SCIREA Journal of Mathematics
ISSN: 2995-5823
http://www.scirea.org/journal/Mathematics
April 16, 2025
Volume 10, Issue 2, April 2025
https://doi.org/10.54647/mathematics110536

A Note On “A New Method For Calculating the Distance Between Two Points On A Plane”

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Abstract:

This article proves that using generalized cosine and generalized sine to calculate the distance between two points in a good affine coordinate system is simpler and more accurate than using tangent and cotangent and table look-up in a Cartesian coordinate system.

Keywords: affine coordinate system, Cartesian coordinate system, generalized cosine, generalized sine, tangent, cotangent

In [1], a new method for calculating the distance between two points in a planar affine coordinate system was proposed by using generalized cosine and generalized sine. It was proved that this method is simpler than the Pythagorean theorem commonly used in a Cartesian coordinate system. However, there is another method for calculating the distance between two points in a Cartesian coordinate system, which is to use tangent and cotangent and table look-up methods . This article proves that, in general, using generalized cosine and generalized sine to calculate the distance between two points in a good planar affine coordinate system, i.e. using the method in [1], is simpler and more accurate than using tangent and cotangent and table look-up ! Here are three Examples to illustrate these two methods,the method using in Example 2 and Example 3 can be found in [1].

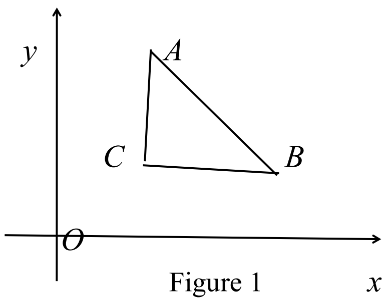

Figure 1

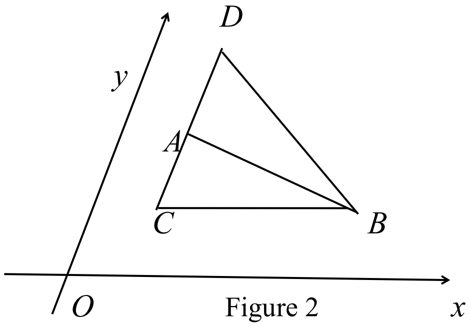

Figure 2

Example 1 In the Cartesian coordinate system xoy shown in Figure 1, the coordinates of two points A,B are $A(2,5),B(10,1.2)$ respectively,find the distance between the two points A,B by using tangent and cotangent and table look-up.

Solution $AC=5-1.2=3.8, BC=10-2=8, \cot\angle A=\frac{AC}{BC}=\frac{3.8}{8}=0.475$, according to the table $\angle A\approx 64.5922818910\ 51^{0}$, , $\sin(2,\angle A)\approx 0.9032775043\ 542$

$$AB=\frac{BC}{\sin(2,\angle A)}\approx 8.8566359301\ 946$$

Example 2: In the affine coordinate system xoy shown in Figure 2, the angle between the positive directions of the two coordinate axes is $\alpha=2\sin^{-1}\frac{7}{25}<60^{0}$, and the coordinates of the two points A,B are $A(2,5),B(10,1.2)$ respectively. Use generalized cosine and generalized sine to calculate the distance between the two points A,B

Solution AC is parallel to the y axis and BC is parallel to the x axis,

$$AC=5-1.2=3.8, BC=10-2=8, \alpha=\angle C=2\sin^{-1}\frac{7}{25}<60^{0}$$

So $\angle C$ is not the largest angle of triangle ABC , so it has a generalized cosine and a generalized sine.

$\cos(r,\angle C)=\frac{AC}{BC}=\frac{3.8}{8}=\frac{1.9}{4}$ .According to the table

$\sin(r,\angle C)\approx 0.6516018723116$, $AB=BC\sin(r,\angle C)\approx 5.2128149784928$

Example 3: In the affine coordinate system xoy shown in Figure 2, the angle between the positive directions of the two coordinate axes is $\alpha = 2\sin^{-1}\frac{4}{5}$, the coordinates of the two points A, B are $A(1,3.6), B(6,1.6)$ respectively. Use generalized cosine and generalized sine to calculate the distance between the two points A, B

Solution AC is parallel to the y axis and BC is parallel to the x axis, $AC = 3.6 - 1.6 = 2, BC = 6 - 1 = 5, \alpha = \angle C = 2\sin^{-1}\frac{4}{5} > 60^0$

Because $\angle C > 60^0$, so $\angle C$ may be the maximum angle of the triangle ABC, and thus in the triangle ABC, $\angle C$ may not have generalized cosine and generalized .Extended CA to the point D, make $CD = CB$,the triangle BCD is an isosceles triangle, $\angle D < 60^0$, so, $\angle D$ is not the largest angle of the triangle ABD , it has a generalized cosine generalized sine,below,we use the generalized cosine generalized sine to solve triangle ABD,

$$AD = BC - AC = 5 - 2 = 3, BD = 2BC\sin\frac{\angle C}{2} = 1.6 \times 5 = 8,\ \cos(r, \angle D) = \frac{AD}{BD} = \frac{3}{8}$$

According to the table ,

$$\sin(r, \angle D) \approx 0.7352720584\ 926, AB = BD\sin(r, \angle D) \approx 5.8821764679\ 408$$

The actual process of this method starts with solving the triangle ABD by using generalized cosine and generalized sine, which has a total of three lines! Example 1 has a total of three lines also,and requires two table look-up operations, while Example 2 and Example 3 only require one table look-up operation as shown below. The main advantage of the method in Example 2 and Example 3 over the method in Example 1 is that it transforms division(for AB) into multiplication(for AB) ! This is the most crucial aspect of this method!

If $\alpha > 60^0$ and $\sin\frac{\alpha}{2} = \sin\frac{\angle C}{2}$ is simple, the affine coordinate system is highly likely to be a good coordinate. At this point, it is not only $BD = 2BC\sin\frac{\alpha}{2}$ easy to find , but also checking the table and finding $\sin(r, \angle D)$ is also simple. This is because the mathematical tables used for checking the table and finding $\sin(r, \angle D)$ are based on formulas $\sin^2(r, \angle D) = \cos^2(r, \angle D) - 2\cos(r, \angle D)\cos\angle D + 1$,where, $\cos(r, \angle D)$ is known in this formula, $\cos\angle D=\cos(\frac{\pi}{2}-\frac{\angle C}{2})=\sin\frac{\angle C}{2}=\sin\frac{\alpha}{2}$,of course, simple, so the angle $\angle D$ does not need to be calculated at all, which can make the $\sin(r,\angle D)$ more accurate,this is Example 1 can't do.Reference to [1-2] for the detailed procedure.

The angles between the two coordinate axes of an affine coordinate system is $\alpha,\beta,\alpha+\beta=\pi$ if , $\alpha\le\frac{\pi}{3}$,then $\beta\ge\frac{2\pi}{3}$,in this case, as long as $\sin\frac{\beta}{2}$ is simple, the affine coordinate system is a good coordinate system. For Example, when $\beta=2\sin^{-1}\frac{24}{25}$,the affine coordinate system is a good coordinate system, this is that affine coordinate system established in Example 2. If $\alpha>\frac{\pi}{3}$ and $\beta>\frac{\pi}{3}$,at this time ,when both $\sin\frac{\alpha}{2}$ and $\sin\frac{\beta}{2}$ are simple, the affine coordinate system is a good coordinate system. For Example, when $\alpha=2\sin^{-1}\frac{4}{5}$,the affine coordinate system is a good coordinate system, this is that affine coordinate system established in Example 3, and the coordinate transformation formula from the affine coordinate system in Example 3 to the Cartesian coordinate system is also simple.According to Example 2 and Example 3,using Pythagorean numbers can establish a good coordinate system,which is one of the uses of Pythagorean numbers! The article [1] proves that using generalized cosine and generalized sine to calculate the distance between two points in a good coordinate system is simpler than using the Pythagorean theorem in a Cartesian coordinate system. This method can be divided into two cases, corresponding to Example 2 and Example 3 respectively. The following explains that this method is also simpler than using tangent and cotangent and table look-up like in Example 1, that is, the method of Example 2 and Example 3 is better than the method of Example 1!

Generally speaking, division is harder than multiplication because firstly, it is more difficult to find the quotient each time, and secondly, the quotient is found more times, otherwise the result will not be accurate enough. There is no such thing as multiplication.So finding $AB=\frac{BC}{\sin(2,\angle A)}$ is harder than finding $AB=BC\sin(r,\angle C)$ or finding $AB=BD\sin(r,\angle D)$, even with the addition of calculating $AD=BC-AC$ and $BD=1.6BC$. That is, finding $AB=\frac{BC}{\sin(2,\angle A)}$ is still harder than finding $AB=BD\sin(r,\angle D)$ and $AD=BC-AC$ and $BD=1.6BC$ all together ,because this only requires finding the quotient one or two more times ,even , there's no need to find again! And if you want to calculate $AB = \frac{BC}{\sin(2, \angle A)}$, you also need to find the quotient more times to make the result accurate! Note that the key here is to calculate the numbers $AD = BC - AC$ and $BD = 1.6BC$ simple!

The last point is that the operation of finding $\cot\angle A = \frac{AC}{BC}$ in Example 1 is canceled out by the operation of finding $\cos(r, \angle C) = \frac{AC}{BC}$ in Example 2 or is canceled out by the operation of finding $\cos(r, \angle D) = \frac{AD}{BD}$ in Example 3.

Because $\sin(r, \angle A) < 1$, if $\sin(r, \angle A)$ has an error, then the error of $\sin(r, \angle A)$ is smaller than the error of $\frac{1}{\sin(r, \angle A)}$, that is, $AB = BD\sin(r, \angle D)$ is more accurate than $AB = \frac{BC}{\sin(2, \angle A)}$.

In summary, using the method described in [1] to calculate the distance between two points on a plane is simpler and more accurate than using tangent and cotangent and table look-up in a Cartesian coordinate system!

The result obtained by using the Pythagorean theorem in Example 1 is 8.8566359301938, with an error of 0.0000000000008 compared to the result in this Example. The result obtained by using the cosine theorem in Example 2 is 5.2128149784929, with an error of 0.0000000000001 compared to the result in this Example. The result obtained by using the cosine theorem in Example 3 is 5.8821764679410, with an error of 0.0000000000002 compared to the result in this Example. This verifies that the method proposed in [1] is more accurate than using tangent and cotangent and table look-up like in Example 1.

What is the total required quantity for Example 3 are

$$AD = BC - AC = 5 - 2 = 3, BD = 2BC\sin\frac{\angle C}{2} = 1.6 \times 5 = 8,$$
$$AB = BD\sin(r, \angle D) \approx 8 \times 0.7352720584\ 926\ .$$

Adding together the solutions for these three quantities is obviously simpler than finding the one quantity $AB = \frac{BC}{\sin(2, \angle A)} = \frac{8}{0.9032775043\ 542}$ in Example 1!Thus ,Example 3 is simpler than Example 1.

Compared with Example 3, there is no need to calculate AD, BD in Example 2 ,so,Example 2 is simpler than Example 3, and of course, it is also simpler than Example 1!

This verifies that the method proposed in [1] is simpler than using tangent and cotangent and table look-up like in Example 1.

Note that the methods of Example 1, Example 2, and Example 3 are general! ,thus, the conclusion of this article has been verified with examples!

According to this article and [1-2], solving a triangle only requires four operations: addition, subtraction, multiplication, division, and table look-up(simpler than operation), this is simpler than solving a right triangle using classical methods. Therefore, the Pythagorean theorem and cosine theorem will be eliminated by the generalized Pythagorean theorem, and a Cartesian coordinate system will be eliminated by a good affine coordinate system. This is a continuous line!

Reference

[1] Zhou Zhongwang.A New Method For Calculating the Distance Between Two Points On A Plane[J].SCIREA Journal of Mathematics.2025,10(1),8-14.

[2] Zhou Zhongwang.Generalized Pythagorean Theorem[J].SCIREA Journal of Mathematics.2025,10(1),1-7.