SCIREA Journal of Mathematics
ISSN: 2995-5823
http://www.scirea.org/journal/Mathematics
April 1, 2024
Volume 9, Issue 2, April 2024
https://doi.org/10.54647/mathematics110479

Knowledge Reasoning about the Aristotelian Syllogism *IAI-4*

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Abstract

On the basis of set theory, propositional logic and generalized quantifier theory, this paper indicates that the other 23 valid syllogisms can be only derived from the syllogism *IAI-4*. These derivations use the symmetry of quantifiers *no* and *some*, the definitions of inner and outer negation of Aristotelian quantifiers, deductive rules of propositional logic, and some relevant facts, and so on. Moreover, this paper establishes a concise formalized axiomatic system for Aristotelian syllogistic logic and puts forward a research paradigm for the study of other syllogistic. This formal method aligns with the idea of knowledge reasoning and knowledge mining in artificial intelligence.

Key words: Aristotelian syllogisms; Aristotelian quantifiers; symmetry; reducibility

1. Introduction

Aristotelian syllogisms are common forms of reasoning in natural language and logic since Aristotle. This paper focuses on the reducibility of Aristotelian syllogisms which means that the validity of another syllogism can be derived from one valid syllogism. It is known that only 24 are valid out of the 256 Aristotelian syllogisms (Xiaojun and Baoxiang, 2021). Some scholars have already deduced the other valid syllogisms from different basic axioms. In deriving all other valid syllogisms, at least two valid syllogisms have been used as basic axioms in previous research, such as Shushan (1988), Xiaojun (2018), and Beihai et al. (2018). Łukasiewicz (1957) takes the two syllogisms *AAA-1* and *AII-3* as basic axioms to deduce the remaining 22 valid syllogisms, while the two syllogisms *AAA-1* and *EAE-1* as basic axioms in Xiaojun and Sheng (2016) to do the same work. Xiaojun et al. (2022), Cheng (2022), Long (2023), and Hui (2023) used merely one syllogism (that is, *EIO-1*, *IAI-3*, *AEE-4*, and *EIO-2* respectively) as a basic axiom to infer the other 23 valid ones. Inspired by previous works, this paper takes just one syllogism *IAI-4* as a basic axiom to infer the other 23 valid syllogisms.

2. Related Basic Knowledge

Throughout this paper, let Q be any of the four Aristotelian quantifiers (that is, *all*, *no*, *some* and *not all*). w, v and z represent lexical variables. The sets composed of w, v, z are respectively W, V, Z. Besides, D indicates the domain of lexical variables. 'wff' is an abbreviation for 'well-formed formula'. m, n, s and t are wffs, '$\vdash m$' means that the wff m is provable. The others are similar.

An Aristotelian syllogism consists of categorical propositions which involve the following four types of Propositions A, E, I and O. Proposition A means that all ws are z, and can be formalized as $all(w, z)$. Proposition E indicates that no ws are z, and can be symbolized as $no(w, z)$. Proposition I denotes that some ws are z, and can be formalized as $some(w, z)$. Proposition O means that not all ws are z, and can be written as $not\ all(w, z)$.

The definitions of the figures in Aristotelian syllogisms are as usual. For example, the Aristotelian syllogism '*some* zs are v, and *all* vs are w, then *some* ws are z' can be abbreviated as *IAI-4* and formalized as $some(z, v) \wedge all(v, w) \rightarrow some(w, z)$, the others are similar.

3. Axiomatization of Aristotelian Syllogistic Logic

This formalized axiomatic system of Aristotelian syllogistic logic is based on the following four components: primitive symbols, formation rules of well-formed formulas (short for wff), basic axioms and inference rules.

3.1 Primitive Symbols

(3.1.1) lexical variables: w, v, z

(3.1.2) quantifier: *no*

(3.1.3) unary negative connective: $\neg$

(3.1.4) binary implication connective: $\rightarrow$

(3.1.5) brackets: (,)

3.2 Formation Rules

(3.2.1) If Q is a quantifier, w and z are lexical variables, then $Q(w, z)$ is a wff.

(3.2.2) If m is a wff, then $\neg m$ is a wff.

(3.2.3) If m and n are wffs, then $m\wedge n$ and $m\rightarrow n$ are wffs.

(3.2.4) Only the formulas obtained from the above three rules are wffs.

3.3 Basic Axioms

(3.3.1) A1: if m is a valid formula in propositional logic, then $\vdash m$.

(3.3.2) A2: $\vdash some(z, v)\wedge all(v, w)\rightarrow some(w, z)$ (that is, the syllogism *IAI-4*).

3.4 Deductive Rules

(3.4.1) Rule 1: $\vdash(m\wedge n\rightarrow t)$ can be obtained from $\vdash(m\wedge n\rightarrow s)$ and $\vdash(s\rightarrow t)$.

(3.4.2) Rule 2: $\vdash(\neg s\wedge m\rightarrow\neg n)$ can be obtained from $\vdash(m\wedge n\rightarrow s)$.

3.5 Relevant Definitions

(3.5.1) Definition of connective$\leftrightarrow$: $(m\leftrightarrow n)=_{\text{def}}(m\rightarrow n)\wedge(n\rightarrow m)$;

(3.5.2) Definition of conjunction connective $\wedge$: $(m\wedge n)=_{def}\neg(m\rightarrow\neg n)$;

(3.5.3) Definition of inner negative quantifier: $(Q\neg)(w, z)=_{def}Q(w, D\text{-}z)$;

(3.5.4) Definition of outer negative quantifier: $(\neg Q)(w, z)=_{def}$ It is not that $Q(w, z)$;

(3.5.5) Definition of truth value of the quantifier: *all*$(w, z)=_{def}W\subseteq Z$;

(3.5.6) Definition of truth value of the quantifier: *no*$(w, z)=_{def}W\cap Z=\varnothing$;

(3.5.7) Definition of truth value of the quantifier: *some*$(w, z)=_{def}W\cap Z\neq\varnothing$;

(3.5.8) Definition of truth value of the quantifier: *not all*$(w, z)=_{def}W\nsubseteq Z$.

3.6 Relevant Facts

Fact 1 (inner negation):

(1.1) $\vdash$*all*$(w, z)\leftrightarrow$*no*$\neg(w, z)$; (1.2) $\vdash$*no*$(w, z)\leftrightarrow$*all*$\neg(w, z)$;

(1.3) $\vdash$*some*$(w, z)\leftrightarrow$*not all*$\neg(w, z)$; (1.4) $\vdash$*not all*$(w, z)\leftrightarrow$*some*$\neg(w, z)$.

Fact 2 (outer negation):

(2.1) $\vdash\neg$*not all*$(w, z)\leftrightarrow$*all*(w, z); (2.2) $\vdash\neg$*all*$(w, z)\leftrightarrow$*not all*(w, z);

(2.3) $\vdash\neg$*no*$(w, z)\leftrightarrow$*some*(w, z); (2.4) $\vdash\neg$*some*$(w, z)\leftrightarrow$*no*(w, z).

Fact 3 (symmetry):

(3.1) $\vdash$*some*$(w, z)\leftrightarrow$*some*(z, w); (3.2) $\vdash$*no*$(w, z)\leftrightarrow$*no*(z, w).

Fact 4 (assertoric subalternations):

(4.1) $\vdash$*all*$(w, z)\rightarrow$*some*(w, z); (4.2) $\vdash$*no*$(w, z)\rightarrow$*not all*(w, z).

4. The Other 23 Valid Syllogisms are Derived from the Syllogism *IAI-4*

In the following Theorem 2, *IAI-4*→*AII-1* means that the validity of the syllogism *AII-1* can be derived from that of the syllogism *IAI-4*. One can prove the validity of the syllogism *IAI-4* on the basis of the truth values in Definition 3.5.

Theorem 1 (*IAI-4*): The Aristotelian syllogism *some*$(z, v)\wedge$*all*$(v, w)\rightarrow$*some*(w, z) is valid.

Proof: Suppose that *some*(z, v) and *all*(v, w) are true, then $Z\cap V\neq\varnothing$ and $V\subseteq W$ are true according to Definition (3.5.7) and (3.5.5), respectively, in which the sets composed of *w*, *v*, *z* are respectively *W*, *V*, *Z*. Thus, it can be seen that $W \cap Z \neq \emptyset$ is true. Then *some*(*w*, *z*) is true in the light of Definition (3.5.7). It follows that *some*(*z*, *v*)∧*all*(*v*, *w*)→*some*(*w*, *z*) is valid, just as expected.

Theorem 2: The other 23 valid syllogisms can be just deduced from the syllogism *IAI-4*. According to the steps of proof, one can obtain the following:

(1) *IAI-4→AII-1*

(2) *IAI-4→IAI-3*

(3) *IAI-4→IAI-3→AII-3*

(4) *IAI-4→EIO-4*

(5) *IAI-4→EIO-4→EIO-3*

(6) *IAI-4→EIO-4→EIO-2*

(7) *IAI-4→EIO-4→EIO-2→EIO-1*

(8) *IAI-4→EIO-4→AEE-4*

(9) *IAI-4→EIO-4→AEE-4→AEE-2*

(10) *IAI-4→EIO-4→AEE-4→EAE-1*

(11) *IAI-4→EIO-4→AEE-4→EAE-1→EAE-2*

(12) *IAI-4→EIO-4→AEE-4→AEO-4*

(13) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2*

(14) *IAI-4→EIO-4→AEE-4→AEO-4→AAI-4*

(15) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2→AAI-1*

(16) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2→AAI-1→EAO-3*

(17) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2→AAI-1→EAO-3→EAO-4*

(18) *IAI-4→IAI-3→OAO-3*

(19) *IAI-4→EIO-4→EIO-2→AOO-2*

(20) *IAI-4→EIO-4→AEE-4→EAE-1→AAA-1*

(21) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2→EAO-2*

(22) *IAI-4→EIO-4→AEE-4→AEO-4→AEO-2→AAI-1→EAO-1*

(23) $IAI\text{-}4 \rightarrow EIO\text{-}4 \rightarrow AEE\text{-}4 \rightarrow AEO\text{-}4 \rightarrow AEO\text{-}2 \rightarrow AAI\text{-}1 \rightarrow EAO\text{-}3 \rightarrow AAI\text{-}3$

Proof:

[1] $\vdash some(z, v) \wedge all(v, w) \rightarrow some(w, z)$ (i.e. *IAI-4*, basic axiom A2)

[2] $\vdash some(w, z) \leftrightarrow some(z, w)$ (by Fact (3.1))

[3] $\vdash some(z, v) \wedge all(v, w) \rightarrow some(z, w)$ (i.e. *AII-1*, by [1] and [2])

[4] $\vdash some(z, v) \leftrightarrow some(v, z)$ (by Fact (3.1))

[5] $\vdash some(v, z) \wedge all(v, w) \rightarrow some(w, z)$ (i.e. *IAI-3*, by [1] and [4])

[6] $\vdash some(v, z) \wedge all(v, w) \rightarrow \mathrm{some}(z, w)$ (i.e. *AII-3*, by [2] and [5])

[7] $\vdash \neg some(w, z) \wedge some(z, v) \rightarrow \neg all(v, w)$ (by [1] and Rule (3.4.2))

[8] $\vdash no(w, z) \wedge some(z, v) \rightarrow not\ all(v, w)$ (i.e. *EIO-4*, by [7], Fact (2.2) and Fact (2.4))

[9] $\vdash no(z, w) \wedge some(z, v) \rightarrow not\ all(v, w)$ (i.e. *EIO-3*, by [8] and Fact (3.2))

[10] $\vdash no(w, z) \wedge some(v, z) \rightarrow not\ all(v, w)$ (i.e. *EIO-2*, by [8] and Fact (3.1))

[11] $\vdash no(z, w) \wedge some(v, z) \rightarrow not\ all(v, w)$ (i.e. *EIO-1*, by [10] and Fact (3.2))

[12] $\vdash \neg not\ all(v, w) \wedge no(w, z) \rightarrow \neg some(z, v)$ (by [8] and Rule (3.4.2))

[13] $\vdash all(v, w) \wedge no(w, z) \rightarrow no(z, v)$ (i.e. *AEE-4*, by [12], Fact (2.1) and Fact (2.4))

[14] $\vdash all(v, w) \wedge no(z, w) \rightarrow no(z, v)$ (i.e. *AEE-2*, by [13] and Fact (3.2))

[15] $\vdash all(v, w) \wedge no(w, z) \rightarrow no(v, z)$ (i.e. *EAE-1*, by [13] and Fact (3.2))

[16] $\vdash all(v, w) \wedge no(z, w) \rightarrow no(v, z)$ (i.e. *EAE-2*, by [15] and Fact (3.2))

[17] $\vdash all(v, w) \wedge no(w, z) \rightarrow not\ all(z, v)$ (i.e. *AEO-4*, by [13], Rule (3.4.1) and Fact (4.2))

[18] $\vdash all(v, w) \wedge no(z, w) \rightarrow not\ all(z, v)$ (i.e. *AEO-2*, by [17] and Fact (3.2))

[19] $\vdash \neg not\ all(z, v) \wedge all(v, w) \rightarrow \neg no(w, z)$ (by [17] and Rule (3.4.2))

[20] $\vdash all(z, v) \wedge all(v, w) \rightarrow some(w, z)$ (i.e. *AAI-4*, by [19], Fact (2.1) and Fact (2.3))

[21] $\vdash \neg not\ all(z, v) \wedge all(v, w) \rightarrow \neg no(z, w)$ (by [18] and Rule (3.4.2))

[22] $\vdash all(z, v) \wedge all(v, w) \rightarrow some(z, w)$ (i.e. *AAI-1*, by [21], Fact (2.1) and Fact (2.3))

[23] $\vdash \neg some(z, w) \wedge all(z, v) \rightarrow \neg all(v, w)$ (by [22] and Rule (3.4.2))

[24] $\vdash no(z, w) \wedge all(z, v) \rightarrow not\ all(v, w)$ (i.e. *EAO-3*, by [23], Fact (2.2) and Fact (2.4))

[25] $\vdash no(w, z) \wedge all(z, v) \rightarrow not\ all(v, w)$ (i.e. *EAO-4*, by [24] and Fact (3.2))

[26] $\vdash not\ all\neg(v, z) \wedge all(v, w) \rightarrow not\ all\neg(w, z)$ (by [5] and Fact (1.3))

[27] $\vdash not\ all(v, D\text{-}z) \wedge all(v, w) \rightarrow not\ all(w, D\text{-}z)$ (i.e. *OAO-3*, by [26] and Definition (3.5.3))

[28] $\vdash all\neg(w, z) \wedge not\ all\neg(v, z) \rightarrow not\ all(v, w)$ (by [10], Fact (1.2) and Fact (1.3))

[29] $\vdash all(w, D\text{-}z) \wedge not\ all(v, D\text{-}z) \rightarrow not\ all(v, w)$ (i.e. *AOO-2*, by [28] and Definition (3.5.3))

[30] $\vdash all(v, w) \wedge all\neg(w, z) \rightarrow all\neg(v, z)$ (by [15] and Fact (1.2))

[31] $\vdash all(v, w) \wedge all(w, D\text{-}z) \rightarrow all(v, D\text{-}z)$ (i.e. *AAA-1*, by [30] and Definition (3.5.3))

[32] $\vdash no\neg(v, w) \wedge all\neg(z, w) \rightarrow not\ all(z, v)$ (by [18], Fact (1.1) and Fact (1.2))

[33] $\vdash no(v, D\text{-}w) \wedge all(z, D\text{-}w) \rightarrow not\ all(z, v)$ (i.e. *EAO-2*, by [32] and Definition (3.5.3))

[34] $\vdash all(z, v) \wedge no\neg(v, w) \rightarrow not\ all\neg(z, w)$ (by [22], Fact (1.1) and Fact (1.3))

[35] $\vdash all(z, v) \wedge no(v, D\text{-}w) \rightarrow not\ all(z, D\text{-}w)$ (i.e. *EAO-1*, by [34] and Definition (3.5.3))

[36] $\vdash all\neg(z, w) \wedge all(z, v) \rightarrow some\neg(v, w)$ (by [24], Fact (1.2) and Fact (1.4))

[37] $\vdash all(z, D\text{-}w) \wedge all(z, v) \rightarrow some(v, D\text{-}w)$ (i.e. *AAI-3*, by [36] and Definition (3.5.3))

So far, the remaining 23 valid syllogisms are inferred from the syllogism *IAI-4* with the help of 37 reasoning steps.

5. Conclusion and Future Work

On the basis of propositional logic and generalized quantifier theory, this paper proves that the other 23 valid syllogisms are only derived from the syllogism *IAI-4*. To be specific, this paper takes full advantage of deductive rules in propositional logic, the definitions of inner and outer negation of Aristotelian quantifiers, and the symmetry of quantifiers *no* and *some*, and then constructs a simple formal axiom system for Aristotelian syllogistic logic. This formal method aligns with the idea of knowledge reasoning and knowledge mining in artificial intelligence.

However, how to reinforce the research results of syllogistic logic and generalized quantifier theory, and further to enhance their roles in logical reasoning and natural language processing?

These problems still need in-depth discussion.

Acknowledgements

This work was supported by the National Social Science Foundation of China under Grant No.22&ZD295.

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