □*not all(c, z),* □*most(c, z),* □*fewer than half of the(c, z),* □*at most half of the(c, z)* and □*at least half of the(c, z)*, and are called: Proposition □A, □E, □I, □O, □M, □F, □H, and *S,* respectively; (3)◇*all(c, z)*, ◇*no(c, z)*, ◇*some(c, z),* ◇*not all(c, z),* ◇*most(c, z),* ◇*fewer than half of the(c, z),* ◇*at most half of the(c, z),* and ◇*at least half of the(c, z),* and are called: Proposition ◇A, ◇E, ◇I, ◇O, ◇M, ◇F, ◇H, and◇S, respectively. The definition of figures in generalized modal syllogisms are similar to that of Aristotelian syllogisms (Chen, 2020). Then, for example, the syllogism E□M◇O-1 is the abbreviation of the first figure syllogism *no*(*n, z)*∧□*most*(*c, n)*→◇*not all*(*c, z).* The others are similar.

Example 1:

Major premise: No person is a dog.

Minor premise: Most animals that can recognize numbers are necessarily person.

Conclusion: Not all animals that can recognize numbers are possibly dogs.

This syllogism can be formalized as '*no*(*n, z)*∧□*most*(*c, n)*→◇*not all*(*c, z)'*, which is abbreviated as E□M◇O-1.

3. Generalized Modal Syllogism System Including *Most*

Among the above eight quantifiers involved in the syllogisms studied in this paper, *no*, *some* and *not all* can be defined by *all*, and *fewer than half of the, at most half of the* and *at least half of the* defined by *most*. More specifically, according to the following Definition D3 and D4, *no*$=_{\text{def}}$*all*¬, *not all*$=_{\text{def}}$¬*all*, *some*$=_{\text{def}}$¬*all*¬, *fewer than half of the*$=_{\text{def}}$*most*¬, *at most half of the*$=_{\text{def}}$¬*most*, and *at least half of the*$=_{\text{def}}$¬*most*¬, therefore, the initial quantifiers of this system are only *all* and *most*.

3.1 Primitive Symbols

(1) lexical variables: *c*, *n*, z

(2) quantifier: *all*

(3) quantifier: *most*

(4) modality: $\Box$

(5) unary negative operator: $\neg$

(6) binary implication operator: $\rightarrow$

(7) brackets: (,)

3.2 Formation Rules

(1) If Q is a quantifier, c and z are lexical variables, then $Q(c, z)$ is a wff.

(2) If δ is a wff, then so are $\neg\delta$ and $\Box\delta$.

(3) If γ and δ are wffs, then so is $\gamma\rightarrow\delta$.

(4) Only the formulas obtained by the above three rules are wffs.

3.3 Basic Axioms

A1: If α is a valid formula in first-order logic, then $\vdash\alpha$.

A2: $\vdash no(n, z)\wedge\Box most(c, n)\rightarrow\Diamond not\ all(c, z)$(that is, the syllogism E$\Box$M$\Diamond$O-1).

3.4 Deductive Rules

Rule 1: From $\vdash(\gamma\wedge\delta\rightarrow\phi)$ and $\vdash(\phi\rightarrow\psi)$ infer $\vdash(\gamma\wedge\delta\rightarrow\psi)$.

Rule 2: From $\vdash(\gamma\wedge\delta\rightarrow\phi)$ infer $\vdash(\neg\phi\wedge\gamma\rightarrow\neg\delta)$.

Rule 3: From $\vdash(\gamma\wedge\delta\rightarrow\phi)$ infer $\vdash(\neg\phi\wedge\delta\rightarrow\neg\gamma)$.

3.5 Relevant Definitions

D1: $(\gamma\wedge\delta)=_{def}\neg(\gamma\rightarrow\neg q)$;

D2: $(\gamma\leftrightarrow\delta) =_{def} (\gamma\rightarrow\delta)\wedge(\delta\rightarrow\gamma)$;

D3: $(Q\neg)(c, z)=_{def} Q(c, D-z)$;

D4: $(\neg Q)(c, z)=_{def}$ It is not that $Q(c, z)$;

D5: $\Diamond Q(c, z)=_{def}\neg\Box\neg Q(c, z)$;

D6: *all*(c, z) is true iff $C\subseteq Z$ is true in any real world;

D7: *some*(c, z) is true iff $C\cap Z\neq\varnothing$ is true in any real world;

D8: *no*(c, z) is true iff $C \cap Z = \varnothing$ is true in any real world;

D9: *not all*(c, z) is true iff $C \nsubseteq Z$ is true in any real world;

D10: *most*(c, z) is true iff $|C \cap Z| \geq 0.6|C|$ is true in any real world;

D11: *at most half of the*(c, z) is true iff $|C \cap Z| < 0.4|C|$ is true in any real world;

D12: $\Box$*most*(c, z) is true iff $|C \cap Z| \geq 0.6|C|$ is true in any possible world;

D13: $\Diamond$*at most half of the*(c, z) is true iff $|C \cap Z| < 0.4|C|$ is true in at least one possible world.

The true value definitions of other quantifiers can be given similarly.

3.6 Relevant Facts

Fact 1 (Inner Negation):

(1.1) *all*(c, z)=*no*¬(c, z);

(1.2) *no*(c, z)=*all*¬(c, z);

(1.3) *some*(c, z)=*not all*¬(c, z);

(1.4) *not all*(c, z)=*some*¬(c, z);

(1.5) *most*(c, z)=*fewer than half of the*¬(c, z);

(1.6) *fewer than half of the*(c, z)=*most*¬(c, z);

(1.7) *at least half of the*(c, z)=*at most half of the*¬(c, z);

(1.8) *at most half of the*(c, z)=*at least half of the*¬(c, z).

Fact 2 (Outer Negation):

(2.1) ¬*all*(c, z)=*not all*(c, z);

(2.2) ¬*not all*(c, z)=*all*(c, z);

(2.3) ¬*no*(c, z)=*some*(c, z);

(2.4) ¬*some*(c, z)=*no*(c, z);

(2.5) ¬*most*(c, z)=*at most half of the*(c, z);

(2.6) ¬*at most half of the*(c, z)=*most*(c, z);

(2.7) $\neg$*fewer than half of the*(c, z)=*at least half of the*(c, z);

(2.8) $\neg$*at least half of the*(c, z)=*fewer than half of the*(c, z).

Fact 3 (Symmetry):

(3.1) *some*$(c, z) \leftrightarrow$*some*(z, c);

(3.2) *no*$(c, z) \leftrightarrow$*no*(z, c).

Fact 4 (Dual):

(4.1) $\neg\Box Q(c, z)=\Diamond\neg Q(c, z)$;

(4.2) $\neg\Diamond Q(c, z)=\Box\neg Q(c, z)$.

Fact 5 (Subordination):

(5.1) $\vdash\Box Q(c, z)\rightarrow Q(c, z)$;

(5.2) $\vdash\Box Q(c, z)\rightarrow\Diamond Q(c, z)$;

(5.3) $\vdash Q(c, z)\rightarrow\Diamond Q(c, z)$.

The above facts are common knowledge of first-order logic and generalized quantifier theory, and their proofs are omitted.

4. How to Screen Valid Generalized Modal Syllogisms

The basic rule that a valid modal syllogism should satisfy is that its conclusion is determined by the weakest premise. The simplest way to screen for valid generalized syllogisms is to add modalities to valid generalized syllogisms, and then delete all invalid syllogisms in line with this basic rule, the rest is valid. Xu and Zhang (2023) have proved that 12 valid generalized modal syllogisms can be obtained by adding modalities to any valid generalized syllogism.

Theorem 1: If the generalized syllogism $Q_1(n, z)\wedge Q_2(c, n)\rightarrow Q_3(c, z)$ is valid, in which Q_1, and Q_2 and Q_3 are generalized quantifiers, then the following 12 valid generalized modal syllogisms can be obtained by adding modalities to this one:

(1.1) $\Box Q_1(n, z)\wedge\Box Q_2(c, n)\rightarrow\Box Q_3(c, z)$;

(1.2) $\Box Q_1(n, z)\wedge\Box Q_2(c, n)\rightarrow Q_3(c, z)$;

(1.3) $\Box Q_1(n, z)\wedge\Box Q_2(c, n)\rightarrow\Diamond Q_3(c, z)$;

(1.4) $\Box Q_1(n, z) \wedge Q_2(c, n) \rightarrow Q_3(c, z)$;

(1.5) $\Box Q_1(n, z) \wedge Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.6) $Q_1(n, z) \wedge \Box Q_2(c, n) \rightarrow Q_3(c, z)$;

(1.7) $Q_1(n, z) \wedge \Box Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.8) $\Box Q_1(n, z) \wedge \Diamond Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.9) $\Diamond Q_1(n, z) \wedge \Box Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.10) $Q_1(n, z) \wedge \Diamond Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.11) $\Diamond Q_1(n, z) \wedge Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$;

(1.12) $Q_1(n, z) \wedge Q_2(c, n) \rightarrow \Diamond Q_3(c, z)$.

In order to find a universal method for screening generalized modal syllogisms, we first prove the validity of the specific generalized syllogism EMO-1. By adding modalities to it, one can obtain 12 valid generalized modal syllogisms according to Theorem 1. Then, other valid generalized modal syllogisms can be deduced from the 12 syllogisms by means of the above definitions, facts and rules.

Theorem 2 (EMO-1): The generalized syllogism *no*$(n, z) \wedge$*most*$(c, n) \rightarrow$*not all*(c, z) is valid.

Proof: Suppose that *no*(n, z) and *most*(c, n) are true, then $N \cap Z = \varnothing$ and $|C \cap N| \geq 0.6|C|$ are true in any real world according to Definition D8 and D10, respectively. Hence it can be concluded that $|C \cap Z| < 0.4|Z|$. And it follows that *at most half of the*(c, z) is true in any real world in the light of Definition D11. Thus, it can be seen that *not all*(c, z) are in line with Fact (5.10), just as desired.

Theorem 3: The following 12 valid generalized modal ones can be obtained by adding modalities to the generalized syllogism EMO-1:

(3.1) □E□M□O-1: $\Box$*no*$(n, z) \wedge \Box$*most*$(c, n) \rightarrow \Box$*not all*(c, z);

(3.2) □E□MO-1: $\Box$*no*$(n, z) \wedge \Box$*most*$(c, n) \rightarrow$*not all*(c, z);

(3.3) □E□M◇O-1: $\Box$*no*$(n, z) \wedge \Box$*most*$(c, n) \rightarrow \Diamond$*not all*$(c, z)$;

(3.4) □EMO-1: $\Box$*no*$(n, z) \wedge$*most*$(c, n) \rightarrow$*not all*(c, z);

(3.5) □EM◇O-1: $\Box$*no*$(n, z) \wedge$*most*$(c, n) \rightarrow \Diamond$*not all*$(c, z)$;

(3.6) E□MO-1: *no*(*n, z*)∧□*most*(*c, n*)→*not all*(*c, z*);

(3.7) E□M◇O-1: *no*(*n, z*)∧□*most*(*c, n*)→◇*not all*(*c, z*);

(3.8) □E◇M◇O-1: ☑*no*(*n, z*)∧◇*most*(*c, n*)→◇*not all*(*c, z*);

(3.9) ◇E□M◇O-1: ◇*no*(*n, z*)∧□*most*(*c, n*)→◇*not all*(*c, z*);

(3.10) E◇M◇O-1: *no*(*n, z*)∧◇*most*(*c, n*)→◇*not all*(*c, z*);

(3.11) ◇EM◇O-1: ◇*no*(*n, z*)∧*most*(*c, n*)→◇*not all*(*c, z*);

(3.12) EM◇O-1: *no*(*n, z*)∧*most*(*c, n*)→◇*not all*(*c, z*).

Proof: The proof can be obtained according to Theorem 1 and Theorem 2.

Theorem 4 (E□M◇O-1): *no*(*n, z*)∧□*most*(*c, n*)→◇*not all*(*c, z*) is valid.

Proof: Suppose that *no*(*n, z*) and □*most*(*c, n*) are true, then it is clear that $N∩Z=\varnothing$ is true in any real world according to Definition D8, and $|C∩N|\geq 0.6|C|$ is true in any possible world according to Definition D12. Due to the fact that a real world is also a possible world, one can conclude that $|C∩Z|<0.4|Z|$ is true in at least one possible world. And it follows that ◇*at most half of the*(*c, z*) is true in line with Definition D13. Thus, it can be seen that ◇*not all*(*c, z*) is true in the light of Fact (5.10), just as desired.

Theorem 4 is Theorem (3.7). In other words, the other syllogisms in Theorem 3 can be similarly proven by means of the above definition and facts. In fact, there is reducibility between different generalized modal syllogisms. Thus, other valid generalized modal syllogisms can be derived from a valid one. Now, taking the syllogism E□M◇O-1 as an example, the following Theorem 5 elaborates on this in detail.

Theorem 5: The validity of the following 17 syllogisms can be inferred from E□M◇O-1:

(1) ⊢E□M◇O-1→E□M◇O-2

(2) ⊢E□M◇O-1→E□A◇H-2

(3) ⊢E□M◇O-1→E□A◇H-2→E□A◇H-1

(4) ⊢E□M◇O-1→□A□MI-3

(5) ⊢E□M◇O-1→□A□MI-3→□M□AI-3

(6) ⊢E□M◇O-1→A□M◇I-1

(7) ⊢E□M◇O-1→□MA◇I-4

(8) ⊢E□M◇O-1→E□M◇O-2→A□F◇O-2

(10) $\vdash$E□M◇O-1→E□A◇H-2→A□E◇H-2

(11) $\vdash$E□M◇O-1→E□A◇H-2→A□E◇H-2→A□E◇H-4

(12) $\vdash$E□M◇O-1→E□A◇H-2→E□M◇O-1

(13) $\vdash$E□M◇O-1→E□A◇H-2→E□M◇O-1→E□M◇O-2

(14) $\vdash$E□M◇O-1→E□A◇H-2→E□A◇H-1→A□A◇S-1

(15) $\vdash$E□M◇O-1→□A□MI-3→□E□MO-3

(16) $\vdash$E□M◇O-1→□A□MI-3→□E□MO-3→□E□MO-4

(17) $\vdash$E□M◇O-1→□M□AI-3→□F□AO-3

Proof:

[1] $\vdash$*no*(*n, z*)∧□*most*(*c, n*)→◇*not all*(*c, z*) (i.e.E□M◇O-1, basic axiom A2)

[2] $\vdash$*no*(*z, n*)∧□*most*(*c, n*)→◇*not all*(*c, z*) (i.e.E□M◇O-2, by [1] and Fact (3.2))

[3] $\vdash$¬◇*not all*(*c, z*)∧*no*(*n, z*)→¬□*most*(*c, n*) (by [1] and Rule 2)

[4] $\vdash$□¬*not all*(*c, z*)∧*no*(*n, z*)→◇¬*most*(*c, n*) (by [3], Fact (4.2) and (4.1))

[5] $\vdash$□*all*(*c, z*)∧*no*(*n, z*)→◇*at most half of the*(*c, n*)

(i.e.E□A◇H-2, by [1], Fact (2.2) and (2.5))

[6] $\vdash$□*all*(*c, z*)∧*no*(*z, n*)→◇*at most half of the*(*c, n*) (i.e.E□A◇H-1, by [5] and Fact (3.2))

[7] $\vdash$¬◇*not all*(*c, z*)∧□*most*(*c, n*)→¬*no*(*n, z*) (by [1] and Rule 3)

[8] $\vdash$□¬*not all*(*c, z*)∧□*most*(*c, n*)→¬*no*(*n, z*) (by [7] and Fact (4.2))

[9] $\vdash$□*all*(*c, z*)∧□*most*(*c, n*)→*some*(*n, z*) (i.e.□A□MI-3, by [8], Fact (2.2) and (2.3))

[10] $\vdash$□*all*(*c, z*)∧□*most*(*c, n*)→*some*(*z, n*) (i.e.□M□AI-3, by [9] and Fact (3.1))

[11] $\vdash$*all*¬(*n, z*)∧□*most*(*c, n*)→◇*some*¬(*c, z*) (by [1], Fact (1.2) and (1.4))

[12] $\vdash$*all*(*n, D–z*)∧□*most*(*c, n*)→◇*some*(*c, D–z*) (i.e.A□M◇I-1, by [11] and Definition D3)

[13] $\vdash$*all*(*n, D–z*)∧□*most*(*c, n*)→◇*some*(*D–z, c*) (i.e.□MA◇I-4, by [12] and Fact (3.1))

[14] $\vdash$*all*¬(*z, n*)∧□*fewer than half of the*¬(*c, n*)→◇*not all*(*c, z*) (by [2], Fact (1.2) and (1.5))

[15] $\vdash$*all*(*z, D–n*)∧□*fewer than half of the*(*c, D–n*)→◇*not all*(*c, z*)

(i.e.A□F◇O-2, by [15] and Definition D3)

[16] $\vdash$□*no*¬(*c, z*)∧*all*¬(*n, z*)→◇*at most half of the*(*c, n*) (by [5], Fact (1.1) and (1.2))

[17] $\vdash$□*no*(*c, D–z*)∧*all*(*n, D–z*)→◇*at most half of the*(*c, n*)

(i.e.A□E◇H-2, by [17] and Definition D3)

[18] $\vdash$□*no*(*D–z, c*)∧*all*(*n, D–z*)→◇*at most half of the*(*c, n*)

(i.e.A□E◇H-4, by [17] and Fact (3.2))

[19] $\vdash$¬◇*at most half of the*(*c, n*)∧*no*(*n, z*)→¬□*all*(*c, z*) (by [5] and Rule 3)

[20] $\vdash$□¬*at most half of the*(*c, n*)∧*no*(*n, z*)→◇¬*all*(*c, z*) (by [19], Fact (4.2) and (4.1))

[21] $\vdash\Box most(c, n)\wedge no(n, z)\rightarrow\Diamond not\ all(c, z)$ (i.e.E□M◇O-1, by [20], Fact (2.6) and (2.1))

[22] $\vdash\Box most(c, n)\wedge no(z, n)\rightarrow\Diamond not\ all(c, z)$ (i.e.E□M◇O-2, by [21], and Fact (3.2))

[23] $\vdash\Box all(c, z)\wedge all\neg(z, n)\rightarrow\Diamond at\ least\ half\ of\ the\neg(c, n)$ (by [6], Fact (1.2) and (1.8))

[24] $\vdash\Box all(c, z)\wedge all(z, D-n)\rightarrow\Diamond at\ least\ half\ of\ the(c, D-n)$

(i.e.A□A◇S-1, by [23] and Definition D3)

[25] $\vdash\Box no\neg(c, z)\wedge\Box most(c, n)\rightarrow not\ all\neg(n, z)$ (by [9], Fact (1.1) and (1.3))

[26] $\vdash\Box no(c, D-z)\wedge\Box most(c, n)\rightarrow not\ all(n, D-z)$ (i.e.□E□MO-3, by [25] and Definition D3)

[26] $\vdash\Box no(D-z, c)\wedge\Box most(c, n)\rightarrow not\ all(n, D-z)$ (i.e.□E□MO-4, by [26] and Fact (3.2))

[27] $\vdash\Box all(c, z)\wedge\Box fewer\ than\ half\ of\ the\neg(c, n)\rightarrow not\ all\neg(z, n)$ (by [6], Fact (1.5) and (1.3))

[28] $\vdash\Box all(c, z)\wedge\Box fewer\ than\ half\ of\ the(c, D-n)\rightarrow not\ all(z, D-n)$

(i.e.□F□AO-3, by [23] and Definition D3)

Theorem 5 says that the other 17 valid generalized modal syllogisms can be deduced from the syllogism E□M◇O-1. Similarly, by using different valid generalized modal syllogisms as basic axioms, one can derive other generalized modal syllogisms with different figures and forms. In other words, this research method has universality.

5. Conclusion and Future Work

In summary, the simple and feasible methods for screening valid generalized modal syllogisms are as follows: (1) First, prove the validity of a generalized syllogism, then add at least one necessary modality (□) or possible modality (◇) to this syllogism, (2) Secondly, according to the basic fact that the conclusion of a modal syllogism is determined by the weakest premise, 12 valid generalized modal syllogisms in the same figure can be obtained by deleting invalid syllogisms; (3) Finally, taking each of these 12 syllogisms as a basic axiom, one can derive generalized modal syllogisms with different figures and forms by means of the above definitions, facts, and deductive rules. The above results obtained by this deductive method are logically consistent.

Undoubtedly, this method provides a concise and unified mathematical research paradigm for the study of other generalized modal syllogisms. Theorem 2 not only reveals the reducible relationship between different syllogisms, but also highlights the idea of universal connections between/among different knowledge or propositions. This study not only has important theoretical value for us to deeply reveal the connections between/among things, but also has important practical significance for knowledge reasoning in artificial intelligence.

Therefore, it is necessary to conduct in-depth research on the validity and reducibility of other generalized modal syllogisms. For example, how many of the 46592 syllogisms mentioned at the beginning are valid? Can we establish a sound and complete axiomatic system for them?

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