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Generalized Pythagorean Theorem and Generalized Derivative

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Abstract:

This paper extends the derivative to affine coordinate systems， introduces the generalized derivative, presents its applications, and expands the theoretical foundation of mathematics!

Keywords: generalized Pythagorean theorem， generalized derivative,rank of triangle， generalized sine， generalized cosine,generalized tangent,generalized secant

1. Generalized Pythagorean theorem 、generalized sine 、 generalized cosine、generalized tangent and generalized cotangent

Theorem 1 [1] For positive real numbers a,b,c , if satisfied $a \le b < c$, there must exist a unique positive real number n , such that $a^n + b^n = c^n$,when $n<1$, a,b,c do not form a triangle, when $n=1$, a,b,c form a straight triangle, when $1<n<2$, a,b,c form an obtuse triangle, when $n=2$, a,b,c form a right triangle, and when $n>2$, a,b,c form an acute triangle,where n is called the rank of the triangle.

Definition 1 For any triangle ABC of rank r, let its maximum angle be $\angle C$ and $AB=c, BC=a, CA=b$,define the generalized sine $\sin\left(r,\angle A\right)=\frac{a}{c}$ 、the generalized cosine $\cos(r,\angle A)=\frac{b}{c}$ 、the generalized tangent $\tan(r,\angle A)=\frac{a}{b}$ and the generalized cotangent $\cot(r,\angle A)=\frac{b}{a}$ with a smaller acute angle $\angle A$. Note that only smaller acute angles have generalized sine 、cosine 、tangent and cotangent.

Theorem 2 [1] For any rank $r>1$ and any acute angle α , their generalized sine $\sin\left(r,\alpha\right)$ and generalized cosine $\cos\left(r,\alpha\right)$ exist and are unique, $\cos\left(r,\alpha\right)$ satisfies the equation $x^r+(x^2-2x\cos\alpha+1)^{\frac{r}{2}}-1=0(1)$

According to this theorem, it can be known that the only solution of the equation $x^r+(x^2-2x\cos\alpha+1)^{\frac{r}{2}}-1=0$ over $(0,1)$ is $\cos(r,\alpha)$, so $\cos(r,\alpha)$ is solely determined by r,α and $\cos(r,\alpha)$ can be calculated using Matlab.Obviously $\cos(2,\alpha)=\cos\alpha, \sin(2,\alpha)=\sin\alpha$. Therefore, the generalized cosine and generalized sine are natural extensions of the cosine and sine. To the given rank $r>1, \cos(r,\alpha)$ is a univariate function of α , which is a new function with almost no expression.

2. Rediscussing solving triangles

Reference [1-2] presents a quick method for solving triangles using generalized cosine and generalized sine. Here, some additional explanations are provided.

According to Theorem 1 and Theorem 2, the rank of a triangle and its smaller angle uniquely determine the generalized cosine of the smaller angle, and conversely, the smaller angle of a triangle and its generalized cosine uniquely determine its rank ,at last the rank of a triangle and the generalized cosine uniquely determined that acute angle .So three tables of electronic mathematics are proposed based on equation $x^r+(x^2-2x\cos\alpha+1)^{\frac{r}{2}}-1=0$:(1) $\cos(r,\alpha)=x$,(2) $\cos^{-1}(x,r)=\alpha$,(3) $\cos^{-1}(x,\alpha)=r$ .Similarly the following nine electronic mathematical tables can be obtained:

(4) $\sin(r,\alpha)=y$,(5) $\sin^{-1}(y,r)=\alpha$,(6) $\sin^{-1}(y,\alpha)=r$.(7) $\tan(r,\alpha)=y$,(8) $\tan^{-1}(y,r)=\alpha$,(9) $\tan^{-1}(y,\alpha)=r$.(10) $\cot(r,\alpha)=y$,(11) $\cot^{-1}(y,r)=\alpha$,(12) $\cot^{-1}(y,\alpha)=r$ .Add three more electronic mathematics tables below:(13) $r(\alpha,\beta)=r$ .That is, given two smaller acute angles α,β of a triangle ,its rank r can be determined .This table is proposed based on equation $\sin^r\alpha+\sin^r\beta=\sin^r(\alpha+\beta)$.(14) $r^{-1}(r,\alpha)=\beta$ .That is, given the rank r and a smaller acute α angle of a triangle, the other smaller acute angle β can be determined .This table is proposed based on equation $\sin^r\alpha+\sin^r\beta=\sin^r(\alpha+\beta)$ also.(15) $r(x,y)=r$ .That is, given the generalized sine and cosine y,x of a smaller acute angle α of a triangle ,its rank r can be determined .This table is proposed based on equation $x^r+y^r=1$ .With the above fifteen electronic math tables,solving triangles will be very simple.Basically,just look up the tables!

Based on the above electronic mathematical tables, it is known that three mutually independent conditions (parameters) of a triangle are sufficient to solve the triangle, and the triangle can always be solved (though the answer may sometimes not be unique). Moreover, solving any triangle only requires the four basic arithmetic operations—addition, subtraction, multiplication, and division—and table lookup (which is simpler than computation). This makes it even easier than solving right triangles using classical methods. Therefore, with the concept of the rank of a triangle, right triangles no longer hold any advantage over other triangles and may even be less convenient!

Note: Knowing the three angles of a triangle does not constitute three independent conditions but only two. Knowing the rank 2 and the right angle of a right triangle does not count as two independent conditions but only one.

For example:

- Given one acute angle and one side of a right triangle, the triangle can be solved—here, there are three independent conditions (two angles and one side).

- Given two sides and one angle of any triangle, the triangle can be solved—here, there are also three independent conditions (two sides and one angle).

- Given the rank of a triangle along with one angle and one side, the triangle can be solved—here, there are clearly three independent conditions as well!

3 Generalized trigonometric function theory

Definition 2: In a plane Cartesian coordinate system xoy,consider a ray originating from the coordinate origin with an angle x relative to the positive direction of the x-axis. Take a point B on the ray with coordinates (p,q), and take a point A on the x-axis with coordinates $(z,0)$. Let the rank of triangle OAB be r. Let $\cos(r,x)=\frac{z}{OB}$.When $q\geq 0$,let $\sin(r,x)=\frac{AB}{OB}$,when $q<0$,let $\sin(r,x)=-\frac{AB}{OB}$

Thus, the generalized cosine and generalized sine are defined for any angle x. This introduces the generalized cosine function and generalized sine function with respect to the variable x (treating r as a constant). Let $\tan(r,x)=\frac{\sin(r,x)}{\cos(r,x)},\cot(r,x)=\frac{\cos(r,x)}{\sin(r,x)}$,this introduces the generalized tangent function and generalized cotangent function with respect to the variable x (treating r as a constant).

When $r=2$, these reduce to the ordinary cosine sine、 sine、 tangent、 cotangent functions. It is easy to see that the generalized cosine、 sine、 tangent、 cotangent functions are also periodic with a period of 2π. Here, OB must be the longest side of the triangle OAB.In general,

$$\sin(r,\pi-x)=\sin(r,x),\sin(r,\pi+x)=-\sin(r,x),\cos(r,\pi-x)=\cos(r,x),\cos(r,\pi+x)=-\cos(r,x)$$
,

$$\sin(r,\frac{\pi}{2}-x)\neq\cos(r,x),\sin(r,\frac{\pi}{2}+x)\neq\cos(r,x),\cos(r,\frac{\pi}{2}-x)\neq\sin(r,x),\cos(r,\frac{\pi}{2}+x)\neq-\sin(r,x)$$
.

$$\sin(r,0)=0,\cos(r,0)=1,\sin(r,\frac{\pi}{2})=1,\cos(r,\frac{\pi}{2})=0,\sin^r(r,x)+\cos^r(r,x)\neq 1,\left|\sin(r,x)\right|^r+\left|\cos(r,x)\right|^r=1.$$

Theorem 3 when α is an acute angle, $\sin(r,\alpha)\geq\sin\alpha$ .If $\sin(r,\alpha)=\sin\alpha$,then $r=2$ .If $\cos(r,\alpha)=\cos\alpha$,then $r=2$.

Proof $x^r+(x^2-2x\cos\alpha+1)^{\frac{r}{2}}-1=0$,where $x=\cos(r,\alpha)$.

$$\sin^2(r,\alpha)=\cos^2(r,\alpha)-2\cos(r,\alpha)\cos\alpha+1=(\cos(r,\alpha)-\cos\alpha)^2+\sin^2\alpha\geq\sin^2\alpha$$

If $\sin(r,\alpha)=\sin\alpha$,then $\cos(r,\alpha)=\cos\alpha$, $\cos^r\alpha+\sin^r\alpha=1,\cos^2\alpha+\sin^2\alpha=1$ .According to generalized Pythagorean(Theorem 1) $r=2$.

When angle $\alpha(\alpha\neq k\pi,k\pi+\frac{\pi}{2})$ is not acute, it is not difficult to derive the corresponding conclusion. Thus, the graph of $y=\sin(r,x)$ on the closed interval [0, π] must lie above the graph of $y=\sin x$, with three points of intersection.,on the closed interval [π , 2 π], it must lie below the curve $y=\sin x$, also with three points of intersection. This gives a rough idea of the graph of function $y=\sin(r,x)$.

4 Generalized derivative

Definition 3 Let a function $y=f(x)$ be defined in a neighborhood of x_0, and let $x_0+\Delta x$ also lie within this neighborhood. If the limit of the ratio of the increment of the function $\Delta y=f(x_0+\Delta x)-f(x_0)$ to the increment of the independent variable Δx , as $\Delta x\to 0$, exists, that is, if $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ exists, then this limit is called the generalized derivative of the function $f(x)$ at the point x_0 . It is denoted by $f'(x_0)$ or $\frac{dy}{dx}|_{x=x_0}$. At this point, the function $y=f(x)$ is said to be generalized differentiable at x_0 .

Here, the function $y=f(x)$ is the equation of a curve C in an affine coordinate system xoy , in which the positive angle between the two coordinate axes ox and oy is α

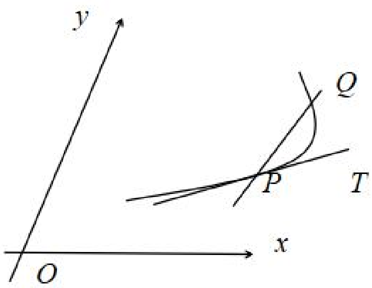

Figure 1

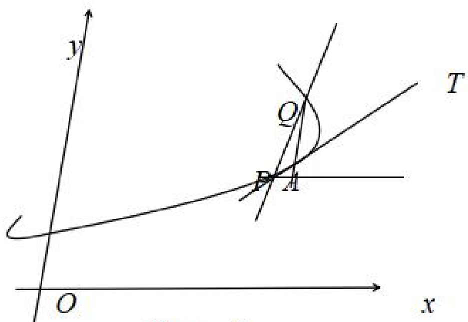

Figure 2

Definition 4 Let P be a fixed point on a curve L , and let Q be a moving point on L . If, as Q approaches P along L , the secant line PQ has a limiting position PT , then the line PT is called the tangent line to the curve L at the point P , as shown in Figure 1.

When the independent variable x takes the values x_0 and $x_0+\Delta x$, the corresponding points $P(x_0, y_0)$ and $Q(x_0+\Delta x, y_0+\Delta y)$ on the curve L are obtained (see Figure 2),where PA is parallel to the x-axis, QA is parallel to y-axis,and $PA=|\Delta x|, QA=|\Delta y|$. The inclination angle of the secant line PQ is $\angle QPA=\varphi$,the inclination angle of the tangent line PT is $\angle TPA=\theta$.According to the sine theorem, $\frac{\Delta y}{\Delta x}=\frac{AQ}{PA}=\frac{\sin\varphi}{\sin(\alpha-\phi)}$.The generalized slope of the secant line PQ is given by $\frac{\Delta y}{\Delta x}=\frac{\sin\varphi}{\sin(\alpha-\varphi)}$. As $\Delta x\to 0$, the point Q approaches P along the curve. Since the limit $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ exists, the secant line PQ has a limiting position PT. At this point, the inclination angle φ of the secant line PQ approaches the inclination angle θ of the tangent line PT. Thus, we have:

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{\sin\theta}{\sin(\alpha-\theta)}$,which means $y'=\frac{\sin\theta}{\sin(\alpha-\theta)}$,where ΔPQA is no longer a right triangle. In other words, the generalized derivative of the function $y=f(x)$ at x_0, geometrically, still represents the generalized slope k of the tangent line to the curve $y=f(x)$ at the point P,it is just that the slope is no longer $\tan\theta$,but $k(\theta)=\frac{\sin\theta}{\sin(\alpha-\theta)}$ instead!When $\alpha=\frac{\pi}{2}$ (i.e., when the affine coordinate system xoy is a Cartesian coordinate system), the slope $k=\frac{\sin\theta}{\sin(\frac{\pi}{2}-\theta)}=\frac{\sin\theta}{\cos\theta}=\tan\theta$, which exactly matches the classical result!This is the geometric meaning of the generalized derivative!

Let r be the rank of the triangle TPA,the following discusses the generalized derivative y' of a function $y=f(x)$ at a certain point P, i.e., the generalized slope k of the tangent line by generalized trigonometric function, under three scenarios:

①. When $\angle PAQ$ is the largest angle in ΔPAQ, $k=y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\tan(r,\theta)(2)$.

②. When $\angle PQA$ is the largest angle in ΔPAQ, $k=y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\sin(r,\theta)(3)$.

③. When $\angle QPA$ is the largest angle in ΔPAQ , $k = y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \sec(r, \alpha)(4)$.

Using formula $k(\theta) = y' = \frac{\sin\theta}{\sin(\alpha - \theta)}(5)$ to find the generalized slope of the tangent line requires division, whereas using the ①②③ method to find the generalized slope only involves consulting the previously mentioned electronic mathematical tables(4)(7),etc!

In particular, when $\theta = 0, k(\theta) = 0$; when $\theta = \frac{\alpha}{2}, k(\theta) = 1$; when $\theta = \alpha, k(\theta) = \infty$; and when $\theta = \frac{\pi}{2}, k(\theta) = -\frac{1}{\cos\alpha}$. Note that $k(\frac{\pi}{2})$ is meaningful, whereas in the Cartesian coordinate system, $k(\frac{\pi}{2}) = \infty$, which aligns with the fact that in the affine coordinate system, $k(\alpha) = \infty$.

$k(\theta) = \frac{\sin\theta}{\sin(\alpha - \theta)}, k'(\theta) = \frac{\sin\alpha}{\sin^2(\alpha - \theta)}$ .From this formula, it is straightforward to derive the variation pattern of $k(\theta)$ with respect to θ.

The generalized derivative and the classical derivative are identical in form or function, meaning that the rules, properties, and theorems of differentiation still hold for generalized derivatives. However, their geometric interpretations are fundamentally different. The geometric meaning of the classical derivative is merely a special case of the generalized derivative—specifically, when the affine coordinate system is orthogonal. In future research, affine coordinate systems can be adopted without being confined to orthogonal ones. When problems cannot be addressed using orthogonal coordinates and require an affine coordinate system, the generalized derivative becomes essential!

Example 1

As shown in Figure 3, in the Cartesian coordinate system XOY , find the equation of the tangent line PT to the parabola $X = Y^2$ at the point $P(\frac{1}{3}, \frac{\sqrt{3}}{3})$.

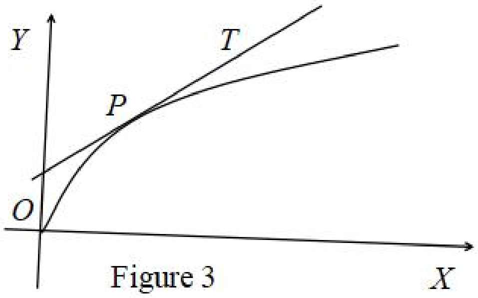

Figure 3

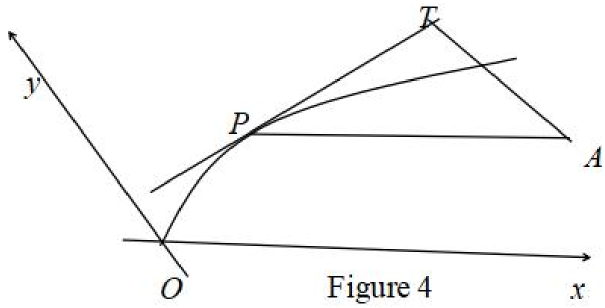

Figure 4

Figure 4 is essentially the same as Figure 3, except that it is obtained by rotating the coordinate axis OY of Figure 3 30 degrees counterclockwise (with the OX axis remaining unchanged). The affine coordinate system in Figure 4 is denoted as xoy, in which the positive angle between the two coordinate axes ox and oy is 120^0. Figures 3 and Figure 4 can be merged together!In the affine coordinate system xoy in Figure 4 find the equation of the same tangent line PT.

Solution In the Cartesian coordinate system XOY, $X=Y^2, 1=2YY', Y'=\frac{1}{2Y}, Y'(\frac{1}{3})=\frac{\sqrt{3}}{2}=k$ **.**Hence the equation of the tangent line at P is

$\frac{Y-\frac{\sqrt{3}}{3}}{X-\frac{1}{3}}=k=\frac{\sqrt{3}}{2}$ .Simplifying the equation,we obtain, $Y=\frac{\sqrt{3}}{2}X+\frac{\sqrt{3}}{6}$ (6) .The inclination angle θ of the tangent line PT equals $\arctan\frac{\sqrt{3}}{2}\approx 40.8934^0$(7)

In Figure 4, the coordinate transformation formula from the affine coordinate system xoy to the Cartesian coordinate system XOY is:

$$\begin{cases} X = x + y\cos\alpha = x + y\cos 120^0 = x - \frac{1}{2}y \\ Y = y\sin\alpha = y\sin 120^0 = \frac{\sqrt{3}}{2}y \end{cases} \qquad (8)$$

Substitute (8) into the parabola $X=Y^2$,we obtain, $x-\frac{1}{2}y=\frac{3}{4}y^2$,thus, the equation of the parabola in the affine coordinate system is: $4x=2y+3y^2$, differentiating both sides yields $2=y'+3yy'$ (9)

The coordinates of point P in the Cartesian coordinate system XOY are $P(\frac{1}{3},\frac{\sqrt{3}}{3})$,according to (8) the coordinates of point P in the affine coordinate system xoy are $P(\frac{2}{3},\frac{2}{3})$,substitute $P(\frac{2}{3},\frac{2}{3})$ into (9),we obtain $y'=\frac{2}{3}$,which is the generalized slope of tangent line of the parabola $4x=2y+3y^2$ in the affine coordinate system xoy at point $P(\frac{2}{3},\frac{2}{3})$ .Hence the equation of the tangent line at P in Figure 4 is $\frac{y-\frac{2}{3}}{x-\frac{2}{3}}=k=\frac{2}{3}$,simplifying the equation we obtain $y=\frac{2}{3}x+\frac{2}{9}$ (10).Substituting the coordinate transformation formula (8) into equation (6) gives, $\frac{\sqrt{3}}{2}y=\frac{\sqrt{3}}{2}(x-\frac{1}{2}y)+\frac{\sqrt{3}}{6}$, and after simplification, we obtain. $y=\frac{2}{3}x+\frac{2}{9}$, which is consistent with equation (10)!

The generalized slope of the parabola curve at point P is $y'=\frac{2}{3}$ as mentioned above.In figure 4 $\frac{TA}{PA}=\frac{\Delta y}{\Delta x}=k=\frac{2}{3}$,so $PA>TA$,and because $\angle A=60^0,\angle PTA$ is the largest angle in triangle PTA . According to Equation (5), we have $\frac{2}{3}=y'=\frac{\sin\theta}{\sin(\alpha-\theta)}=\frac{\sin\theta}{\sin 120^0\cos\theta-\cos 120^0\sin\theta}=\frac{\tan\theta}{\frac{\sqrt{3}}{2}+\frac{1}{2}\tan\theta}$

, $\tan\theta=\frac{\sqrt{3}}{3}+\frac{1}{3}\tan\theta,\tan\theta=\frac{\sqrt{3}}{2}$, this result is consistent with (7).

According to Equation (4), $\cos(r,\angle PAT)=\cos(r,60^0)=\frac{TA}{PA}=\frac{2}{3}$, and according to Equation (1), $\sin^2(r,60^0)=\cos^2(r,60^0)-2\cos(r,60^0)\cos 60^0+1=\frac{7}{9},\sin(r,60^0)=\frac{\sqrt{7}}{3}$,therefore, $\sin(r,\theta)=\cos(r,60^0)$. $\frac{2}{3}$, $\cos(r,\theta)=\sin(r,60^0)=\frac{\sqrt{7}}{3}$,then, based on Equation (1), $\sin^2(r,\theta)=\cos^2(r,\theta)-2\cos(r,\theta)\cos\theta+1$, we obtain $\cos\theta=\frac{2}{\sqrt{7}}$, so $\tan\theta=\frac{\sqrt{3}}{2}$. This result also agrees with (7)!

In fact, if we establish electronic mathematical tables (1)-(15), we wouldn't need to go through all this trouble—we could simply look it up in the tables!

5 Generalized integral

Same as before ,the concepts of generalized definite integrals and generalized indefinite integrals can be introduced. The relationship between generalized definite integrals and generalized indefinite integrals is exactly the same as that between classical definite integrals and indefinite integrals—that is, the Newton-Leibniz formula holds in exactly the same form, and of course, it remains valid! Below, we illustrate its application in calculating the area of an oblique curvilinear trapezoid.

It is stipulated that the area of a parallelogram is the product of its two adjacent sides. Multiplying this product by $\sin\alpha$ gives the classical area of the parallelogram, where α is one of its interior angles.

Example 2 In an affine coordinate system xoy where the angle between the positive x-axis and y-axis is 60 degrees, find the area of the region bounded by the curves, $x=\frac{3}{4}y^2-\frac{1}{2}y, x=0, x=\frac{1}{4}, y=0$ and lying above the x-axis.

Method 1 As shown in the figure 5.

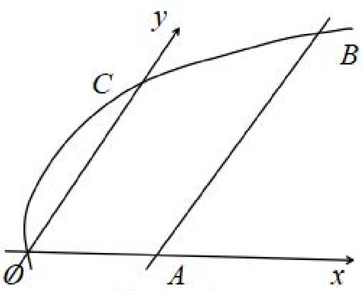

Figure 5

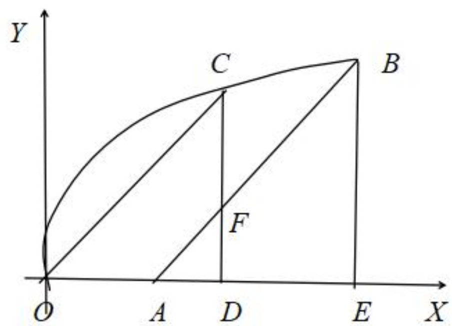

Figure 6

This oblique curved-edge trapezoid is bounded by the line segments OA, AB, OC, and the arc BC. The equation of OC is $x=0$, the equation of AB is $x=\frac{1}{4}$, the equation of OA is $y=0$, and the equation of the arc BC is $y=\frac{1+\sqrt{1+12x}}{3}$. According to the definition of the generalized definite integral, the area T of this oblique curved-edge trapezoid is

$$\int_0^{\frac{1}{4}}\frac{1+\sqrt{1+12x}}{3}dx=\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\int_0^{\frac{1}{4}}\sqrt{1+12x}dx$$

$=\frac{1}{12}+\frac{1}{3\times12}\int_0^{\frac{1}{4}}(1+12x)^{\frac{1}{2}}d(12x+1)=\frac{1}{12}+\frac{1}{36}\times\frac{(1+12x)^{1+\frac{1}{2}}}{1+\frac{1}{2}}|_0^{\frac{1}{4}}=\frac{1}{12}+\frac{1}{36}\times(\frac{8}{1.5}-\frac{1}{1.5})=\frac{23}{108}$ .So,the classical area of this curved trapezoid $S=T\times\sin 60^0=\frac{23}{108}\times\frac{\sqrt{3}}{2}=\frac{23\sqrt{3}}{216}$.

Method 2 Use the classical method by establishing a Cartesian coordinate system XOY as shown in the figure 6.Corresponding to Figure 5, The equation of OC is $Y=\sqrt{3}X$, the equation of AB is $Y=\sqrt{3}(X-\frac{1}{4})$, the equation of OA is $Y=0$, and the equation of the arc BC is $X=Y^2$. The coordinates of A、B、C、D、E、F are individually $A(\frac{1}{4},0)$、$B(\frac{3}{4},\frac{\sqrt{3}}{2})$、$C(\frac{1}{3},\frac{1}{\sqrt{3})}$、$D(\frac{1}{3},0)$、

$E(\frac{3}{4},0)$、$F(\frac{1}{3},\frac{\sqrt{3}}{12})$. The area of the curved-edge trapezoid $DEBC$ $S_1=\int_{\frac{1}{3}}^{\frac{3}{4}}\sqrt{X}\,dX$

$=\frac{X^{1+\frac{1}{2}}}{1+\frac{1}{2}}|_{\frac{1}{3}}^{\frac{3}{4}}=\frac{2}{3}((\frac{3}{4})^{\frac{3}{2}}-(\frac{1}{3})^{\frac{3}{2}})=\frac{19\sqrt{3}}{108}$ .The area of the trapezoid $DEBF$ $S_2=\frac{1}{2}(\frac{\sqrt{3}}{12}+\frac{\sqrt{3}}{2})\times(\frac{3}{4}-\frac{1}{3})=\frac{35\sqrt{3}}{288}$. The area of the trapezoid $OAFC$ $S_3=\frac{1}{6\sqrt{3}}-(\frac{1}{3}-\frac{1}{4})\times\frac{\sqrt{3}}{12}\times\frac{1}{2}=\frac{5\sqrt{3}}{96}$ .Hence The area of the oblique curved-edge trapezoid $OABC$ $S=S_1-S_2+S_3=\frac{23\sqrt{3}}{216}$ .This matches the result of Method 1.Clearly, Method 1 is simpler than Method 2,as it at least avoids the need to calculate the coordinates of points A、B、C、D、E、F . This example can also be solved using a double integral, but that would be quite cumbersome. This demonstrates the advantage of generalized definite integrals!Based on this paper and references [1-3], whenever a theory involves coordinate systems, there is no need to rigidly adhere to Cartesian coordinates as in classical theory,using (well-chosen) affine coordinate systems is better, simpler, and naturally applicable to a wider range of scenarios! Together, they expand the theoretical foundation of mathematics and have broad prospects for application!

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