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The Reducibility of the Generalized Syllogism *MMI-4* with the Quantifiers in Square{*most*} and Square{*some*}

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Abstract

This paper firstly proves the validity of the generalized syllogism *MMI-4* with the quantifiers in Square{*most*} and Square{*some*}, and then making full use of the relevant definitions, facts, and reasoning rules to infer the other 20 valid generalized ones from the syllogism *MMI-4*. In other words, there are reducible relationships between/among these valid generalized syllogisms. The reason for this is because any quantifier in Square{*some*} can define the other three quantifiers, and so can any quantifier in Square{*most*}. This study has important theoretical value for natural language information processing.

Keywords: generalized quantifiers; generalized syllogisms; reducibility; validity

1.Introduction

Syllogism reasoning is a common form of reasoning in deductive reasoning, which plays an important role in natural language and human society. And there are many studies on Aristotelian syllogisms (Łukasiewicz, 1957; Zhang and Li, 2016; Hao, 2023), Aristotelian modal syllogisms (Johnson 2004; Malink, 2013; Zhang, 2023) and rational syllogisms (Ivanov and Vakarelov, 2012). But there are few works on the generalized syllogisms (Endrullis and Moss, 2015).

Thus this paper focuses on the study of non-trivial generalized syllogisms, which contain at least one and at most three of non-trivial generalized quantifiers. There are trivial generalized quantifiers (i.e. Aristotelian quantifiers, that is, *not all*, *some*, *all*, *no*) and non-trivial generalized quantifiers (such as *both*, *most*, *few, several,* and so on), which has infinite number in natural language. Thus, there is infinite number of non-trivial generalized syllogisms. Therefore, this study has important theoretical value for natural language information processing.

2. Preliminaries

In the following, let *n*, *t* and *v* be lexical variables, *D* be the domain of lexical variable. The sets composed of *n*, *t* and *v* are respectively *N*, *T* and *V*. '$|N\cap V|$' indicates the cardinality for the intersection of *N* and *V*. And *Q* represents a generalized quantifier, $\neg Q$ and $Q\neg$ its outer and inner quantifier, respectively. Let φ, γ, ε and δ be well-formed formulas (abbreviated as wff). '$\varphi =_{def} \gamma$' shows that φ can be defined by γ. '$\vdash \varphi$' means φ is provable. The others are similar. And the common operators '$\wedge$, $\neg$, $\rightarrow$, $\leftrightarrow$' are respectively symbols of conjunction, negation, conditionality, biconditionality in mathematical logic (Hamilton, 1978).

The generalized syllogisms discussed in this paper only involve Aristotelian quantifiers (namely, *not all*, *some*, *all*, *no*) and the four common non-trivial generalized quantifiers as follows: *most*, *fewer than half of the*, *at least half of the*, *at most half of the*. These four Aristotelian quantifier forms Square{*some*}, and the last four quantifiers Square{*most*}. These 8 quantifiers correspond to the following 8 types of propositions: *all*(*n*, *v*), *no*(*n*, *v*), *some*(*n*, *v*), *not all*(*n*, *v*), *most*(*n*, *v*), *fewer than half of the*(*n*, *v*), *at least half of the*(*n*, *v*), *at most half of the*(*n*, *v*), and they are respectively abbreviated as Proposition *A*, *E*, *I*, *O*, *M*, *F*, *S*, and *H*. The generalized syllogism as the basis for reasoning in this paper is the first figure syllogism *most*(*n*, *t*)$\wedge$*most*(*t*, *v*)$\rightarrow$*some*(*v*, *n*) ,which can be shortened as *MMI-4*. An instance of the syllogism *MMI-4* is as follows:

Major premise: Most students in this class are boys.

Minor premise: Most boys like maths.

Conclusion: Some students who like maths are students in this class.

Let t be boys, v be students in this class, and n be students who like math. Then the instance can be formalized as $most(n, t) \wedge most(t, v) \rightarrow some(v, n)$, and denoted by *MMI-4*. Others are similar to this.

3. The Generalized Syllogism Formal System

The system consists of the following parts: primitive symbols, basic axioms, formative and deductive rules, relevant definitions and facts.

3.1 Primitive Symbols

(3.1.1) lexical variables: n, t , v

(3.1.2) quantifiers: *most*, *some*

(3.1.3) operators: $\neg$, $\rightarrow$

(3.1.4) brackets: (,)

3.2 Formative Rules

(3.2.1) If Q is a quantifier, n and v are lexical variables, then $Q(n, v)$ is a wff.

(3.2.2) If φ is a wff, then so is $\neg\varphi$.

(3.2.3) If γ and φ are wffs, then so is $\gamma\rightarrow\varphi$.

(3.2.4) Merely the formulas constructed by the above rules are wffs.

3.3 Basic Axioms

A1: If φ is a valid formula in classical logic, then $\vdash \varphi$.

A2: $\vdash most(n, t) \wedge most(t, v) \rightarrow some(v, n)$ (that is, the syllogism *MMI-4*).

3.4 Deductive Rules

R1: (subsequent weakening): From $\vdash (\varphi\wedge\gamma\rightarrow\varepsilon)$ and $\vdash (\varepsilon\rightarrow\delta)$ infer $\vdash (\varphi\wedge\gamma\rightarrow\delta)$.

R2: (anti-syllogism): From $\vdash (\varphi\wedge\gamma\rightarrow\varepsilon)$ infer $\vdash (\neg\varepsilon\wedge\varphi\rightarrow\neg\gamma)$.

R3: (anti-syllogism): From $\vdash(\varphi\wedge\gamma\rightarrow\varepsilon)$ infer $\vdash(\neg\varepsilon\wedge\gamma\rightarrow\neg\varphi)$.

R4: (antecedent strengthening): From $\vdash(\varepsilon\rightarrow\delta)$ and $\vdash(\delta\wedge\gamma\rightarrow\varphi)$ infer $\vdash(\varepsilon\wedge\gamma\rightarrow\varphi)$.

R5: (antecedent strengthening): From $\vdash(\varepsilon\rightarrow\gamma)$ and $\vdash(\delta\wedge\gamma\rightarrow\varphi)$ infer $\vdash(\delta\wedge\varepsilon\rightarrow\varphi)$.

3.5 Relevant Definitions

D1 (conjunction): $(\varphi\wedge\gamma)=_{def}\neg(\varphi\rightarrow\neg\gamma)$;

D2 (bicondition): $(\varphi\leftrightarrow\gamma)=_{def}(\varphi\rightarrow\gamma)\wedge(\gamma\rightarrow\varphi)$;

D3 (inner negation): $(Q\neg)(n, v)=_{def}Q(n, D-v)$;

D4 (outer negation): $(\neg Q)(n, v)=_{def}$ It is not that $Q(n, v)$;

D5 (truth value): *all*$(n, v)=_{def}N\subseteq V$;

D6 (truth value): *some*$(n, v)=_{def}N\cap V\neq\varnothing$;

D8 (truth value): *no*$(n, v)=_{def}N\cap V=\varnothing$;

D9 (truth value): *not all*$(n, v)=_{def}N\nsubseteq V$;

D10 (truth value): *most*(n, v) is true iff $|N\cap V|>0.5|N|$ is true;

D11 (truth value): *at most half of the*(n, v) is true iff $|N\cap V|\leq0.5|N|$;

D12 (truth value): *fewer than half of the*(n, v) is true iff $|N\cap V|<0.5|N|$ is true;

D13 (truth value): *at least half of the*(n, v) is true iff $|N\cap V|\geq0.5|N|$ is true.

3.5 Relevant Facts

Fact 1 (inner negation):

(1.1) *all*(n, v)=*no*$\neg(n, v)$;

(1.2) *no*(n, v)=*all*$\neg(n, v)$;

(1.3) *some*(n, v)=*not all*$\neg(n, v)$;

(1.4) *not all*(n, v)=*some*$\neg(n, v)$;

(1.5) *most*(n, v)=*fewer than half of the*$\neg(n, v)$;

(1.6) *fewer than half of the*(n, v)=*most*$\neg(n, v)$;

(1.7) *at least half of the*(n, v)=*at most half of the* (n, v);

(1.8) *at most half of the*(*n, v*)=*at least half of the* (*n, v*).

Fact 2 (outer negation):

(2.1) ¬*all*(*n, v*)=*not all*(*n, v*);

(2.2) ¬*not all*(*n, v*)=*all*(*n, v*);

(2.3) ¬*no*(*n, v*)=*some*(*n, v*);

(2.4) ¬*some*(*n, v*)=*no*(*n, v*);

(2.5) ¬*most*(*n, v*)=*at most half of the*(*n, v*);

(2.6) ¬*at most half of the*(*n, v*)=*most*(*n, v*);

(2.7) ¬*fewer than half of the*(*n, v*)=*at least half of the*(*n, v*);

(2.8) ¬*at least half of the*(*n, v*)=*fewer than half of the*(*n, v*).

Fact 3 (symmetry):

(3.1) *some*(*n, v*)↔*some*(*v, n*);

(3.2) *no*(*n, v*)↔*no*(*v, n*).

Fact 4 (Subordination) :

(4.1) ⊢ *no*(*n, v*)→*not all*(*n, v*);

(4.2) ⊢ *all*(*n, v*)→*some*(*n, v*);

(4.3) ⊢ *all*(*n, v*)→*most*(*n, v*);

(4.4) ⊢ *most*(*n, v*)→*some*(*n, v*);

(4.5) ⊢ *all*(*n, v*)→*at least half of the*(*n, v*);

(4.6) ⊢ *at least half of the*(*n, v*)→*some*(*n, v*);

(4.7) ⊢ *fewer than half of the*(*n, v*)→*not all*(*n, v*);

(4.8) ⊢ *most*(*n, v*)→*at least half of the*(*n, v*);

(4.9) ⊢ *at most half of the*(*n, v*)→*fewer than half of the*(*n, v*).

The above facts are the basic knowledge in generalized quantifier theory (Peters and Westerståhl, 2006), so their proofs are omitted.

4. The Reducible Relationships between/among Valid Generalized Syllogisms

The following Theorem 1 shows the generalized syllogism *MMI-4* is valid. '*MMI-4* *MMI-1*' in Theorem 2 indicates that the validity of the syllogism *MMI-1* can be deduced from that of the syllogism *MMI-4*. That is to say that there is a reducible relationship between these two syllogisms. The others are similar.

Theorem 1(*MMI-4*)：The generalized syllogism $most(n, t) \wedge most(t, v) \rightarrow some(v, n)$ is valid.

Proof: Suppose that $most(n, t)$ and $most(t, v)$ are true, then it is easy to obtain that $|N \cap T| > 0.5|N|$ and $|T \cap V| > 0.5|T|$ is true by Definition D10. Therefore, it can be concluded that $N \cap V \neq \varnothing$ is true. According to Definition D6, $some(v, n)$ is true. This proves that the syllogism $most(n, t) \wedge most(t, v) \rightarrow some(v, n)$ is valid.

Theorem 2: The validity of the following 20 generalized syllogisms can be derived from the syllogism *MMI-4*:

(2.1) ⊢*MMI-4*→*MMI-1*

(2.2) ⊢*MMI-4*→*AMI-4*

(2.3) ⊢*MMI-4*→*AMI-4*→*MAI-1*

(2.4) ⊢*MMI-4*→*MAI-4*

(2.5) ⊢*MMI-4*→*MAI-4*→*AMI-1*

(2.6) ⊢*MMI-4*→*EMH-4*

(2.7) ⊢*MMI-4*→*EMH-4*→*EMH-3*

(2.8) ⊢*MMI-4*→*EMH-4*→*EMF-4*

(2.9) ⊢*MMI-4*→*EMH-4*→*EMH-3*→*EMF-3*

(2.10) ⊢*MMI-4*→*EMH-4*→*EAM-4*

(2.11) ⊢*MMI-4*→*EMH-4*→*EAM-4*→*EAM-3*

(2.12) ⊢*MMI-4*→*EMH-4*→*EAM-4*→*EAF-4*

(2.13) ⊢*MMI-4*→*EMH-4*→*EAM-4*→*EAM-3*→*EAF-3*

(2.14) ⊢*MMI-4*→*MMI-1*→*FMO-1*

(2.15) ⊢*MMI-4*→*MMI-1*→*FMO-1*→*FAO-1*

(2.16) ⊢*MMI-4*→*MMI-1*→*FMO-1*→*HMO-1*

(2.17) ⊢*MMI-4*→*AMI-4*→*EAH-4*

(2.18) ⊢*MMI-4*→*AMI-4*→*EAH-4*→*EAH-3*

(2.19) ⊢*MMI-4*→*AMI-4*→*MEO-4*

(2.20) ⊢*MMI-4*→*AMI-4*→*MEO-4*→*MEO-2*

Proof:

[1] ⊢*most*(*n, t*)∧*most*(*t, v*)→*some*(*v, n*) (i.e. *MMI-4*, basic axiom A2)

[2] ⊢*most*(*n, t*)∧*most*(*t, v*)→*some*(*n, v*) (i.e. *MMI-1*, by [1] and Fact(3.1))

[3] ⊢*all*(*n, t*)∧*most*(*t, v*)→*some*(*v, n*) (i.e. *AMI-4*, by [1], Fact(4.3) and R4)

[4] ⊢*all*(*n, t*)∧*most*(*t, v*)→*some*(*n, v*) (i.e. *MAI-1*, by [4] and Fact(3.1))

[5] ⊢*most*(*n, t*)∧*all*(*t, v*)→*some*(*v, n*) (i.e. *MAI-4*, by [1], Fact(4.3) and R5)

[6] ⊢*most*(*n, t*)∧*all*(*t, v*)→*some*(*n, v*) (i.e. *AMI-1*, by [5] and Fact(3.1))

[7] ⊢¬*some*(*v, n*)∧*most*(*n, t*)→¬*most*(*t, v*) (by [1] and R2)

[8] ⊢*no*(*v, n*)∧*most*(*n, t*)→*at most half of the*(*t, v*) (i.e. *EMH-4*, by [7], Fact(2.4) and Fact(2.5))

[9] ⊢*no*(*n, v*)∧*most*(*n, t*)→*at most half of the*(*t, v*) (i.e. *EMH-3*, by [8] and Fact(3.2))

[10] ⊢*no*(*v, n*)∧*most*(*n, t*)→*fewer than half of the*(*t, v*) (i.e. *EMF-4*, by [8], R1 and Fact(4.9))

[11] ⊢*no*(*n, v*)∧*most*(*n, t*)→*fewer than half of the*(*t, v*) (i.e. *EMF-3*, by [9], R1 and Fact(4.9))

[12] ⊢*no*(*v, n*)∧*all*(*n, t*)→*at most half of the*(*t, v*) (i.e. *EAM-4*, by [8], Fact(4.3) and R5)

[13] ⊢*no*(*n, v*)∧*all*(*n, t*)→*at most half of the*(*t, v*) (i.e. *EAM-3*, by [12] and Fact(3.2))

[14] ⊢*no*(*v, n*)∧*all*(*n, t*)→*fewer than half of the*(*t, v*) (i.e. *EAF-4*, by [12], R1 and Fact(4.9))

[15] ⊢*no*(*n, v*)∧*all*(*n, t*)→*fewer than half of the*(*t, v*) (i.e. *EAF-3*, by [13] , R1 and Fact(4.9))

[16] ⊢*most*(*n, t*)∧*fewer than half of the*¬(*t, v*)→*not all*¬(*n, v*) (by [2], Fact(1.2) and Fact(1.3))

[17] ⊢*most*(*n, t*)∧*fewer than half of the*(*t, D-v*)→*not all*(*m, D-v)* (i.e. *FMO-1*, by [16] and D3)

[18] ⊢*all*(*n, t*)∧*fewer than half of the*(*t, D-v*)→*not all*(*m, D-v)*

(i.e. *FAO-1*, by [17], Fact(4.3) and R4)

[19] ⊢*most*(*n, t*)∧*at most half of the*(*t, D-v*)→*not all*(*m, D-v)*

(i.e. *HMO-1*, by [17], Fact(4.3) and R5)

[20] ⊢¬*some*(*v, n*)∧*all*(*n, t*)→¬*most*(*t, v*) (by [3] and R2)

[21] $\vdash no(v, n) \land all(n, t) \rightarrow at\ most\ half\ of\ the(t, v)$ (i.e. *EAH-4*, by [20], Fact (2.4) and Fact(2.5))

[22] $\vdash no(n, v) \land all(n, t) \rightarrow at\ most\ half\ of\ the(t, v)$ (i.e. *EAH-3*, by [21] and Fact(3.2))

[23] $\vdash \neg some(v, n) \land most(t, v) \rightarrow \neg all(n, t)$ (by [3] and R3)

[24] $\vdash no(v, n) \land most(t, v) \rightarrow not\ all(n, t)$ (i.e. *MEO-4*, by [23], Fact(2.4) and Fact(2.1))

[25] $\vdash no(n, v) \land most(t, v) \rightarrow not\ all(n, t)$ (i.e. *MEO-2*, by [23], Fact(3.2))

More valid generalized syllogisms can be obtained if one continues to reason according to the above deductive methods. So far, on the basis of the valid generalized syllogism *MMI-4*, there are at least 20 valid generalized syllogisms deduced through the above reduction operations.

5. Conclusion and Future Work

To sum up, this paper firstly proves the validity of the generalized syllogism *MMI-4* on the basis of generalized quantifier theory and set theory. Then, the other 20 valid generalized syllogisms are deduced from reduction operations. It can be concluded that there are reducible relationships between/among the above 21 generalized syllogisms. The reason for this is because any quantifier in Square{*some*} can define the other three quantifiers, and so can any quantifier in Square{*most*}.

Does this generalized syllogism fragment have soundness and completeness? This question deserves further research.

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