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Optimization of the Part Manufacturing Process Using Analytical and Simulation Models of a Closed Queueing System

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Abstract

The production of a specific type of part entails a lengthy assembly process, concluding with a short firing period in a furnace. Given the high operational costs of the furnace, multiple assemblers share a single furnace, which can fire only one part at a time. The objective of this study is to determine the optimal number of assemblers, m, that maximizes the furnace utilization factor, K. We define the optimal value of m as the smallest quantity that meets the specified condition $K \geq 0.990$. To this end, we develop an analytical and simulation model based on a closed queueing system. Using an analitycal model and a GPSS World simulation model, we investigate the dependencies of the optimal number of assemblers on the following parameters: the coefficients of variation $V(X)$, $V(Y)$, and the ratio $\rho = E(Y)/E(X)$. Here X represents the assembly time for a part, and Y denotes the furnace firing time of a part. We validate the simulation model by comparing its results with those obtained from an analytical method.

Keywords: closed queueing system, simulation model, GPSS World, part manufacturing process, optimization

1. Introduction

In modern manufacturing industries, the optimization of production processes has become increasingly important due to rising market demands, competition, and the need for cost efficiency. One of the key challenges faced by manufacturers is to maximize productivity while minimizing resource wastage and lead times. Achieving this balance requires a deep understanding of how different components of the manufacturing system interact with one another and how these interactions impact overall system performance.

Queueing theory, which studies the behavior of waiting lines or queues, provides valuable insights for modeling such complex interactions in manufacturing processes [1, 2]. Specifically, closed queueing systems, where a fixed number of jobs circulate within the system, offer a robust framework for analyzing the flow of parts through various stages of production. These systems allow for a detailed examination of factors such as machine utilization, job scheduling, and process bottlenecks, all of which are critical to optimizing production [3, 4].

The application of queueing theory to manufacturing systems has been widely studied. For example, Gershwin [5] highlighted the importance of queueing models in optimizing production processes by addressing system variability and resource constraints. Similarly, Papadopoulos et al. [6] explored the role of closed queueing networks in manufacturing, showing their effectiveness in modeling production lines and assembly processes. These works demonstrate the potential of queueing-based models to reduce lead times, improve throughput, and enhance overall manufacturing efficiency.

The manufacturing of a certain type of parts involves an extended assembly process, which ends with a brief firing period in a furnace. Since the operation of the furnace is very costly, several assemblers use a single furnace that can only fire one part at a time. An assembler cannot start a new assembly until the previous part has been removed from the furnace. Therefore, the assembler works in the following mode:

1) Assembles the next part;
2) Waits for the opportunity to use the furnace on a FIFO basis;
3) Uses the furnace;
4) Returns to step 1.

In this paper, we present analytical and simulation models based on a closed queueing system to optimize the part manufacturing process. The proposed models aim to provide manufacturers with a tool for analyzing and improving system performance under various operational conditions. By simulating various configurations and process parameters, the models assist in determining the optimal number of assemblers, thereby enhancing throughput and improving overall manufacturing efficiency.

One of the methods for studying queuing systems is the simulation method. In this approach, the model simulates the operation of a real system, reproducing the process of functioning of a real system over time. In this paper, we use the GPSS World simulation system [7, 8].

GPSS (General-Purpose Simulation System) is a general process-oriented simulation software environment. GPSS World is a Microsoft Windows application designed to run on various Windows operating systems.

2. The Queuing Model

2.1. The Analytical Model

Let us introduce the notation: m is the number of assemblers, X represents the assembly time for a part, and Y is the furnace firing time of a part. The discrete random process $Z(t)$ represents the number of parts in the assembly stage at time t. The values of $Z(t)$ from the set $\{0, 1, ... , m\}$ correspond to the states of a single-channel closed queueing system. We denote by $E(T)$ the mean of the random variable T, and $\rho=E(Y)/E(X)$. Let us denote by p_k the stationary probability that there are k parts in the assembly stage. The furnace is idle if m parts are in the assembly stage, meaning the random process $Z(t)$ is in state "m". Therefore, the stationary value of the furnace utilization factor is determined by the formula

$K = 1 - p_m.$

For exponential distributions of random variables X and Y, the stationary distribution of random variable Z is known [9]:

$$p_k = p_0 \tilde{p}_k, \quad \tilde{p}_k = \frac{m!\rho^k}{(m-k)!}, \quad 1 \le k \le m; \quad p_0 = \frac{1}{1+\sum_{k=1}^{m} \tilde{p}_k}; \quad \rho = \frac{\lambda}{\mu}. \tag{1}$$

Here, λ and μ are the parameters of exponential distributions of the random variables X and Y, respectively. Formulas (1) are valid for any distribution of the random variable X if we take $\rho=E(Y)/E(X)$.

The goal is to find the optimal number of assemblers m that maximizes the furnace utilization factor K. We will consider optimal the smallest value of m for which the condition c is satisfied.

Graphs illustrating the dependence of K on m are presented in Figures 1 to 3 for different values of ρ. It is evident that as ρ increases, the optimal value of m decreases.

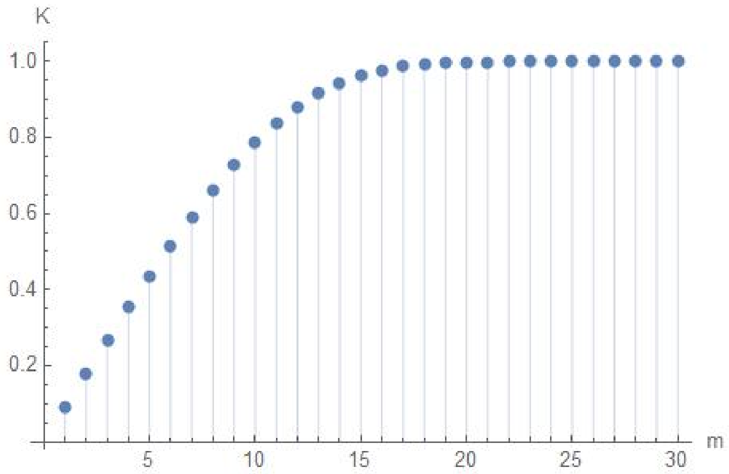

Figure 1. The dependence of the furnace utilization factor on the number of assemblers in the case when ρ=0.1 and the random variable Y follows an exponential distribution

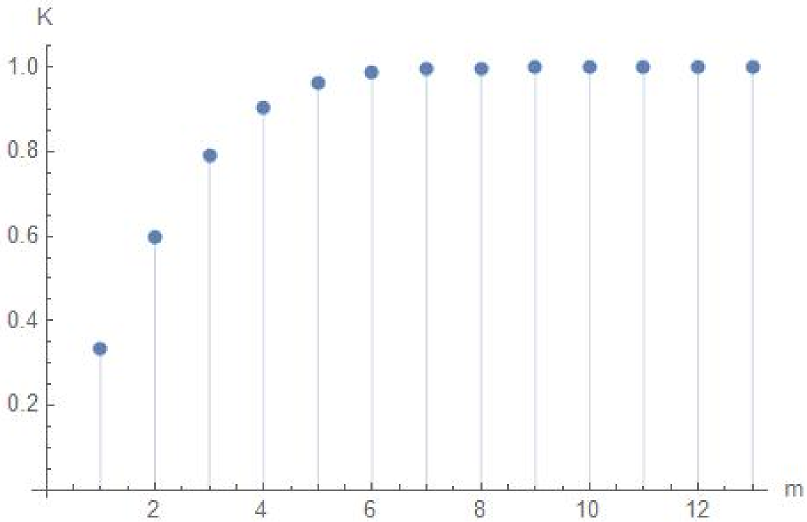

Figure 2. The dependence of the furnace utilization factor on the number of assemblers in the case when ρ=0.5 and the random variable Y follows an exponential distribution

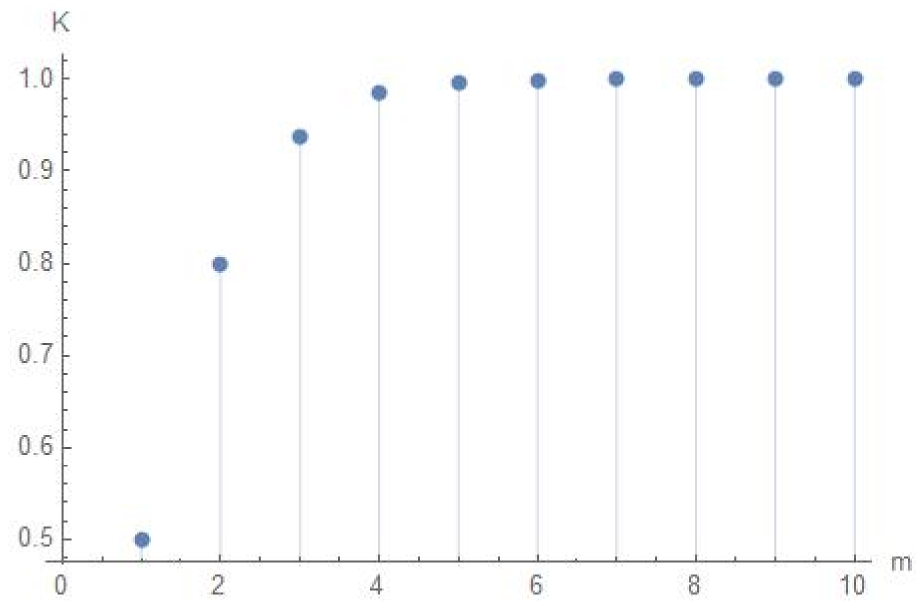

Figure 3. The dependence of the furnace utilization factor on the number of assemblers in the case when ρ=1.0 and the random variable Y follows an exponential distribution

2.2. The Gamma Distribution

In our simulation model, we can consider any distributions of random variables X and Y, but we will only examine examples with exponential, degenerate and gamma distributions of these random variables.

Let us denote for the random variable T the probability density function, variance and coefficient of variation as $f_T(t)$, $D(T)$, and $V(T)$, respectively, then for the gamma distribution, we have

$$f_T(t) = \frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{t}{\beta}}, \quad \alpha > 0, \quad \beta > 0, \quad t \geq 0, \quad \Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx;$$

$$E(T) = \alpha\beta, \qquad D(T) = \alpha\beta^2, \qquad V(T) = \frac{\sqrt{D(T)}}{E(T)} = \frac{1}{\sqrt{\alpha}}.$$

For the gamma distribution, fixing $E(T)$ and varying the parameters α and β, we can consider distributions with different values of the coefficient of variation $V(T)$.

For the degenerate distribution, the coefficient of variation V=0, while for the exponential distribution, V=1.

In a single-channel closed queueing system with exponential distributions of the random variables X and Y, we use the notation M/M. The degenerate distribution is represented by the letter D, while the gamma distribution with a coefficient of variation V is denoted as $G(V)$. For example, $D/G(5)$ refers to a single-channel closed queueing system where X follows a degenerate distribution, and Y follows a gamma distribution with $V(Y) = 5$.

3. The Simulation Model

3.1. Description of the Model

Below we provide a GPSS simulation model in the case when $m = 18$, $E(X) = 10/7$, $E(Y) = 1$, and $\rho = 0.7$. We consider gamma distributions of random variables X and Y, with the flexibility to easily adjust the parameters of these distributions. The coefficients of variation for the random variables X and Y are 0.7 and 5, respectively. The simulation time, $t_{\text{mod}} = 10^5$.

The simulation model reproduces the functioning of the closed queue system over time and is built with the minimum number of blocks necessary to calculate the furnace utilization factor.

```
GENERATE ,,,18
ASS ADVANCE (Gamma(1,0,49/70,100/49))
SEIZE Furnace
ADVANCE (Gamma(1,0,25,1/25))
RELEASE Furnace
TRANSFER ,ASS
GENERATE 100000
SAVEVALUE Koef,FR$Furnace
TERMINATE 1
START 1
```

The standard GPSS World report is provided below. Since K>0.990 for m=18 and K<0.990 for m=17, we accept m=18 as the optimal value for the given problem parameters. The value of the furnace utilization factor is highlighted in red.

```
START TIME          END TIME  BLOCKS  FACILITIES  STORAGES
               0.000        100000.000     9         1          0

 LABEL               LOC  BLOCK TYPE     ENTRY COUNT CURRENT COUNT RETRY
                     1    GENERATE            18             0       0
ASS                  2    ADVANCE         100214            17       0
                     3    SEIZE           100197             0       0
                     4    ADVANCE         100197             1       0
                     5    RELEASE         100196             0       0
                     6    TRANSFER        100196             0       0
                     7    GENERATE             1             0       0
                     8    SAVEVALUE            1             0       0
                     9    TERMINATE            1             0       0

FACILITY         ENTRIES  UTIL.   AVE. TIME AVAIL. OWNER PEND INTER RETRY DELAY
FURNACE          100197   0.990      0.988  1        7     0    0     0     17

SAVEVALUE              RETRY        VALUE
 KOEF                    0         990.116
```

The GPSS World uses random number generators to sample random numbers for ADVANCE blocks. We can select which random number generator number is to be used as the source of the random number. The results obtained for different values of the random number generator may differ slightly from each other. In this work, we use the number of the random number generator, which is equal to 1.

This model offers flexibility to easily change not only the number of assemblers but also the probability distributions of the random variables X and Y, which are defined in the ADVANCE blocks.

3.2. Checking the Simulation Model

Let us use analytical results from paragraph 2.1 to test the constructed simulation model and to find the optimal number of assemblers, m. We assume that $E(Y)=1,$ and consider the exponential distribution of the random variable Y. We take into account that for such a distribution of Y, the stationary distribution of the random variable Z does not depend on the distribution of the random variable X, but only on the value of $\rho=E(Y)/E(X)$. For the simulation time, $t_{\text{mod}}=10^5,$ we have almost identical analytical and simulation modeling results for fixed values of ρ (see Table 1).

Table 1. Comparison of Analytical and Simulation Modeling Results in Calculating the Optimal Value of *m*

ρ	m, analitycal model for M/M	m, GPSS World for M/M	K, analitycal model for M/M	K, GPSS World for M/M	m, for D/M	K, for D/M	m, for $G(2)/M$	K, for $G(2)/M$
0.1	18	18	0.993	0.993	18	0.992	18	0.994
0.2	11	11	0.992	0.991	11	0.992	11	0.992
0.3	9	9	0.995	0.995	9	0.995	9	0.995
0.4	7	8	0.990	0.997	7	0.991	7	0.990
0.5	7	7	0.997	0.997	7	0.997	7	0.997
0.6	6	6	0.994	0.995	6	0.995	6	0.995
0.7	6	6	0.997	0.997	6	0.997	6	0.997
0.8	5	5	0.993	0.993	5	0.993	5	0.993
0.9	5	5	0.995	0.995	5	0.995	5	0.996
1.0	5	5	0.997	0.997	5	0.997	5	0.997
2.0	4	4	0.998	0.998	4	0.998	4	0.998

Table 1 lists the optimal values of m and the corresponding values of K found for various values of ρ. In all simulation models presented in this article, we consider the case where $E(Y)=1$ and use a simulation time of $t_{\text{mod}}=10^5$.

4. A Study of Dependencies of the Optimal Number of Assemblers on Various Process Configurations and Parameters

4.1. Dependencies *m*(*ρ*) for the *G*(*V*)/*M* Systems

The dependence $m(\rho)$ for the $G(V)/M$ Systems is obtained using the analytical model, since for this system the stationary distribution of the random variable Z does not depend on the distribution of the random variable X, but only on the value of ρ. Graph illustrating the dependence of $m(\rho)$ is presented in Figure 4.

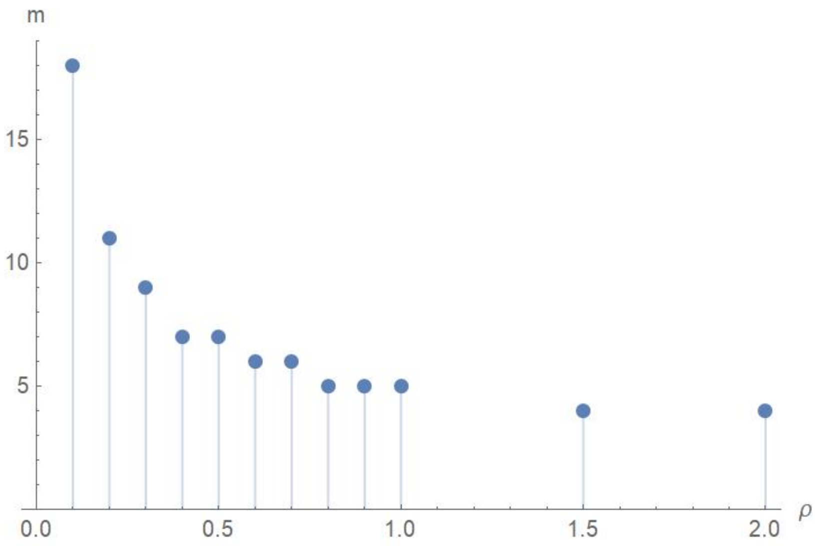

Figure 4. The dependence $m(\rho)$ for the $G(V)/M$ Systems

As ρ increases from 0 to 2, the value of m decreases sharply at the beginning and then stabilizes. For small values of ρ, m starts at a high value but quickly declines, showing an inverse relationship between m and ρ, particularly in the range $0<\rho<1$. After this range, the values of m become relatively stable.

4.2. Dependencies *m*(*ρ*) for Different Values of *V*(*Y*)

We examine the dependencies of m on ρ for the $D/G(V)$, $M/G(V)$, and $G(5)/G(V)$ systems. The results are presented in Figures 5 to 7. The different curves are shown, corresponding to different values of the parameter $V(Y)$.

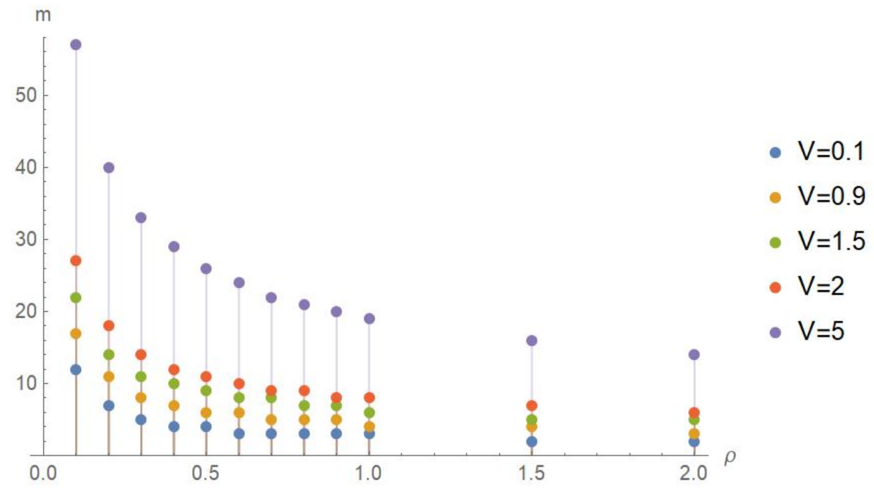

Figure 5. The dependencies $m(\rho)$ for the $D/G(V)$ Systems

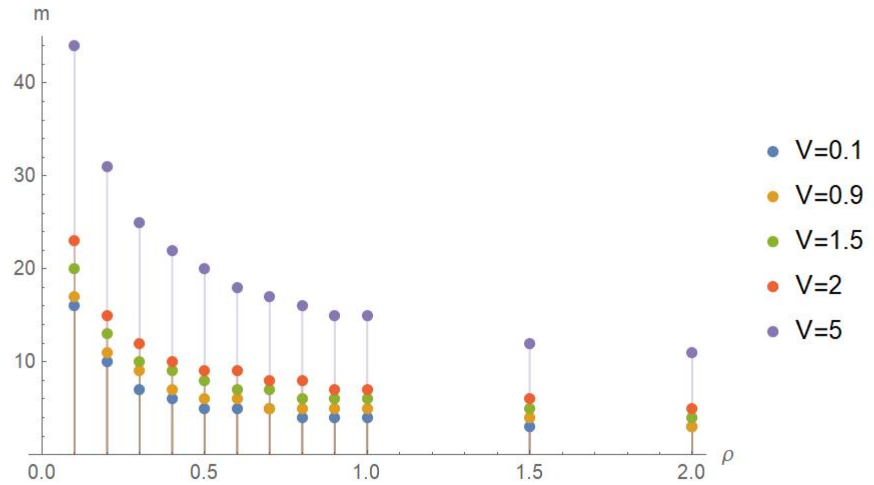

Figure 6. The dependencies $m(\rho)$ for the $M/G(V)$ Systems

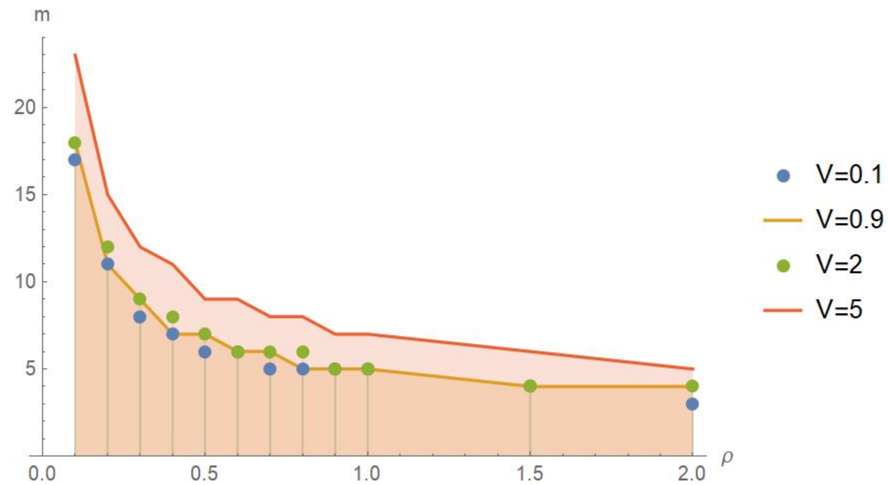

Figure 7. The dependencies $m(\rho)$ for the $G(5)/G(V)$ Systems

Overall, the value of m decreases as ρ increases. This indicates that as ρ grows, the optimal number of assemblers (m) diminishes. For a fixed value of ρ, m increases as $V(Y)$ rises. For larger values of $V(Y)$ (e.g., $V(Y)=5$), m is significantly higher compared to smaller $V(Y)$ values. However, at high ρ values, the m values for different $V(Y)$ converge. As $V(X)$ increases, the values of m decrease and converge for varying $V(Y)$.

4.3. Dependencies *m*(*V*) for the *D*/*G*(*V*), *M*/*G*(*V*), and *G*(5)/*G*(*V*) Systems

We study the dependencies of m on $V(Y)$ for ρ values of 0.1, 0.7, and 2. The results are presented in Figures 8 through 10. The various curves correspond to the $D/G(V)$, $M/G(V)$, and $G(5)/G(V)$ systems.

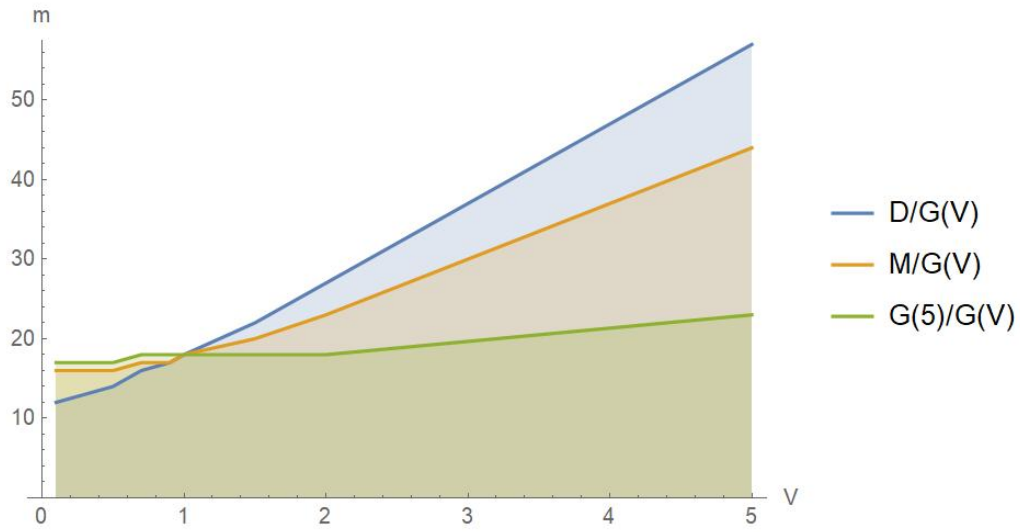

Figure 8. The dependencies $m(V)$ for ρ=0.1

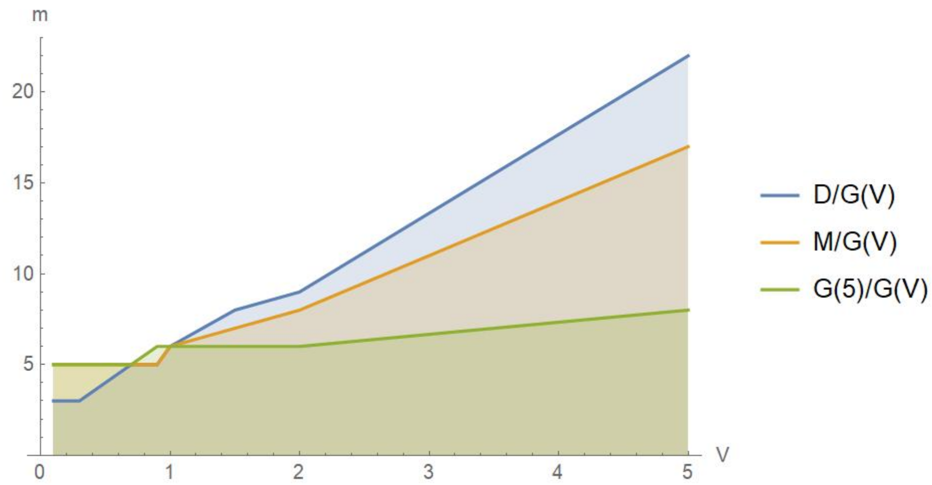

Figure 9. The dependencies $m(V)$ for ρ=0.7

We observe that $m(V)$ is an increasing function. For $V(Y)<1$, m increases as $V(X)$ rises. However, for $V(Y)>1$, m decreases with increasing $V(X)$. In the $G(V)/M$ system, insensitivity to V for is observed, causing all curves to intersect at a single point on the line $V=1$. Additionally, the property of decreasing $m(\rho)$ is also evident for these systems.

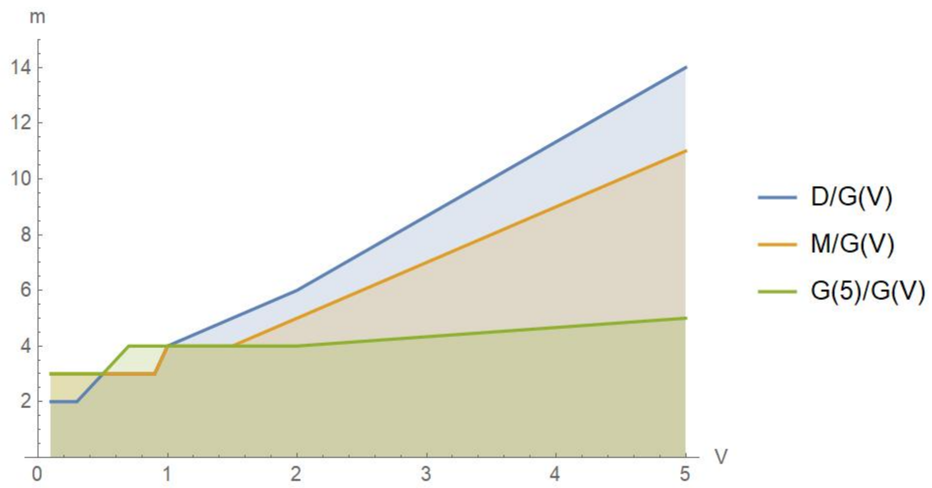

Figure 10. The dependencies $m(V)$ for ρ=2

4.4. Dependencies $m(V)$ for the $G(V)/D$, $G(V)/M$, and $G(V)$/G(5) Systems

We study the dependencies of m on $V(X)$ for ρ values of 0.1, 0.7, and 2. The results are presented in Figures 11 through 13. The various curves correspond to the $G(V)/D$, $G(V)/M$, and $G(V)$/G(5) systems.

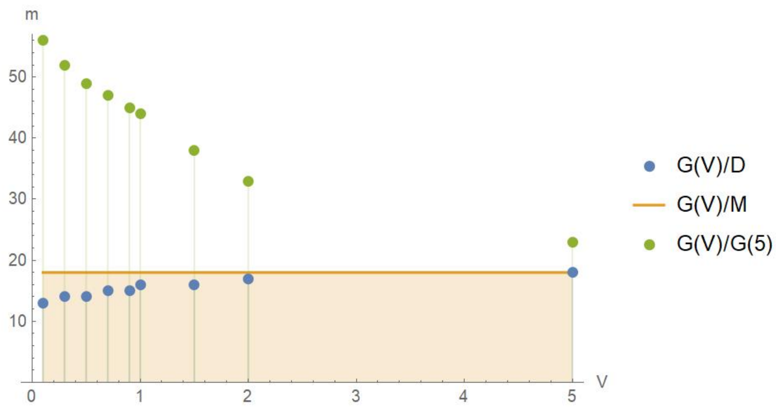

Figure 11. The dependencies $m(V)$ for ρ=0.1

We observe that $m(V)$ is a decreasing function for $V(Y)<1$, while m increases as $V(X)$ rises for $V(Y)>1$, and m does not depend on $V(X)$ in the $G(V)/M$ system. These systems also exhibit the property of decreasing $m(\rho)$.

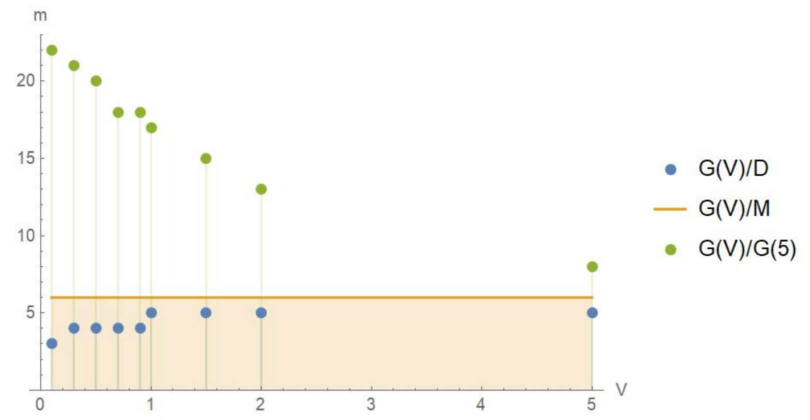

Figure 12. The dependencies $m(V)$ for ρ=0.7

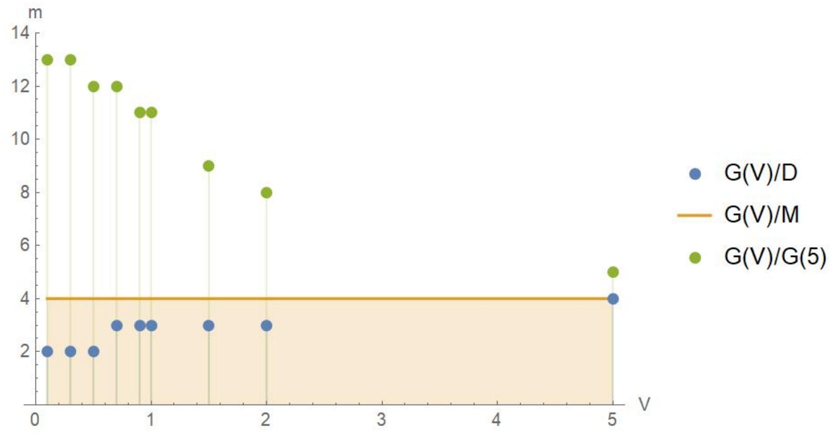

Figure 13. The dependencies $m(V)$ for ρ=2

4.5. Dependencies *m*(*ρ*) for Different Values of *V*(*X*)

We examine the dependencies of m on ρ for the $G(V)/D$, $G(V)/M$, and $G(V)$/G(5) systems. The results are presented in Figures 14 and 15. The different curves are shown, corresponding to different values of the parameter $V(X)$. The points corresponding to the $G(V)/M$ system are located in the upper part of Figure 14 and the lower part of Figure 15. In these figures, we again observe a decrease in $m(\rho)$ and verify that m decreases as $V(X)$ increases for $V(Y) < 1$, while m is an increasing function of $V(X)$ for $V(Y) > 1$.

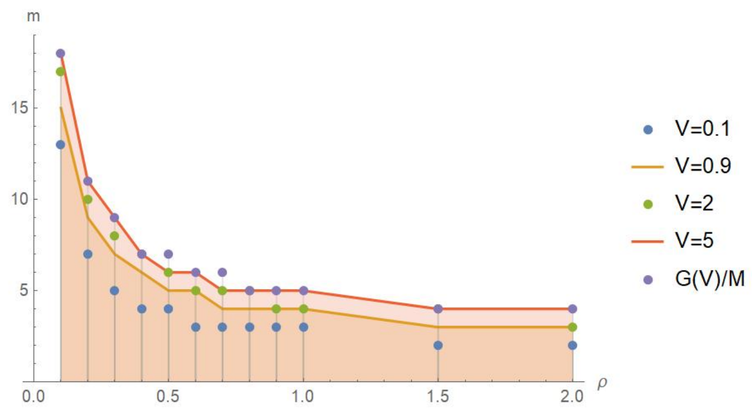

Figure 14. The dependencies $m(\rho)$ for the $G(V)/D$ and $G(V)/M$ Systems

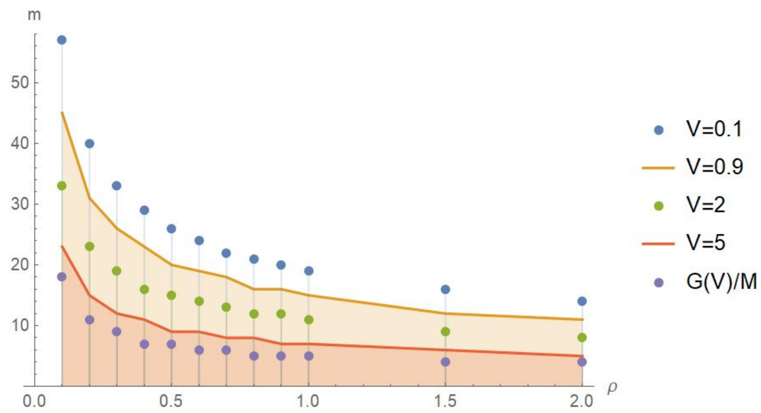

Figure 15. The dependencies $m(\rho)$ for the $G(V)/G(5)$ and $G(V)/M$ Systems

5. Conclusion

In this paper, we have developed and applied analytical and simulation models based on a closed queueing system to optimize the part manufacturing process. These models can serve as valuable tools for manufacturers, offering insights into system performance under various operational conditions. By utilizing the results obtained through the simulation of different configurations and process parameters, it is possible to determine the optimal number of assemblers, thereby increasing throughput and improving overall manufacturing efficiency.

The results indicate a significant dependence of the optimal number of assemblers on the coefficient of variation in assembly time and the furnace firing time for each part. Consequently, incorporating the practical application of analytical models becomes challenging, as these models were derived exclusively for exponential distributions. Thus, the practical application of analytical models becomes challenging, as these models were derived exclusively for exponential distributions. An exception is the analytical model for the case of an exponential distribution of the furnace firing time for each part, which remains applicable for any distribution of the assembly time for a part.

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