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Unsteady MHD natural convection flow past an impulsively moving plate with ramped temperature

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Abstract

A numerical investigation of an unsteady MHD natural convection flow past an impulsively moving plate with ramped temperature. The dimensionless governing coupled, non-linear boundary layer partial differential equations are solved by a finite element method. The effects of the various dimensionless parameters entering into the problem on the primary velocity, secondary velocity, temperature and concentration profiles throughout the boundary layer are investigated through graphs. In addition, the analytical and numerical results of the local skin friction, couple stress coefficients, the rate of heat and mass transfers at the wall are prepared in tabular forms.

Key words:Unsteady, MHD, Natural convection,Ramped temperature,FEM.

1. Introduction

Theoretical/Experimental investigation of hydrodynamic natural convection flow arising near solid bodies with different geometries embedded in a porous medium is of much significance due to its varied and wide applications in several areas of science and technology viz. chemical catalytic reactors, thermal insulators, heat exchanger devices, nuclear waste repositories, drying of porous solids, enhanced oil and gas recovery, underground energy transport etc. Several researchers investigated natural convection flow near a vertical plate embedded in a porous medium considering different aspects of the problem. Mention may be made of research studies of Cheng and Minkowycz [1], Nakayama and Koyama [2], Lai and Kulacki [3], Hsieh *et al* [4], Nield and Kuznetsov [5], Gorla and Chamkha [6]. Comprehensive reviews of thermal convection in porous media along with its wide variety of engineering applications are well presented by Ingham and Pop [7], Vafai [8], Nield and Bejan [9].

Investigation of the problems of hydromagnetic natural convection flow of an electrically conducting fluid within a fluid saturated porous medium is of much significance due to considerable influence of magnetic field on boundary layer control, geothermal energy extraction, enhanced recovery of petroleum products, thermal insulation of buildings, sensible heat storage bed, plasma studies and on the performance of many engineering devices viz. MHD energy generators, MHD pumps, MHD accelerators, MHD flow-meters, Plasma jet engines, controlled thermo-nuclear reactors etc. Keeping in view the importance of such study, Raptis and Kafousias [10] analyzed the effects of magnetic field on steady free convection flow through a porous medium bounded by an infinite vertical plate. Raptis [11] investigated time varying two-dimensional natural convection flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical porous plate embedded in a porous medium. Takhar and Ram [12] considered hydromagnetic free convection flow of water at 40C through porous medium. Chamkha [13] studied MHD free convection from a vertical plate embedded in a thermally stratified porous medium. Chamkha and Khanafer [14] discussed non-similar combined convection flow past a vertical surface embedded in a variable porosity medium. Jha [15] investigated MHD free convection and mass transfer flow past a uniformly accelerated moving vertical plate through porous medium when applied magnetic field is fixed with the moving plate. Aldosset *al* [16] studied combined free and forced convection flow past a vertical plate embedded in a porous medium in the presence of a magnetic field. Kim [17] considered hydromagnetic natural convection flow past a vertical moving plate embedded in a porous medium. Makinde and Sibanda [18] considered steady

hydromagnetic heat transfer by mixed convection flow past a vertical plate embedded in a uniform porous medium in the presence of uniform transverse magnetic field. Makinde [19] studied MHD mixed convection flow and mass transfer past a vertical porous plate with constant heat flux embedded in a porous medium.

Fluid heat generation or absorption effects are of much significance in certain porous medium applications such as fluids undergoing exothermic and/ or endothermic chemical reaction (Vajravelu and Nayfeh [20]), applications in the field of nuclear energy (Crepeau and Clarksean [21]), convection in Earth's mantle (McKenzie et al. [22]), post-accident heat removal (Baker et al. [23]), fire and combustion modeling (Delichatsios [24]), development of metal waste from spent nuclear fuel (Westphal et al. [25]) etc. It is noticed that exact modeling of internal heat generation/ absorption is much complicated. It is found that some simple mathematical models yet idealized may express their average behavior for most of the physical situations. Taking into consideration this fact, investigation of such fluid flow problems is carried out by many researchers [26-30] in the past.

In most cases, the Hall term is ignored in applying Ohm's law as it has no marked effect for small magnetic fields. However, to study the effects of strong magnetic fields on the electrically conducting fluid flow, we see that the influence of the electromagnetic force is noticeable and causes anisotropic electrical conductivity in the plasma. This anisotropy in the electrical conductivity of the plasma produces a current known as the Hall current. The Hall effect is important when the magnetic field is high or when the collision frequency is low, causing the Hall parameter to be significant. The effects of Hall current on the fluid flow and heat transfer in rotating channels have many engineering applications in flows of laboratory plasmas, in MHD power generation, in MHD accelerators, and in several astrophysical and geophysical situations. Thus, the effect of Hall current on MHD flow has been investigated by many researchers, Pop and Watanabe [31], Abo – Eldahab and Elbarbary [32], Takhar et al. [33] and Saha et al. [34]. Siva Reddy Sheri et al. [35] studied Thermal-diffusion and diffusion-thermo effects on MHD natural convective flow through porous medium in a rotating system with ramped temperature. Siva Reddy Sheri et al. [36] investigated Heat and mass transfer effects on MHD natural convection Flow past an infinite inclined plate with ramped temperature. Siva Reddy Sheri et al. [37] have found Finite element analysis of heat and mass transfer past an impulsively moving vertical plate with ramped temperature. It is worthy to note that Hall current induces secondary flow in the flow – field which is also the

characteristics of Coriolis force. Therefore, it becomes very important to compare and contrast the effects of these two agencies and also to study their combined effects on such fluid flow problems.

Objective of the present investigation is to study the effect of an unsteady MHD natural convection flow past an impulsively moving plate with ramped temperature. The governing equations are solved numerically using finite element technique. Numerical results are reported for various values of the physical parameters of interest.

2. Mathematical formulation

The equations governing the motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are

Equation of continuity:

$$\nabla \cdot \bar{v} = 0 \quad (1)$$

Momentum equation:

$$\rho \left[\frac{\partial \bar{v}}{\partial t'} + (\bar{v} \cdot \nabla) \bar{v} \right] = -\nabla p + \bar{J} \times \bar{B} + \rho \bar{g} + \mu \nabla^2 \bar{v} - \frac{\mu}{k'} \bar{v} \quad (2)$$

Energy equation:

$$\rho C_p \left[\frac{\partial T'}{\partial t'} + (\bar{v} \cdot \nabla) T' \right] = \kappa \nabla^2 T' - \frac{\partial q_r}{\partial y'} \quad (3)$$

Species continuity equation:

$$\frac{\partial C'}{\partial t'} + (\bar{v} \cdot \nabla) C' = D \nabla^2 C' \quad (4)$$

Kirchhoff's first law:

$$\nabla \cdot \bar{J} = 0 \quad (5)$$

General Ohm's law, taking Hall Effect into account:

$$\bar{J} + \frac{\varpi_e \tau_e}{B_0} (\bar{J} \times \bar{B}) = \sigma \left(\bar{E} + \bar{v} \times \bar{B} + \frac{1}{e \eta_e} \nabla p_e \right) \quad (6)$$

Gauss's law of magnetism:

$$\bar{\nabla} \cdot \bar{B} = 0 \quad (7)$$

We now consider an unsteady flow of an electrically conducting fluid past an infinite vertical porous flat plate coinciding with the $X -$ axis $y' = 0$, taking into account the Hall current in presence of a uniform transverse magnetic field. Our investigation is restricted to the following assumptions:

- i. All the fluid properties except the density in the buoyancy force term are constant.
- ii. The plate is electrically non – conducting.
- iii. The magnetic Reynolds number is so small that the induced magnetic field may be neglected.
- iv. p_e is constant
- v. $\bar{E} = 0$

We introduce a coordinate system (x', y', z') with $X -$ axis vertically upwards, $Y -$ axis normal to the plate directed into the fluid region and $Z -$ axis along the width of the plate. Let $\bar{v} = u'\hat{i} + v'\hat{j} + w'\hat{k}$ be the velocity, $\bar{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$ be the current density at the point $p(x', y', z', t')$ and $\bar{B} = \bar{B}_0\hat{j}$ be the applied magnetic field, $\hat{i}, \hat{j}, \hat{k}$ being unit vectors along $X -$ axis, $Y -$ axis and $Z -$ axis respectively. Since the plate is of infinite length in X and Z direction, therefore all the quantities except possibly the pressure are independent of x' and z' .

$$\text{Now, the Eq. (1) gives } \frac{\partial \bar{v}}{\partial y'} = 0 \quad (8)$$

$$\text{which is trivially satisfied by } \bar{v} = -V_0 \quad (9)$$

where V_0 is a constant and $V_0 > 0$

$$\text{Therefore the velocity vector } \bar{v} \text{ is given by } \bar{v} = u'\hat{i} - V_0\hat{j} + w'\hat{k} \quad (10)$$

$$\text{Again the Eq. (7) is satisfied by } \bar{B} = \bar{B}_0\hat{j} \quad (11)$$

$$\text{Also the Eq. (5) reduces to } \frac{\partial J_y}{\partial y'} = 0 \quad (12)$$

which shows that $J_y = \text{constant}$

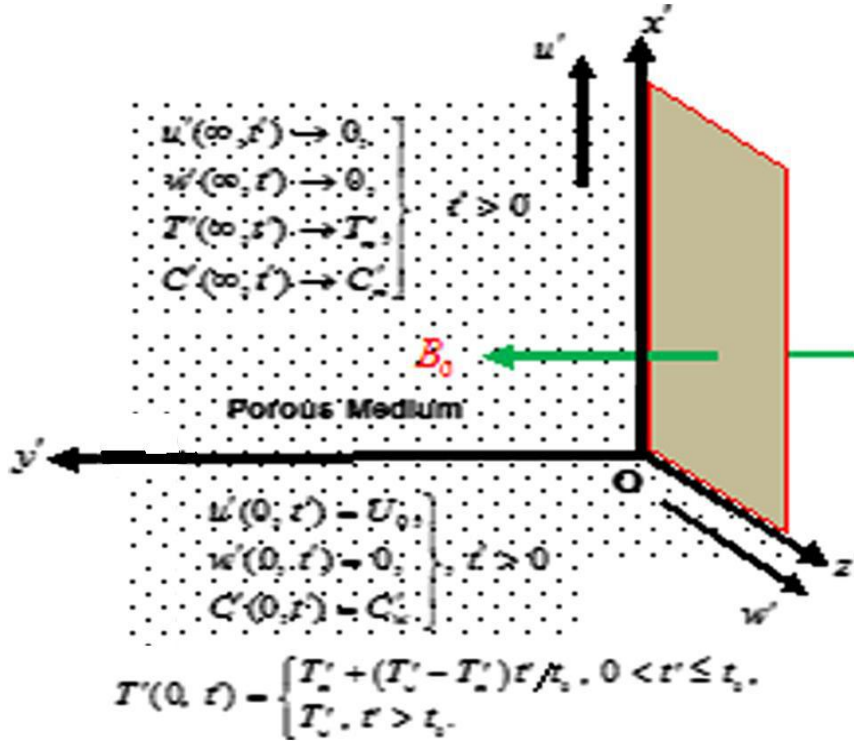


Figure 1. Sketch of the diagram

Since the plate is non – conducting, $J_y = 0$ at the plate and hence $J_y = 0$ at all points in the fluid. Thus the current density is given by $\bar{J} = J_x \hat{i} + J_z \hat{k}$ (13)

Under the assumption (iv) and (v), the Eq. (6) takes the form

$$\bar{J} + \frac{m}{B_0} (\bar{J} \times \bar{B}) = \sigma (\bar{v} \times \bar{B}) \quad (14)$$

Where $m = \varpi_e \tau_e$ is the Hall parameter.

The Eqs. (10), (11), (13), and (14) yield,

$$J_x = \frac{\sigma B_0}{1+m^2} (mu' - w') \text{ and } J_z = \frac{\sigma B_0}{1+m^2} (u' + mw') \quad (15)$$

With the following assumptions and the usual boundary layer and Boussinesq's approximation, the Eqs. (2), (3) and (4) reduce to the following:

$$\frac{\partial u'}{\partial t'} + \nu \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{1}{(1+m^2)} \left[\frac{\sigma B_0^2}{\rho} \right] u' + \frac{m \sigma B_0^2}{\rho(1+m^2)} w' - \left[\frac{\nu}{k'} \right] u' + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) \quad (16)$$

$$\frac{\partial w'}{\partial t'} + \nu \frac{\partial w'}{\partial y'} = \nu \frac{\partial^2 w'}{\partial y'^2} + \frac{\sigma B_0^2 (mu' - w')}{\rho(1+m^2)} - \frac{\nu w'}{k'} \quad (17)$$

$$\frac{\partial(T' - T'_\infty)}{\partial t'} + \nu \frac{\partial(T' - T'_\infty)}{\partial y'} = \frac{\kappa}{\rho C_p} \frac{\partial^2(T' - T'_\infty)}{\partial y'^2} \quad (18)$$

$$\frac{\partial(C' - C'_\infty)}{\partial t'} + \nu \frac{\partial(C' - C'_\infty)}{\partial y'} = D \frac{\partial^2(C' - C'_\infty)}{\partial y'^2} \quad (19)$$

Subject to the boundary conditions

$$\left. \begin{aligned} t' \leq 0 : u' = 0, w' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \\ t' \leq t_o : T' = T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_o} \text{ at } y' = 0 \\ t' > 0 : \begin{cases} u' = U_o, w' = 0, T' = T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_o}, C' = C'_w \text{ at } y' = 0 \\ u' \rightarrow 0, w' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \end{cases} \\ t' > t_o : \begin{cases} T' = T'_w \text{ at } y' = 0 \\ T' \rightarrow T'_\infty \text{ as } y' \rightarrow \infty \end{cases} \end{aligned} \right\} (20)$$

Characteristic time t_o is defined according to the non – dimensional process mentioned above

$$\text{as } t_o = \frac{\nu}{V_o}.$$

Let us introduce the following dimensionless quantities:

$$\left. \begin{aligned} \eta = \frac{V_o y'}{\nu}, t = \frac{V_o^2 t'}{\nu}, u = \frac{u'}{V_o}, w = \frac{w'}{V_o}, \theta = \frac{\theta'}{a}, \phi = \frac{C'}{b}, Gr = \frac{g\beta\nu a}{V_o^3}, Gc = \frac{g\beta^* \nu b}{V_o^3}, \\ M = \frac{B_o^2 \sigma \nu}{\rho V_o^3}, Pr = \frac{\nu \rho C_p}{\kappa}, Sc = \frac{\nu}{D}, K = \frac{V_o^2 k'}{\nu^2} \end{aligned} \right\} (21)$$

All the physical variables are defined in the Nomenclature.

Eqs. (16), (17), (18) and (19) transform to the following non – dimensional forms, respectively

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} - \frac{M}{(1+m^2)}(u + mw) + Gr\theta + Gc\phi - \frac{u}{K} \quad (22)$$

$$\frac{\partial w}{\partial t} - \frac{\partial w}{\partial \eta} = \frac{\partial^2 w}{\partial \eta^2} - \frac{M}{(1+m^2)}(mu - w) - \frac{w}{K} \quad (23)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial \eta} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \eta^2} \quad (24)$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial \eta} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial \eta^2} \quad (25)$$

The Corresponding boundary conditions in non – dimensional forms are

$$\left. \begin{array}{l} t \leq 0: \quad u = 0, w = 0, \theta = 0, \phi = 0 \quad \text{for all } \eta \\ t > 0: \quad \left\{ \begin{array}{l} u = 1, w = 0, \theta = t, \phi = 1 \quad \text{at } \eta = 0 \\ u \rightarrow 0, w \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{array} \right. \\ t \leq 1: \quad \theta = t \quad \text{at } \eta = 0 \\ t > 1: \quad \left\{ \begin{array}{l} \theta = 1 \quad \text{at } \eta = 0 \\ \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{array} \right. \end{array} \right\} \quad (26)$$

For practical engineering applications and the design of chemical engineering systems, quantities of interest include the following Skin – friction coefficient, Couple stress coefficient, Nusselt number and Sherwood number are useful to compute.

The local skin – friction coefficient which signifies the surface shear stress is defined as:

$$C_f = \left[\frac{\partial u}{\partial \eta} \right]_{\eta=0} \quad (27)$$

The couple stress coefficient at the wall is given by

$$C_w = \left[\frac{\partial w}{\partial \eta} \right]_{\eta=0} \quad (28)$$

The local Nusselt number which embodies the ratio of convective to conductive heat transfer across (normal to) the boundary and is a quantification of the surface temperature gradient (heat transfer rate at the wall) is defined as:

Then

$$Nu = - \left[\frac{\partial \theta}{\partial \eta} \right]_{\eta=0} \quad (29)$$

Finally the local Sherwood number which encapsulates the ratio of convective to diffusive mass transport and simulates the surface mass transfer rate is defined as:

Then

$$Sh = - \left[\frac{\partial \phi}{\partial \eta} \right]_{\eta=0} \quad (30)$$

3. Method of Solution

The finite element method has been implemented to obtain numerical solutions of (22) – (25) under boundary conditions (26). This technique is extremely efficient and allows robust solutions of complex coupled, nonlinear multiple degree differential equation systems. The fundamental steps comprising the method are now summarized. An excellent description of finite element formulations is available in Bathe [38] and Reddy [39].

Step – 1 Discretization of the Domain into Elements

The whole domain is divided into finite number of “sub – domains”, a process known as Discretization of the domain. Each sub – domain is termed a “finite element”. The collection of elements is designated the “finite element mesh”.

Step – 2 Derivation of the element

The derivation of finite element *i.e.*, algebraic among the unknown parameters of the finite element approximation, involves the following three steps.

- a. Construct the variational formulation of the differential equation.
- b. Assume the form of the approximate solution over a typical finite element.
- c. Derive the finite element by substituting the approximate solution into variational formulation.

These steps results in a matrix equation of the form $[K^e]\{u^e\} = \{F^e\}$, which defines the finite element model of the original Equation.

Step – 3 Assembly of Elements

The algebraic so obtained are assembled by imposing the “inter – element” continuity conditions. This yields a large number of algebraic constituting the “global finite element model”, which governs the whole flow domain.

Step – 4 Impositions of Boundary Conditions

The physical boundary conditions defined in (28) are imposed on the assembled

Step – 5 Solution of the Assembled

The final matrix equation can be solved by a direct or indirect (iterative) method. For computational purposes, the coordinate η is varied from 0 to $\eta_{\max} = 10$, where η_{\max} represents infinity *i.e.*, external to the momentum, energy and concentration boundary layers.

The whole domain is divided into a set of 100 intervals of equal length 0.1 . At each node 3 functions are to be evaluated. Hence after assembly of the elements we obtain a set of 123. The system of after assembly of elements, are non – linear and consequently an iterative scheme is employed to solve the matrix system, which are solved using the Gauss Elimination method maintaining an accuracy of 0.0005 .

4. Results and discussion

We have obtained a comprehensive range of solutions to the transformed conservation equations. To test the validity of finite element computations, we have compared the local skin – friction coefficient and couple stress coefficient in Tables 1, 2 and 3 with the perturbation Technique. It is clearly seen from Table 1, 2 and 3 that the results are in excellent agreement. As the accuracy of the numerical solutions is very good, the values of local skin – friction coefficient, couple stress coefficient to analytical and numerical solutions are very close to each other. From Table 1, it is observed that the values of local skin – friction coefficient and couple stress coefficient are increases to increase of hall parameter. This is because the effect of transverse magnetic field is to increase the flow by offering additional resistance called magnetic viscosity. Table 2 presents the numerical values of rate of heat transfer in terms of Nusselt number Nu due to variations in the values of Prandtl number Pr . It is observed that rate of heat transfer Nu are least for mercury and highest for water at $4^0 C$. Table 3 presents the numerical values of rate of mass transfer in terms of Sherwood number Sh . The values of Sc for hydrogen, helium and ammonia. It is observed that the values of the rate of mass transfer Sh increase due to increase in Sc .

Hall parameter	Finite Element Method		Perturbation Technique	
	local skin – friction coefficient	Couple stress coefficient	local skin – friction coefficient	Couple stress coefficient
0.5	2.325756798	3.023157864	2.3257567990	3.02315786301
1.0	2.456947862	3.146408538	2.4569478630	3.14640853700
1.5	2.588279843	3.269659212	2.5882798440	3.2696592110
2.0	2.717853215	3.392909882	2.7178532160	3.39290988101

Table – 1 Comparison values of local skin – friction coefficient and couple stress coefficient with $Gr = 6.0, Gc = 5.0, K = 0.5, M = 2.0, Pr = 0.71, Sc = 0.6$ and $t = 0.5$.

Prandtl number	Finite Element Method	Perturbation Technique
0.025	0.549673412	0.5496734130
0.71	0.407029841	0.4070298420
7.0	0.264386268	0.26438626900

Table – 2 Comparison values of local Nusselt number with $Gr = 6.0, Gc = 5.0, K = 0.5, M = 2.0, m = 0.5, Sc = 0.6$ and $t = 0.5$.

Schmidt number	Finite Element Method	Perturbation Technique
0.22	0.4348793152	0.4348793152
0.30	0.3637549217	0.3637549217
0.78	0.121582079	0.121582079

Table – 3 Comparison values of local Sherwood number with $Gr = 6.0, Gc = 5.0, K = 0.5, M = 2.0, m = 0.5, Pr = 0.71,$ and $t = 0.5$.

In order to analyze the effects of Hall current, thermal buoyancy force, concentration buoyancy force, thermal diffusion, mass diffusion, thermal radiation and time on the flow – field, numerical values of the primary and secondary fluid velocities in the boundary layer region, computed from the numerical solutions are displayed graphically versus boundary layer coordinate η in Figs. 2 – 9. In the present study we adopted the following default parameter values of finite element computations: $Gr = 5.0, Gc = 5.0, m = 0.5, M = 2.0, K = 0.5, Pr = 0.71, Sc = 0.6, t = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph. It is revealed from Figs. 2 – 9 that, for both ramped temperature and isothermal plates, primary velocity u and secondary velocity w attain a distinctive maximum value near surface of the plate and then decrease properly on increasing boundary layer coordinate η to approach free stream value. It is also noticed that the primary and secondary fluid velocities are slower in the case of ramped temperature plate than that of isothermal plate.

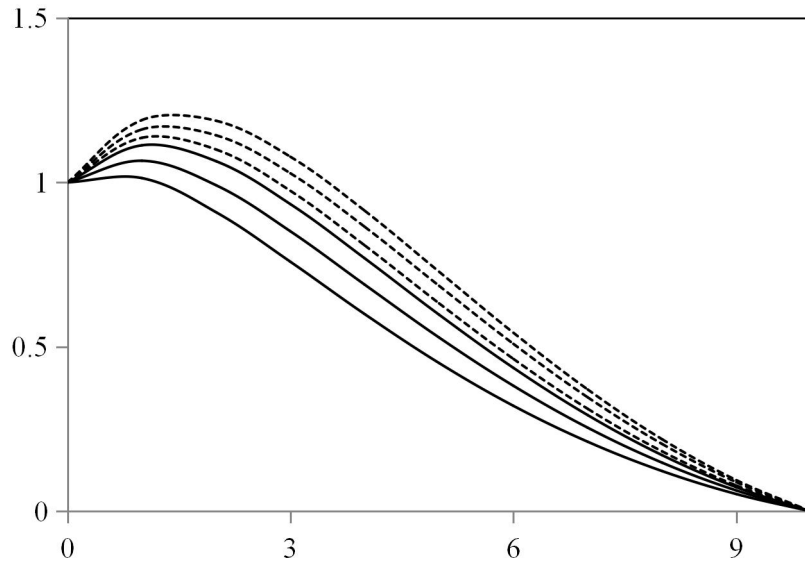


Fig. 2 Primary velocity profiles

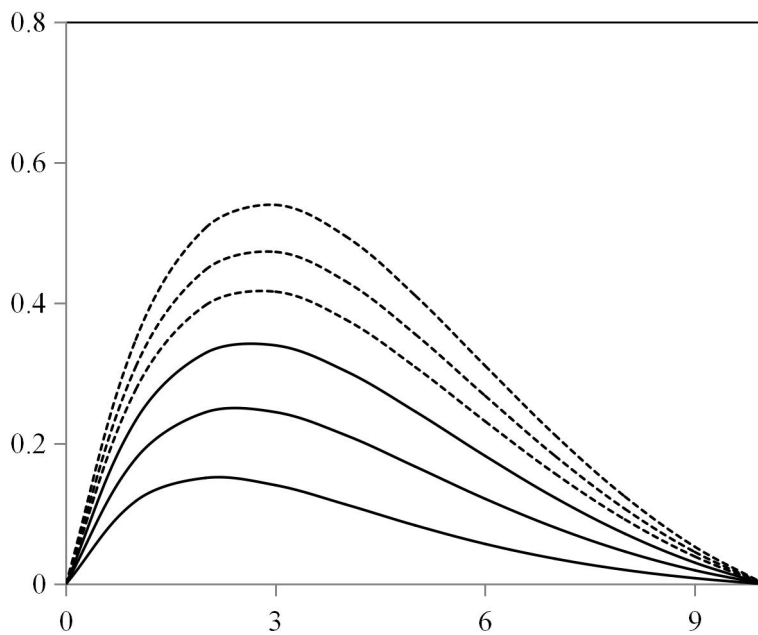


Fig. 3 Secondary velocity profiles

Figs. 2 and 3 depict the influence of Hall current on the primary velocity u and secondary velocity w for both ramped temperature and isothermal plates. It is evident from Figs. 2 and 3 that, for both ramped temperature and isothermal plates, u increases on increasing m in a region near to the plate and it decreases on increasing m in the region away from the plate whereas w increases on increasing m throughout the boundary layer region. This implies that, for both ramped temperature and isothermal plates, Hall current tends to accelerate

secondary fluid velocity throughout the boundary layer region which is consistent with the fact that Hall current induces secondary flow in the flow – field. Hall current tends to accelerate primary fluid velocity in a region close to the plate whereas it has a reverse effect on primary fluid velocity in the region away from the plate.

Figs. 4 and 5 represent the influence of time on the primary and secondary fluid velocities for both ramped temperature and isothermal plates. It is evident from Figs.4 and 5 that, for both ramped temperature and isothermal plates, u and w increase on increasing t . This implies that primary and secondary fluid velocities are getting accelerated with the progress of time throughout the boundary layer region for both ramped temperature and isothermal plates.

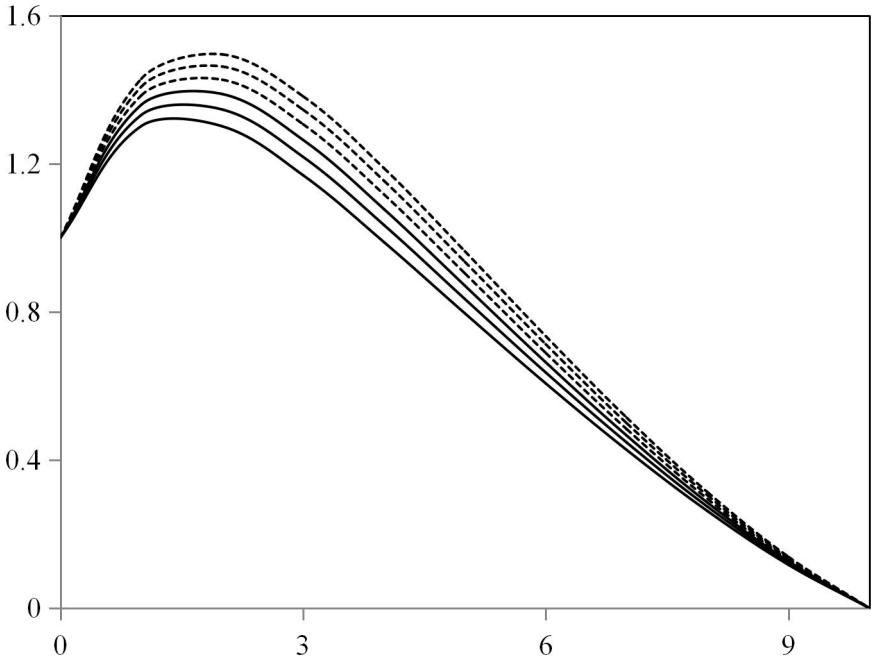


Fig. 4 Primary velocity profiles

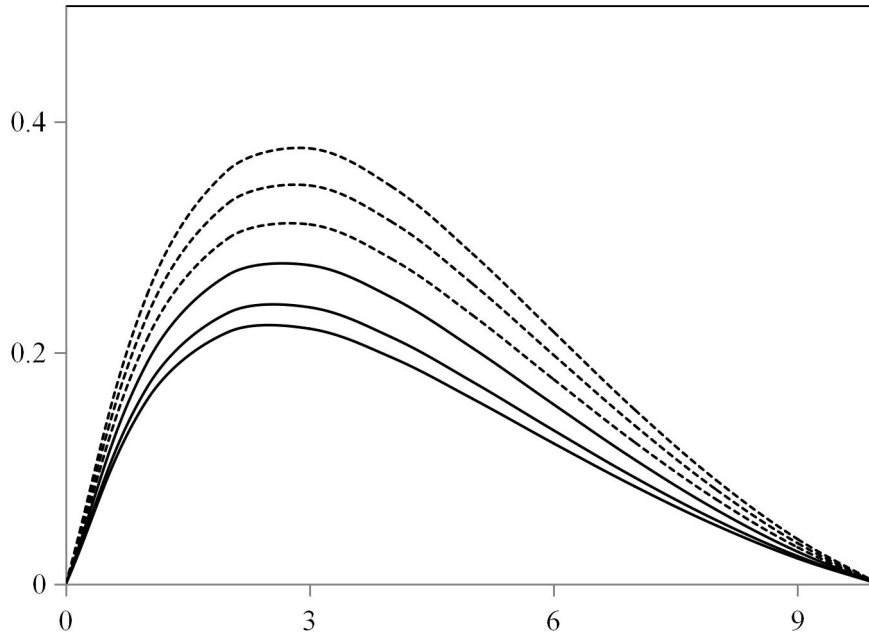


Fig.5 Secondary velocity profiles

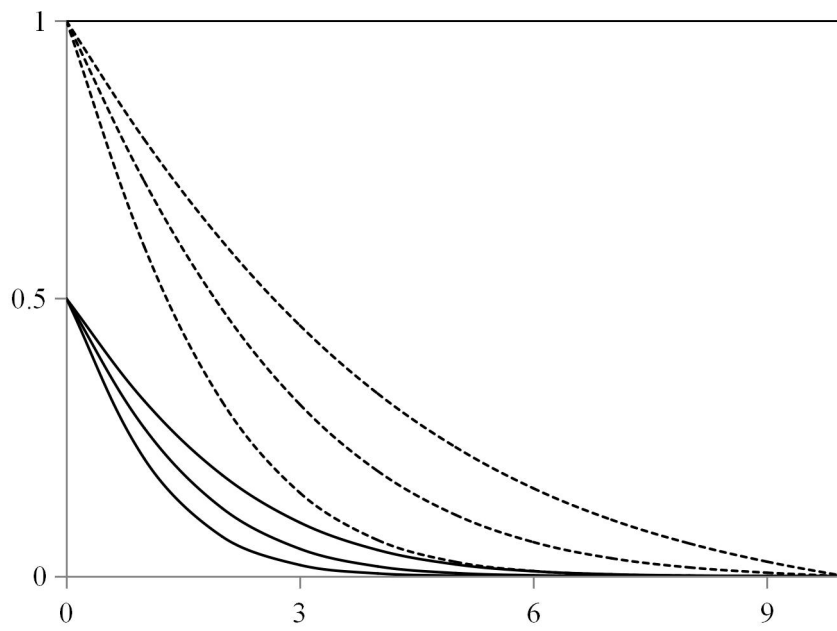


Fig. 6 Temperature profiles

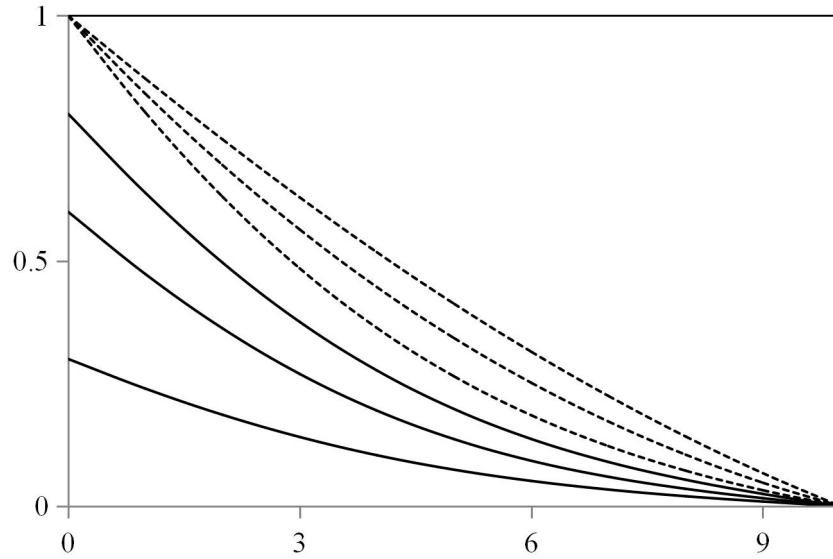


Fig. 7 Temperature profiles

Figs. 6 and 7 reveal that fluid temperature θ decreases on increasing Pr whereas it increases on increasing t for both ramped temperature and isothermal plates. This implies that, for both ramped temperature and isothermal plates, thermal diffusion tends to enhance fluid temperature and there is an enhancement in fluid temperature with the progress of time throughout the boundary layer region. The numerical values of species concentration ϕ , computed from the numerical solution, are presented graphically versus boundary layer coordinate η in Figs. 8 and 9 for various values of Schmidt number Sc and time t . It is evident from Figs. 8 and 9 that species concentration ϕ decreases on increasing Sc whereas it increases on increasing t . This implies that mass diffusion tends to enhance species concentration and there is an enhancement in species concentration with the progress of time throughout the boundary layer region.

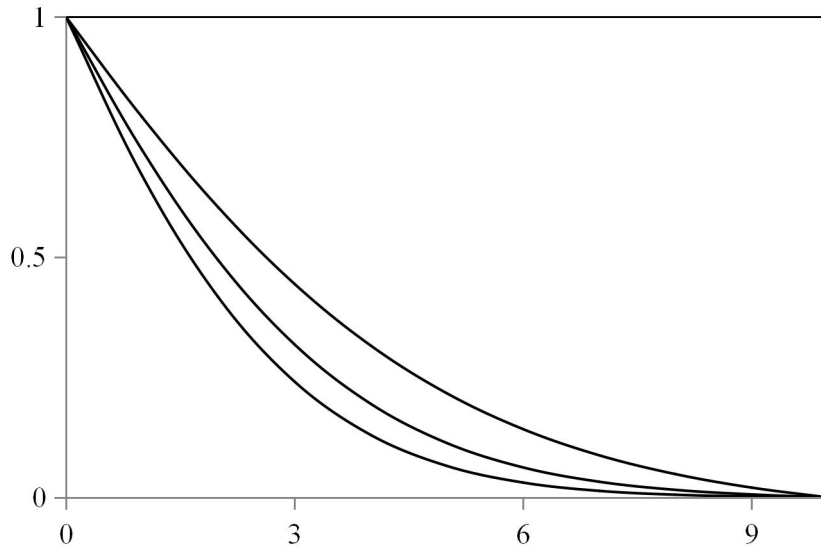


Fig.8 Concentration profiles

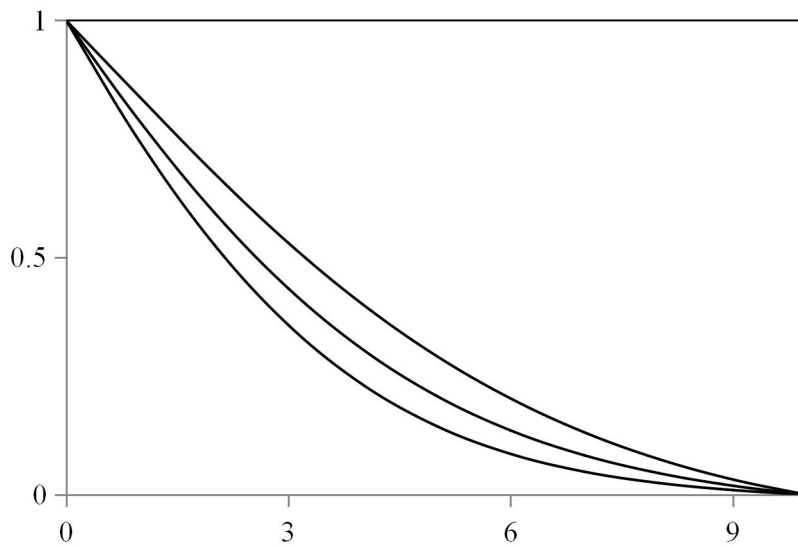


Fig.9 Concentration profiles

5. Conclusions

An investigation of the effect of an unsteady MHD natural convection flow past an impulsively moving plate with ramped temperature. Hall current tends to accelerate secondary fluid velocity throughout the boundary layer region. Hall current tends to accelerate primary fluid velocity in a region close to the plate whereas it has a reverse effect on the primary fluid velocity in the region away from the plate. Mass diffusion tends to reduce species concentration and there is an enhancement in species concentration with the progress of time

obtained throughout the boundary layer. The values of local skin – friction coefficient and couple stress coefficient are increases with increasing of hall parameter. The rate of heat transfer at the plate significantly decreases for high Prandtl number. Schmidt number has tendency to reduce the Sherwood number throughout the boundary layer. The accuracy of the numerical solutions is very good and the values of local skin-friction coefficient, couple stress coefficient, Nusselt number and Sherwood number corresponding to analytical and numerical solutions are very close to each other.

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