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On E_c-(δ-β_c)-Compact Spaces and E_c-(δ-β_c)-Separation Axioms

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Abstract

The purpose of the present paper is to consider new classes of generalized compact spaces called E_c-(*resp.* δ-β_c)-compact spaces via E_c-(*resp.* δ-β_c)-open sets respectively. Several characterizations and fundamental properties concerning of these forms of spaces are obtained. Moreover, some new types of separation axioms in topological spaces namely E_c-(*resp.* δ-β_c)-separation axioms via E_c (*resp.* δ-β_c)-open sets are introduced and studied. Several of their fundamental properties and relationships with other well-known types of spaces are discussed.

Keywords: E_c-(resp. δ-β_c)-Compact space, E_c-(resp. δ-β_c)-open sets, E_c- separation axioms, δ-β_c- separation axioms.

1. Introduction

The notion of compactness is very useful and fundamental notion of general topology also in the other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness. In literature, different classes of generalized compactness such as [1, 2] are studied. Benchalli and Patil [3] introduced and studied a new class of closed sets called αw-closed sets and continuous maps in topological space. In [4] Hariwan Z. Ibrahim presented a new class of space named B_c-compact and gave some properties of B_c-compact space by using B_c- open sets. As well, P. G. Patil, in [5] introduced the concept of $\omega\alpha$-compactness in topological spaces and gave some characterization of $\omega\alpha$-compactness by using $\omega\alpha$-closed sets . On the other hand, recently, Sarika and Rayanagoudar [6] introduced a new concept called αg^*s-compactness in topological spaces and obtained some of their properties by using αg^*s-closed sets. In recent literature, we find many topologists had focused their research in the direction of investigating different types of separation axioms. Some of these have been found to be useful in computer science and digital topology [see for example [7, 8]. Dontcheve and Ganster [7] proved that the digital line is $T_{3/4}$ space but not T_1. Also, Navalagi [9] introduce semi generalized- T_i spaces, i= 0, 1, 2. In addition, in 2011, Ahu Açıkgöz [10] defined two new separation axioms called $\beta^*T_{1/2}$ and β^{**} $T_{1/2}$ spaces as applications of β^*g−closed sets. Hariwan Z. Ibrahim in [11] presented and investigated some weak separation axioms by using the notions of B_c-open sets and the B_c-closure operator. As well, in the same year Hussein A. Khaleefah[12] studied new types of separation axioms termed by, generalized b- R_i, i= 0, 1 and generalized b-T_i, i= 0, 1, 2 by using generalized b-open sets, Relations among these types are investigated, and several properties and characterizations are provided. A.I. EL-Maghrabi and M.A. AL-Juhani [13] introduced and investigated a new class of separation axioms called M-Ti-spaces, i = 0, 1, 2. Also, the M-regularity and the M-normality are examined in the context of these new concepts. This work is devoted to introduce a new classes of generalized compact spaces called E_c-*(resp.* δ-β_c)-compact spaces. Several characterizations and fundamental properties concerning of these forms of spaces are obtained. As well as to introduce and study new notions of separation axioms called E_c-*(resp.* δ-β_c)-separation axioms. Several fundamental properties and preservation properties concerning of these kinds of weak separation axioms are provided. Furthermore, the relationships among these types of separation axioms and other well-known types of spaces are discussed.

2. Preliminaries

Throughout this paper, *(X, T)* and *(Y, T^*)* (or simply *X* and *Y*) mean topological spaces on which no separation axioms are assumed unless explicitly stated. For any subset A of *X*, The closure and interior of A are denoted by *Cl(A)* and *Int(A)*, respectively.

We recall the following required results, which will be used often throughout this paper.

Definition 2.1: Let *(X, T)* be a topological space. A subset *A* of *X* is said to be:

a) Regular open (resp. regular closed) [14] if A = Int(Cl(A)) (resp. A = Cl(Int(A))).

b) δ-open [15] if for each $\varkappa \in A$ there exists a regular open set V such that $\varkappa \in V \subseteq A$. The **$\delta$**-interior of A is the union of all regular open sets contained in A and is denoted by $\mathrm{Int}_\delta(A)$. The subset A is called δ-open [15] if $A = \mathrm{Int}_\delta(A)$. A point $\varkappa \in X$ is called a δ-cluster points of A [15] if $A \cap \mathrm{Int}(\mathrm{Cl}(V)) \neq \varphi$, for each open set V containing $\varkappa$. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $Cl_\delta(A)$. If $A = \mathrm{Cl}_\delta(A)$), then A is said to be δ-closed [15]. The complement of δ-closed set is said to be δ-open set. A subset A of a Topological space X is called δ-open [15] if for each $\varkappa \in A$ there exists an open set G such that, $\varkappa \in G \subseteq \mathrm{Int}(\mathrm{Cl}(G)) \subseteq A$. The family of all δ-open sets in X is denoted by $\delta\Sigma(X, T)$.

Definition 2.2: *Let (X, T)* be a Topological space. Then:

a) A subset A of a space X is called E-open [16] if $A \subseteq \mathrm{Cl}(\delta\text{-Int}(A)) \cup \mathrm{Int}(\delta\text{-Cl}(A))$. The complement of an E-open set is called E-closed. The intersection of all E-closed sets containing A is called the E-closure of A [16] and is denoted by E-Cl(A). The union of all E-open sets of X contained in A is called the E-interior [**16**] of A and is denoted by E-Int(A).

b) A subset A of a space X is called δ-β-open [17] or e^*-open [18], if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\delta\text{-Cl}(A)))$, the complement of a δ-β-open set is called δ-β-closed. The intersection of all δ-β-closed sets containing A is called the δ-β-closure of A [17] and is denoted by δ-β-Cl(A). The union of all δ-β-open sets of X contained in A is called the δ-β-interior [17] of A and is denoted by δ-β-Int(A).

Remark 2.3: The family of all E-open (resp. E-closed, δ-β-open, δ-β-closed) subsets of X containing a point $\varkappa \in X$ is denoted by $E\Sigma(X, \varkappa)$ (resp. $EC(X, \varkappa)$, $\delta\text{-}\beta\Sigma(X, \varkappa)$, $\delta\text{-}\beta C(X, \varkappa)$). The family of all E-open (resp. E-closed, δ-β-open, δ-β-closed) sets in X are denoted by $E\Sigma(X, T)$ (*resp*. $EC(X, T)$, $\delta\text{-}\beta\Sigma(X, T)$, $\delta\text{-}\beta C(X, T)$).

Definition 2.4: Let *(X, T)* be a Topological space. A subset *A* of *X* is said to be:

i) E_c-open set if for each $\varkappa \in A \in E\Sigma(X, T)$, there exists a closed set $\boldsymbol{F}$ such that, $\varkappa \in F \subseteq A$. The family of all E_c-open subsets of *(X, T)* is denoted by $EC\Sigma(X, T)$ *OR* $EC\Sigma(X)$.

ii) δ-β_c-open set if for each $\varkappa \in A \in \delta$-$\beta\Sigma(X, T)$, there exists a closed set $\boldsymbol{F}$ such that, $\varkappa \in F \subseteq A$. The family of all δ-β_c -open subsets of *(X, T)* is denoted by δ-$\beta C\Sigma(X, T)$ *OR* δ-$\beta C\Sigma(X)$.

A subset *F* of a space *(X, T)* is said to be E_c (*resp.* δ-β_c)-closed set when, $X \setminus F \in EC\Sigma(X, T)$ (*resp.* δ-$\beta C\Sigma(X, T)$).

Remark 2.5: The family of all E_c (*resp.* δ-β_c)-closed subsets of *(X, T)* is denoted by *ECC(X, T) OR ECC(X)* (*resp.* δ-$\beta CC(X, T)$ *OR* δ-$\beta CC(X)$).

Theorem 2.6: Every regular closed sub-set in a space *(X, T)* is δ-β_c-open set. [19].

Theorem 2.7: Let *X* be a Topological space, if *X* is Regular space. Then every open set is a E_c (*resp.* δ-β_c)-open set. [19].

Remark 2.8: We have the following figure in which the converses of implications need not be true, see the examples in [20], [16] and [18].

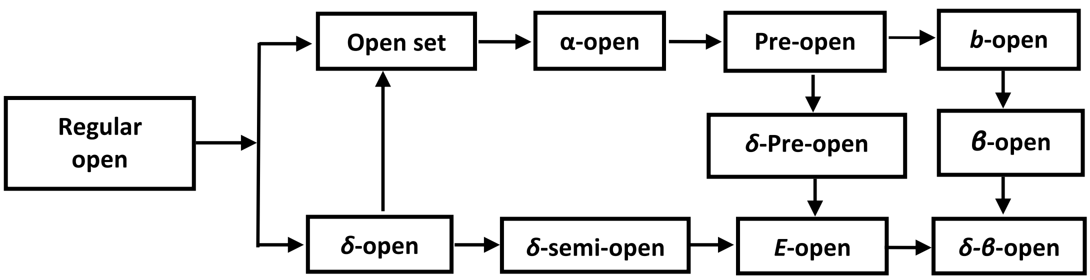

Figure (1): The relationships among some well-known generalized open sets in Topological Spaces

3. Characterizations of *(E_c)* and *(δ-β_c)*-Compact spaces

In this section, several characterizations concerning of new classes of spaces named E_c-(*resp.* δ-β_c)-compact spaces are introduced and investigated utilizing new generalized open sets.

Definition 3.1: A topological space *X* is called:

a) E_c (*resp.* δ-β_c)- Compact if for every E_c (*resp.* δ-β_c)-open cover $\{\mathcal{V}_\lambda : \lambda \in \nabla \text{ of } X\}$ there exists

A finite sub-set $\nabla_o \subseteq \nabla$ $(s.t)$ $X = \cup\{\mathcal{V}_\lambda : \lambda \in \nabla_o\}$.

b) Nearly compact [21] if for each open cover $\mho$ of *X*, there exists a finite sub collection $\mathcal{B} \subseteq \mho\ (s.t)\ \bigcup\{Int(Cl(\mathcal{V}))|\mathcal{V} \in \mathcal{B}\} = X$.

c) Almost regular [22] If for each *T*- regularly closed subset A of X and $\forall$ point $\varkappa \notin A\ \exists$ disjoint T_s- open sets $\mathcal{U}\ and\ \mathcal{V}\ (s.t)\ A \subseteq \mathcal{U}\ and\ \varkappa \in \mathcal{V}$.

d) *E*- (*resp*. *δ*-*β*)-Compact [23] if every *E*-(*resp*. *δ*-*β*)-open cover of *X* has a finite sub-cover.

Remark 3.2: The family of all regular open sets constitutes a base for a Topology T_s on X. This topology T_s is known as the semi-regularization of *T*. We note that $T_s \subseteq T$. [22]

Definition 3.3: A filter base $\mathcal{F}$ in a space *X* is $\boldsymbol{E_c}$ **(***resp*. *δ*-β_c)- Convergence to a point $\varkappa \in X\ if\ \forall$ E_c (*resp*. *δ*-β_c)-open set $\mathcal{V}\ (s.t)\ \varkappa \in \mathcal{V}\ \exists\ F \in \mathcal{F}\ (s.t)\ F \subseteq \mathcal{V}$.

Definition 3.4: A filter base $\mathcal{F}$ in a space *X* is $\boldsymbol{E_c}$ **(***resp*. *δ*-β_c)- Accumulation of a point $\varkappa \in X$ if $F \cap \mathcal{V} \neq \varphi\ \forall\ \mathcal{F} - open\ set\ \mathcal{V}\ (s.t)\ \varkappa \in \mathcal{V}\ \&\ \forall F \in \mathcal{F}$.

Theorem 3.5: Let $\mathcal{F}$ be filter base of a space *X.* If $\mathcal{F}$ is. *δ*-β_c- Convergence to a point $\varkappa \in X$, then $\mathcal{F}$ is $rc -$ Convergence to $\varkappa$.

Proof: Assume that $\mathcal{F}$ ***δ***-$\boldsymbol{\beta_c}$-Converges to $\varkappa \in X$. Let $\mathcal{V}$ be any regular closed set (s. t), $\varkappa \in \mathcal{V} \Longrightarrow \mathcal{V} \in$ ***δ-β****CΣ(X)*. Since $\mathcal{F}$ is ***δ***-$\boldsymbol{\beta_c}$-Convergence to $\varkappa \in X$, thus $\exists$ an $F \in \mathcal{F}\ (s.t)\ F \subseteq \mathcal{V}$. This explains that $\mathcal{F}$ is $rc -$ Convergence to $\varkappa$.

Remark 3.6: The converse of Theorem-(3. 5) is not necessarily true, as shown in the following example:

Example 3.7: Consider $(\mathbb{R}, \mathbb{T}_{\mathcal{U}})$ the space of usual Topology and let

$\mathcal{F} = \{\mathbb{R}, [0 - \varepsilon, 0 + \varepsilon] : \varepsilon > 0 \in \mathbb{R}\}$. So $\mathcal{F}$ is $rc -$ Convergence to 0, but $\mathcal{F}$ doesn't *δ*-β_c-Convergence to 0, because the set of $(0 - \varepsilon, 0 + \varepsilon)$ is *δ*-β_c-open set containing 0, but $\nexists\ F \in \mathcal{F}$, such that, $F \subseteq (0 - \varepsilon, 0 + \varepsilon)$.

Corollary 3.8: Let $\mathcal{F}$ be filter base of a space *X.* If $\mathcal{F}$ *δ*-β_c-Accumulation of a point a point $\varkappa \in X$, then $\mathcal{F}$ is $rc -$ Accumulation to $\varkappa$.

Proof: Similar to that of Theorem-(3.5).

Theorem 3.9: Let $\mathcal{F}$ be filter base of a space *X* and $\mathcal{H}$ be any closed set containing $\varkappa$. If $\exists\ F \in \mathcal{F}$ (s. t) $F \subseteq \mathcal{H}$. Then $\mathcal{F}$ is E_c (*resp*. *δ*-β_c)- Convergence to a point $\varkappa \in X$.

Proof: suppose that $\mathcal{V}$ be any E_c (*resp.* δ-β_c)-open set containing $\varkappa$, so $\forall \varkappa \in \mathcal{V} \exists$ a closed set $\mathcal{H}$ (s. t) $\varkappa \in \mathcal{H} \subseteq \mathcal{V}$. via hypothesis, $\exists F \in \mathcal{F}$ (s.t) $F \subseteq \mathcal{H} \subseteq \mathcal{V} \Longrightarrow F \subseteq \mathcal{V}$. Thus, $\mathcal{F}$ *is* $\boldsymbol{E_c}$ (*resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta_c}$)- Converges to $\varkappa \in X$.

Theorem 3.10: Let $\mathcal{F}$ be filter base of a space X and $\mathcal{H}$ be any closed set containing $\varkappa$, $(s.t)$ $F \cap \mathcal{H} \neq \varphi \ \forall F \in \mathcal{F}$. Then $\mathcal{F}$ *is* E_c (*resp.* δ-β_c)- Accumulation of a point $\varkappa \in X$.

Proof: Similar to that of Theorem-(3.9).

Theorem 3.11: Let (X, T) be a Topological space. If every closed cover of X has a finite sub-cover, then X is E_c (*resp.* δ-β_c)- Compact space.

Proof: Let $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ be any E_c (*resp.* δ-β_c)-open cover of X, and $\varkappa \in X$, so $\forall \varkappa \in \mathcal{V}_\lambda(\varkappa); \lambda \in \nabla$, there exists a closed set $\mathcal{H}_\lambda(\varkappa)(s.t)\ \varkappa \in \mathcal{H}_\lambda(\varkappa) \subseteq \mathcal{V}_\lambda(\varkappa)$. so the collection $\{\mathcal{H}_\lambda(\varkappa) : \varkappa \in X\}$ is a cover of X via closed set, then by hypothesis, this collection has a finite sub-cover (s, t):

$X = \{\mathcal{H}_\lambda(x_i) : i = 1, 2, 3, \dots, n\} \subseteq \{\mathcal{V}_\lambda(x_i) : i = 1, 2, 3, \dots, n\}$. Thus $X = \{\mathcal{V}_\lambda(x_i) : i = 1, 2, 3, \dots, n\}$. therefore X is E_c (*resp.* δ-β_c)-Compact space.

Theorem 3.12: If a Topological space (X, T) is E- (*resp.* δ-β)-Compact, then its E_c (*resp.* δ-β_c)-Compact space.

Proof: Let $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ be any E_c (*resp.* δ-β_c)-open cover of X. So $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ is E (*resp.* δ-β)-open cover of X, since X is E- (*resp.* δ-β)-Compact, so there exists a finite sub-set $\nabla_o \subseteq \nabla$ $(s.t)$ $X = \cup \{\mathcal{V}_\lambda : \lambda \in \nabla_o\}$. Thus X is E_c (*resp.* δ-β_c)-Compact space.

Theorem 3.13: Every E_c (*resp.* δ-β_c)-Compact $T_1 - \text{space}$ is E- (*resp.* δ-β)-Compact space.

Proof: Assume that X is $\boldsymbol{E_c}$ **(***resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta_c}$**)**-Compact and $T_1 - \text{space}$. Let $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ be any $\boldsymbol{E}$ **(***resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta}$**)**-open cover of X. so $\forall \varkappa \in X \ \exists \lambda(\varkappa) \in \nabla$ (s.t) $\varkappa \in \mathcal{V}_\lambda(\varkappa)$. Since X is $T_1 - \text{space}$, and X is $\boldsymbol{E_c}$ **(***resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta_c}$**)**-Compact, thus there exists a finite sub-set $\nabla_o \subseteq \nabla$ in X $(s.t)$ $X = \cup\{\mathcal{V}_\lambda : \lambda \in \nabla_o\}$. Thus X is $\boldsymbol{E}$ **(***resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta}$**)**-Compact space.

The following corollary is directly consequence of Theorem-(3.12 and 3.13).

Corollary 3.14: Let X be a $T_1 - \text{space}$. Then X is $\boldsymbol{E_c}$ (*resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta_c}$)-Compact iff X is $\boldsymbol{E}$- **(***resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta}$**)**-Compact space.

Theorem 3.15: Let (X, T) be a δ-β_c-Compact space, then X is $rc - \text{Compact space}$.

Proof: Suppose that $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ is any regular closed cover of *X*, so via Theorem-(2.6), $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ constitutes a *δ-β*c-open cover of *X*. Since *X* is *δ-β*c-Compact, hence there exists a finite sub-set $\nabla_o \subseteq \nabla$ in X $(s.t)$ $X = \cup\{\mathcal{V}_\lambda : \lambda \in \nabla_o\}$. Thus *X* is $rc - \mathrm{Compact\ space}$.

Theorem 3.16: Let (X, T) be a regular space. If *X* is E_c (*resp*. *δ-β*c)-Compact, then *X* is compact.

Proof: Assume that $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ is any open cover of *X*. Since *X* is regular, so via Theorem-(2.7), $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ constitutes a E_c (*resp*. *δ-β*c)-open cover of *X*. and since *X* is $\boldsymbol{E_c}$ **(***resp*. ***δ-β*c)**-Compact, thus there exists a finite sub-set $\nabla_o \subseteq \nabla$ in X $(s.t)$ $X = \cup\{\mathcal{V}_\lambda : \lambda \in \nabla_o\}$. Thus *X* is Compact space.

Theorem 3.17: Let (X, T) be an almost regular space. If *X* is *δ-β*c-Compact, then *X* is nearly compact.

Proof: Suppose that $\{\mathcal{V}_\lambda : \lambda \in \nabla\}$ is any regular open cover of *X*. Since *X* is almost regular, so $\forall\, \varkappa \in X \text{ and regular open } \mathcal{V}_\lambda(\varkappa) \ \exists \text{ an open set } \mathcal{M}_\varkappa \ (s.t)\ \varkappa \in \mathcal{M}_\varkappa \subseteq Cl(\mathcal{M}_\varkappa) \subset \mathcal{V}_\lambda(\varkappa)$.
but $Cl(\mathcal{M}_\varkappa)$ is regular closed $\forall\, \varkappa \in X$, *this implies* that *the collection* $\{Cl(\mathcal{M}_\varkappa) : \varkappa \in X\}$ via Theorem (2.6), is constitutes a *δ-β*c-open cover of *X*. Since *X* is *δ-β*c-Compact, so there exists a sub-collection $\{Cl(\mathcal{M}(\varkappa_i)) : i = 1, 2, \dots, n\}, (s.t),\ X = \cup_{i=1}^{n} Cl(\mathcal{M}(\varkappa_i)) \subseteq \cup_{i=1}^{n} \mathcal{V}_\lambda(\varkappa_i)$. Thus *X* is nearly compact.

4. Fundamental properties of *E*c *(resp. δ-β*c*)*-$\boldsymbol{T_i - Spaces}$ $(\boldsymbol{i = 0, 1, 2})$

In this part, several characterizations and some basic properties concerning of new kinds of separation axioms called E_c (*resp*. *δ-β*c)- separation axioms such as E_c-T_0-(*resp*. *δ-β*c-T_0-), E_c-T_1-(*resp*. *δ-β*c-T_1-) E_c-T_2-(*resp*. *δ-β*c-T_2-) are given, as well the relationships among these kinds of spaces and other well- known spaces are discussed.

Definition 4.1: *A* mapping $f : (X, T) \rightarrow (Y, T^*)$ is said to be:

i) E_c-Irresolute, if $f^{-1}(\mathcal{V})$ is E_c-open in *X* for every E_c-open sub-set $\mathcal{V}$ of Y.

ii) *δ-β*c-Irresolute, if $f^{-1}(\mathcal{V})$ is *δ-β*c-open in *X* for every *δ-β*c-open sub-set $\mathcal{V}$ of Y.

iii) E_c-open, if the image of each open set of *(X, T)* is E_c-open of *(Y, T*)*.

iv) *δ-β*c-open, if the image of each open set of *(X, T)* is *δ-β*c-open of *(Y, T*)*.

v) E_c-continuous, if $f^{-1}(\mathcal{V})$ is E_c-open in *X* for every open sub-set $\mathcal{V}$ of Y.

vi) δ-β_c-continuous, if $f^{-1}(\mathcal{V})$ is δ-β_c-open in X for every open sub-set $\mathcal{V}$ of Y.

Definition 4.2: *A* topological space (X,T) is called:

a) $E_c(resp.\delta-\beta_c)-T_0-Space$ if for every distinct points x and y of X, there is E_c(*resp.* δ-β_c)-open set containing one of them but not the other.

b) $E_c(resp.\delta-\beta_c)-T_1-Space$ if for each pair of distinct points $\varkappa, y\ (\varkappa \neq y) \in X$, there exist two E_c(*resp.* δ-β_c)-open sets $\mathcal{U}\ \&\ V\ (s.t)\ \varkappa \in \mathcal{U}\ but\ y \notin \mathcal{U}\ and\ y \in \mathcal{V}\ but\ \varkappa \notin \mathcal{V}$.

c) $E_c(resp.\delta-\beta_c)-T_2-Space\ or\ E_c(resp.\delta-\beta_c)-Hausdorff\ Space$ if for each pair of distinct points $\varkappa, y\ (\varkappa \neq y) \in X$, there exist two disjoint E_c(*resp.* δ-β_c)-open sets $\mathcal{U}\ \&\ V\ \ (s.t)\ \varkappa \in \mathcal{U}\ and\ y \in \mathcal{V}$.

Remark 4.3: From the respective definitions, we have the following diagram. However none of these implications is reversible as shown via examples of [19, 24, 25].

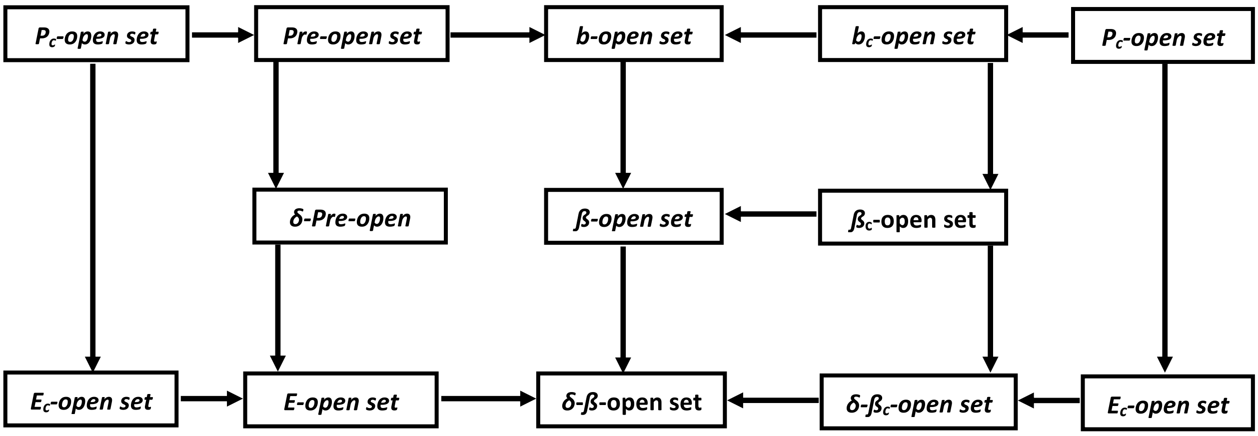

Figure (2): The relationships among other well-known types of generalized open sets in Top-Sp

Theorem 4.4: The following properties are hold in a topological space(X, T):

a) Every $E_c(resp.\delta-\beta_c)-T_2-Space$is $E_c(resp.\delta-\beta_c)-T_1-Space$

b) Every $E_c(resp.\delta-\beta_c)-T_1-Space$ is $E_c(resp.\delta-\beta_c)-T_0-Space$

c) Every $E_cT_2-Space$ is $\delta-\beta_cT_2-Space.$

d) Every $E_cT_1-Space$ is $\delta-\beta_cT_1-Space.$

e) Every $E_cT_0-Space$ is $\delta-\beta_cT_0-Space.$

Proof: The proof is obvious it is follows immediately from their respective definitions.

Remark 4.5: From the respective definitions, the relationships among E_c *(resp.* δ-β_c*)*-$T_i - spaces\ (i = 0, 1, 2)$ and some other well-known forms of spaces shown in the following figure:

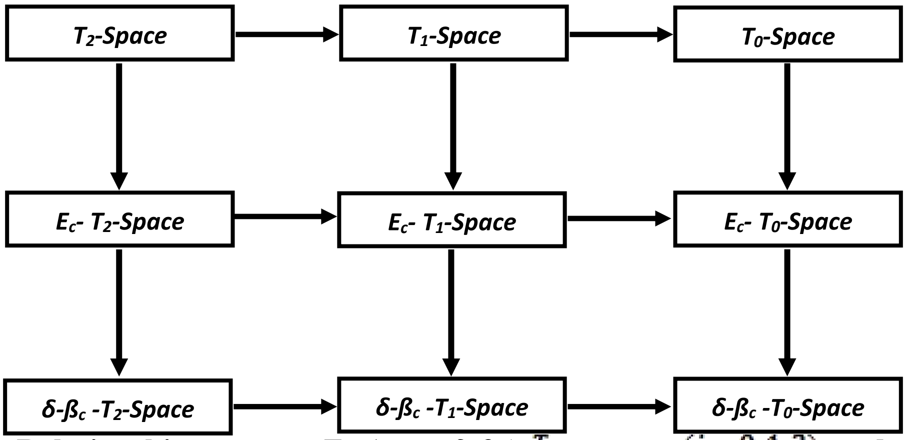

Figure (3): Relationships among E_c *(resp. δ-βc)*-$T_i - spaces\ (i = 0, 1, 2)$ and some other well-known forms of spaces

However none of these implications is reversible as shown in the following examples.

Example 4.6: Let $X = \{a, b, c, d\}$ with a topology $T = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}, X\}$. Then, $EC\Sigma(X, T) = \{\varphi, \{a, c\}, \{b, c, d\}, \{a, c, d\}, X\}$.

And, ***δ-β****CΣ(X, T)* $= \{\varphi, \{a, c\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X\}$. Then X is $E_cT_0(resp. \delta - \beta_cT_0) - Space$, but it is neither $E_c(resp. \delta - \beta_c) - T_1 - Space$ nor $E_c(resp. \delta - \beta_c) - T_2 - Space$

Example 4.7: Consider X any infinite set with the co-finite topology T_c (such that the closed sets are X and the finite sub-sets). Since $X \setminus \{x\}$ is E_c(*resp.* δ-β_c)-open, therefore X is,

$E_c(resp. \delta - \beta_c) - T_1 - Space$. But there is no non empty E_c(*resp.* δ-β_c)-open sets are disjoint, so X cannot be $E_c(resp. \delta - \beta_c) - T_2 - Space$.

Theorem 4.8: the following statements are equivalent for a space(X, T):

a) X is an $E_c(resp. \delta - \beta_c) - T_0 - Space$

b) For every two distinct points $x, y\ (x \neq y) \in X$,

E_c-$Cl(\{x\})$ (*resp.* δ-β_c-$Cl(\{x\})) \neq E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\}))$.

Proof: **Necessity.** Suppose that $(X, T)\ is\ E_c(resp. \delta - \beta_c) - T_0 - Space$ and for each $x, y\ (x \neq y) \in X$, there exists an E_c(*resp.* δ-β_c)-open set $\mathcal{U}\ (s.t)\ x \in \mathcal{U}, y \notin \mathcal{U} \Rightarrow y \in X \setminus \mathcal{U}$, where $X \setminus \mathcal{U}$ is *E_c(resp. δ-β_c)-closed* which does not contain $\varkappa$ but contains y. Since E_c-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) is the smallest E_c(*resp.* δ-β_c)-closed set containing y, so E_c-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) $\subseteq X \setminus \mathcal{U}$ and hence $\varkappa \notin E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$). Consequently, E_c-$Cl(\{\varkappa\})$ (*resp.* δ-β_c-$Cl(\{\varkappa\})$) $\neq E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$).

Sufficiency. Assume that $\varkappa, y\ (\varkappa \neq y) \in X$, and E_c-$Cl(\{\varkappa\})$ (*resp.* δ-β_c-$Cl(\{\varkappa\})$) $\neq E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$). Let $z \in X\ (s.t)\ z \in E_c$-$Cl(\{\varkappa\})$ (*resp.* δ-β_c-$Cl(\{\varkappa\})$) but $z \notin E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$). We prove that $\varkappa \notin E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$). Suppose that $\varkappa \in E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$), so $\{\varkappa\} \subseteq E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$),which implies that, E_c-$Cl(\{\varkappa\})$ (*resp.* δ-β_c-$Cl(\{\varkappa\})$) $\subseteq E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) and thus $z \in E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) which is a contradiction with the fact of $z \notin E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$), hence $\varkappa \notin E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) which implies that, $\varkappa \in X \setminus E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$).

Consequently $X \setminus E_c$-$Cl(\{y\})$ (*resp.* δ-β_c-$Cl(\{y\})$) is an E_c(*resp.* δ-β_c)-open set containing $\varkappa$ but not y. Therefore, X is $E_c(resp.\,\delta-\beta_c)-T_0-Space$.

Theorem 4.9: Let (X, T) be a topological space. Then the following statements are equivalent:

a) X is an $E_c(resp.\,\delta-\beta_c)-T_1-Space$.

b) For each point $\varkappa \in X$ the singleton set $\{\varkappa\}$ is E_c(*resp.* δ-β_c)-closed set,

c) For each point $\varkappa \in X$, E_c-$D(\{\varkappa\})$ (*resp.* δ-β_c-$D(\{\varkappa\})$) $= \varphi$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ Suppose that X is $E_c(resp.\,\delta-\beta_c)-T_1-Space$ For each $\varkappa, y\ (\varkappa \neq y) \in X$, there exists $\boldsymbol{E_c}$(*resp.* δ-β_c)-open set $\mathcal{U}\ (s.t)\ y \in \mathcal{U}\ but\ \varkappa \notin \mathcal{U}$. Consequently, $y \in \mathcal{U} \subseteq X \setminus \{\varkappa\}$. Thus $X \setminus \{\varkappa\} = \cup\{\mathcal{U}: y \in X \setminus \{\varkappa\}\}$ which is the union of an $\boldsymbol{E_c}$(*resp.* δ-β_c)-open sets. Then $X \setminus \{\varkappa\}$ is an $\boldsymbol{E_c}$(*resp.* δ-β_c)-open sets. Thus $\{\varkappa\}$ $\boldsymbol{E_c}$(*resp.* δ-β_c)-closed sets.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Assume that $\{\mathcal{P}\}$ is $\boldsymbol{E_c}$(*resp.* δ-β_c)-closed for each $\mathcal{P} \in X$. So via hypothesis for each $\varkappa, y\ (\varkappa \neq y) \in X, \{\varkappa\}, \{y\}$ are $\boldsymbol{E_c}$(*resp.* δ-β_c)- closed sets. Hence $X \setminus \{\varkappa\}, X \setminus \{y\}$ are $\boldsymbol{E_c}$(*resp.* $\boldsymbol{\delta}$-$\boldsymbol{\beta_c}$)- open sets (s. t) $\varkappa \in X \setminus \{y\}, y \notin X \setminus \{y\}\ and\ y \in X \setminus \{\varkappa\}, \varkappa \notin X \setminus \{\varkappa\}$. Therefore X is $E_c(resp.\,\delta-\beta_c)-T_1-Space$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{c})$ Assume that $\{\varkappa\}$ is E_c(*resp.* δ-β_c)-closed set for each $\varkappa \in X$. Thus,

$\{\varkappa\} = E_c$-$Cl(\{\varkappa\})$ (*resp.* δ-β_c-$Cl(\{\varkappa\})$) $= \{\varkappa\} \cup E_c$-$D(\{\varkappa\})$ (*resp.* δ-β_c-$D(\{\varkappa\})$). Therefore,

E_c-$D(\{\varkappa\})$ (*resp.* δ-β_c-$D(\{\varkappa\})$) $= \varphi$.

$(c) \Longrightarrow (b)$ Let E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\})) = \varphi$ for each $\varkappa \in X$. Since

E_c-$Cl(\{\varkappa\})$ (*resp*. δ-β_c-$Cl(\{\varkappa\})) = \{\varkappa\} \cup E_c$-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\}))$. Thus,

E_c-$Cl(\{\varkappa\})$ (*resp*. δ-β_c-$Cl(\{\varkappa\})) = \{\varkappa\}\ iff\ \{\varkappa\}$ E_c(*resp*. δ-β_c)-closed set.

$(a) \Longrightarrow (c)$ Suppose that X is $E_c(resp.\delta-\beta_c)-T_1-Space$ and assume that E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\})) \neq \varphi$ for some $\varkappa \in X, then\ \exists\ y \in$ E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\}))$ and $(\varkappa \neq y)$. Since X is $E_c(resp.\delta-\beta_c)-T_1-Space, so\ \exists$ E_c(*resp*. δ-β_c)-open set $\mathcal{U}\ (s.t)\ y \in \mathcal{U}\ and\ \varkappa \notin \mathcal{U}$ which implies, $\mathcal{U} \cap \{\varkappa\} = \varphi$, and thus $y \notin E_c$-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\}))$ which a contradiction with the hypothesis. Hence, $\forall\ \varkappa \in X$, E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\})) = \varphi$.

$(c) \Longrightarrow (a)$ Let E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\})) = \varphi, \forall\ \varkappa \in X$, so

E_c-$Cl(\{\varkappa\})$ (*resp*. δ-β_c-$Cl(\{\varkappa\})) = \{\varkappa\} \cup$ E_c-$D(\{\varkappa\})$ (*resp*. δ-β_c-$D(\{\varkappa\})) = \{\varkappa\}$ which implies, $\{\varkappa\}$ is E_c(*resp*. δ-β_c)-closed set and thus via (part (a) & (b)) X is $E_c(resp.\delta-\beta_c)-T_1-Space.$

Theorem 4.10: If (X, T) is a topological space, then the following properties are equivalent:

a) X is an $E_c(resp.\delta-\beta_c)-T_2-Space$

b) If $\varkappa \in X$, then $\forall\ (\varkappa \neq y)$, $\exists$ an E_c(*resp*. δ-β_c)-open set $\mathcal{U}\ containing\ \varkappa\ (s.t)\ y \notin$ E_c-$Cl(\{\mathcal{U}\})$ (*resp*. δ-β_c-$Cl(\{\mathcal{U}\}))$.

Proof: $(a) \Longrightarrow (b)$ since X is $E_c(resp.\delta-\beta_c)-T_2-Space$ so $\forall\ (\varkappa \neq y)\ \exists$ E_c(*resp*. δ-β_c)-open sets.

$\mathcal{U}\ \&\ V\ (s.t)\ \varkappa \in \mathcal{U}\ \&\ y \in \mathcal{V}\ and\ \mathcal{U} \cap \mathcal{V} = \varphi$. Thus, $\varkappa \in \mathcal{U} \subseteq X \setminus \mathcal{V}$, put $X \setminus \mathcal{V} = F$, then F is E_c(*resp*. δ-β_c)-closed set, $\mathcal{U} \subseteq F\ and\ y \notin F \Longrightarrow y \notin \bigcap\{F: F$ is E_c(*resp*. δ-β_c)-closed set & $\mathcal{U} \subseteq F\} = E_c$-$Cl(\{\mathcal{U}\})$ (*resp*. δ-β_c-$Cl(\{\mathcal{U}\}))$.

$(b) \Longrightarrow (a)$ Assume that $\varkappa, y\ (\varkappa \neq y) \in X$, by hypothesis, there exists E_c(*resp*. δ-β_c)-open set $\mathcal{U}\ containing\ \varkappa\ (s.t)\ y \notin E_c$-$Cl(\{\mathcal{U}\})$ (*resp*. δ-β_c-$Cl(\{\mathcal{U}\}))$. Hence $y \in X \setminus$ (E_c-$Cl(\{\mathcal{U}\})$ (*resp*. **δ-β_c**-$Cl(\{\mathcal{U}\})))$ which is E_c(*resp*. δ-β_c)-open and $\varkappa \notin X \setminus$ (E_c-$Cl(\{\mathcal{U}\})$ (*resp*. δ-β_c-$Cl(\{\mathcal{U}\})))$. As well, $\mathcal{U} \cap (X \setminus$ (E_c-$Cl(\{\mathcal{U}\})$ (*resp*. δ-β_c-$Cl(\{\mathcal{U}\}))) = \varphi$. So, X is $E_c(resp.\delta-\beta_c)-T_2-Space$.

Definition 4.11: Let (X, T) be a topological space and $\mathcal{A} \subseteq X$. Then, the intersection of all E_c(*resp*. δ-β_c)-open subsets of X containing $\mathcal{A}$ is called the E_c-$kernal$(*resp*. δ-β_c-$kernal$) of $\mathcal{A}$ and its denoted via E_c-$ker(\mathcal{A})$(*resp*. δ-β_c-$ker(\mathcal{A}))$ *of* $\mathcal{A}$ *(i. e):*

E_c-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) = $\cap\{\mathcal{U} \in EC\Sigma(X)$ (resp. δ-$\beta C\Sigma(X)$): $\mathcal{A} \subseteq \mathcal{U}\}$.

Theorem 4.12: Let (X, T) be a topological space and $\varkappa \in X$. then, $y \in E_c$-$ker(\{\varkappa\})$ (resp. δ-β_c-$ker(\{\varkappa\})$) iff $\varkappa \in E_c$-$Cl(\{y\})$ (resp. δ-β_c-$Cl(\{y\})$).

Proof: Assume that $y \notin E_c$-$ker(\{\varkappa\})$ (resp. δ-β_c-$ker(\{\varkappa\})$). So, there exists E_c(resp. δ-β_c)-open set $\mathcal{U}$ containing $\varkappa$ (s.t) $y \notin \mathcal{U}$. Thus we get $\varkappa \notin E_c$-$Cl(\{y\})$ (resp. δ-β_c-$Cl(\{y\})$). Similarly we can prove the converse case.

Theorem 4.13: Let $\mathcal{A}$ be a sub set of a topological space(X, T). Then,

E_c-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) = $\{\varkappa \in X$: E_c-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$) $\cap \mathcal{A} \neq \varphi\}$.

Proof: Suppose that $\varkappa \in E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) and

E_c-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$) $\cap \mathcal{A} = \varphi$, Thus, $\varkappa \notin X \setminus$ (E_c-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$)) which is an E_c(resp. δ-β_c)-open set containing $\mathcal{A}$. This case is not possible, since $\varkappa \in E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$). therefore E_c-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$) $\cap \mathcal{A} \neq \varphi$. Now suppose that $\varkappa \in X$ (s, t) E_c-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$) $\cap \mathcal{A} \neq \varphi$ and $\varkappa \notin E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$). So, there exists an E_c(resp. δ-β_c)-open set $\mathcal{U}$ containing $\mathcal{A}$ and $\varkappa \notin \mathcal{U}$. Let $y \in \boldsymbol{E_c}$-$Cl(\{\varkappa\})$ (resp. δ-β_c-$Cl(\{\varkappa\})$) $\cap \mathcal{A}$. Hence, $\mathcal{U}$ is an E_c(resp. δ-β_c)-Neighbourhood of y which does not contain $\varkappa$. So via this contradiction we get $\varkappa \in E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) and this is the request.

Theorem 4.14: The following properties hold for the subsets $\mathcal{A}$ and B of a topological space(X, T):

a) $\mathcal{A} \subseteq E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$).

b) $\mathcal{A} \subseteq B \Longrightarrow E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) $\subseteq E_c$-$ker(B)$ (resp. δ-β_c-$ker(B)$).

c) If $\mathcal{A}$ is E_c(resp. δ-β_c)-open of (X, T), then $\mathcal{A} = E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$).

d) E_c-$ker(E_c - ker(\mathcal{A}))$ (resp. δ-β_c-$ker(\delta - \beta_c - ker(\mathcal{A}))$) = E_c-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$).

Proof: The proof of parts (a), (b) and (c) are directly consequences of definition **(4.11).** Now we prove part (d), first via parts (a) and (b) we have:

E_c-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$) $\subseteq$ E_c-$ker(E_c - ker(\mathcal{A}))$ (resp. δ-β_c-$ker(\delta - \beta_c - ker(\mathcal{A}))$). If $\varkappa \notin E_c$-$ker(\mathcal{A})$ (resp. δ-β_c-$ker(\mathcal{A})$). So $\exists\, \mathcal{U} \in EC\Sigma(X)$ (resp. δ-$\beta C\Sigma(X)$) (s. t)

$\mathcal{A} \subseteq \mathcal{U}$ and $\varkappa \notin \mathcal{U}$. Thus, E_c-$ker(\mathcal{A})$ *(resp.* δ-β_c-$ker(\mathcal{A})) \subseteq \mathcal{U}$, and so we get:

$\varkappa \notin E_c$-$ker(E_c - ker(\mathcal{A}))$ *(resp.* δ-β_c-$ker(\delta - \beta_c - ker(\mathcal{A})))$. Therefore,

E_c- $ker(E_c - ker(\mathcal{A}))$ *(resp.* δ-β_c- $ker(\delta - \beta_c - ker(\mathcal{A})))$ = E_c- $ker(\mathcal{A})$ *(resp.* δ-β_c-$ker(\mathcal{A}))$.

Theorem 4.15: for any two distinct points $\varkappa$ *and* y in a topological space (X, T), the following properties hold:

a) E_c-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\}) \neq E_c$-$ker(\{y\})$*(resp.* δ-β_c-$ker(\{y\}))$.

b) E_c-$Cl(\{\varkappa\})$*(resp.* δ-β_c-$Cl(\{\varkappa\})) \neq E_c$-$Cl(\{y\})$*(resp.* δ-β_c-$Cl(\{y\}))$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ Assume that E_c-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\}) \neq E_c$-$ker(\{y\})$*(resp.* δ-β_c-$ker(\{y\}))$. So there exists a point $z \in X$ $(s.t)$ $z \in E_c$-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\})$ and $z \notin E_c$-$ker(\{y\})$*(resp.* δ-β_c-$ker(\{y\}))$. Since $z \in E_c$-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\})$ Consequently that $\{\varkappa\} \cap \boldsymbol{E_c}$-$Cl(\{z\})$*(resp.* δ-β_c-$Cl(\{z\})) \neq \varphi \Longrightarrow \varkappa \in \boldsymbol{E_c}$-$Cl(\{z\})$*(resp.* δ-β_c-$Cl(\{z\}))$. utilize

$z \notin E_c$- $ker(\{y\})$ *(resp.* δ-β_c- $ker(\{y\}))$ we get $\{y\} \cap \boldsymbol{E_c}$- $Cl(\{z\})$*(resp.* δ-β_c- $Cl(\{z\})) = \varphi$. Since $\varkappa \in$ $\boldsymbol{E_c}$- $Cl(\{z\})$ *(resp.* δ-β_c- $Cl(\{z\}))$, so, E_c- $Cl(\{\varkappa\})$ *(resp.* δ-β_c- $Cl(\{\varkappa\})) \subseteq$ $\boldsymbol{E_c}$-$Cl(\{z\})$*(resp.* δ-β_c-$Cl(\{z\}))$ and $\{y\} \cap \boldsymbol{E_c}$-$Cl(\{\varkappa\})$*(resp.* δ-β_c-$Cl(\{\varkappa\})) = \varphi$. Thus, it follows that $\boldsymbol{E_c}$-$Cl(\{\varkappa\})$*(resp.* $\boldsymbol{\delta}$-β_c-$Cl(\{\varkappa\})) \neq E_c$-$Cl(\{y\})$*(resp.* δ-β_c-$Cl(\{y\}))$. So, E_c-$ker(\{\varkappa\})$*(resp.* δ-β_c- $ker(\{\varkappa\}) \neq$ E_c- $ker(\{y\})$ *(resp.* δ-β_c- $ker(\{y\}))$ implies that $\boldsymbol{E_c}$- $Cl(\{\varkappa\})$ *(resp.* δ-β_c-$Cl(\{\varkappa\})) \neq E_c$-$Cl(\{y\})$*(resp.* δ-β_c-$Cl(\{y\}))$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ Suppose that E_c-$Cl(\{\varkappa\})$*(resp.* δ-β_c-$Cl(\{\varkappa\})) \neq E_c$-$Cl(\{y\})$*(resp.* δ-β_c-$Cl(\{y\}))$ So there exists a point $z \in X$ $(s.t)$ $z \in E_c$-$Cl(\{\varkappa\})$*(resp.* δ-β_c-$Cl(\{\varkappa\}))$ and $z \notin E_c$-$Cl(\{y\})$*(resp.* δ-β_c-$Cl(\{y\}))$. Then, there exists an E_c*(resp.* δ-β_c)-open set containing z and $\varkappa$ but not y, namely, $y \notin E_c$-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\}))$ and hence E_c-$ker(\{\varkappa\})$*(resp.* δ-β_c-$ker(\{\varkappa\}) \neq$ E_c-$ker(\{y\})$*(resp.* δ-β_c-$ker(\{y\}))$.

Theorem 4.16: Suppose that $f: (X, T) \rightarrow (Y, T^*)$ is an injective E_c**(***resp.* δ-β_c)-continuous mapping and Y is $T_i - space$, then X is E_c *(resp.* δ-β_c)-$T_i - space$, *where* $(i = 0, 1, 2)$

Proof: We prove that the theorem for $E_c(resp. \delta - \beta_c) - T_0 - Space$ and the other are similar.

Assume that $\varkappa, y$ $(\varkappa \neq y) \in X$, since f is injective, then $f(\varkappa) \neq f(y)$ in Y. But Y is T_0, then there exist an open set $\mathcal{U}$ $(s.t)$ $f(\varkappa) \in \mathcal{U}, f(y) \notin \mathcal{U}$ *OR* $f(y) \in \mathcal{U}, f(\varkappa) \notin \mathcal{U}$, since f is E_c**(***resp.* δ-β_c)-continuous, so $f^{-1}(\mathcal{U})$ is E_c**(***resp.* δ-β_c)-open set of X *(s. t)*: $\varkappa \in f^{-1}(\mathcal{U}), y \notin f^{-1}(\mathcal{U})$ *OR*

$y \in f^{-1}(\mathcal{U}), \varkappa \notin f^{-1}(\mathcal{U})$, Thus X is $E_c(resp.\delta - \beta_c) - T_0 - Space.$

Theorem 4.17: Let $f: (X,T) \rightarrow (Y,T^*)$be an injective E_c(*resp.* δ-β_c)-irresolute mapping and Y is an E_c *(resp.* δ-β_c*)*-$T_i - space,$ then X is E_c *(resp.* δ-β_c*)*-$T_i - space, where\ (i = 0,1,2)$

Proof: We prove that the theorem for $E_c(resp.\delta - \beta_c) - T_2 - Space$ and the other are similar.

Suppose that $\varkappa, y\ (\varkappa \neq y) \in X,$ since f is injective, then $f(\varkappa) \neq f(y)$ in Y. But Y is an $E_c(resp.\delta - \beta_c) - T_2 - Space$ so there exist two disjoint E_c(*resp.* δ-β_c)-open sets $\mathcal{U}\ and\ \mathcal{V}\ (s.t)\ f(\varkappa) \in \mathcal{U}\ \&\ f(y) \in \mathcal{V}.$ Utilizing E_c(*resp.* δ-β_c)-irresolute of f we get, $f^{-1}(\mathcal{U})\ \&\ f^{-1}(\mathcal{V})$ are E_c(*resp.* δ-β_c)-open set of X *(s. t)*:

$\varkappa \in f^{-1}(\mathcal{U}), y \in f^{-1}(\mathcal{V})\ and\ f^{-1}(\mathcal{U}) \cap f^{-1}(\mathcal{V}) = \varphi.$

Hence X is $E_c(resp.\delta - \beta_c) - T_2 - Space.$

Theorem 4.18: Suppose that $f: (X,T) \rightarrow (Y,T^*)$is a bijective E_c(*resp.* δ-β_c)-open mapping and X is $T_i - space,$ then Y is E_c *(resp.* δ-β_c*)*-$T_i - space, where\ (i = 0,1,2)$

Proof: We prove that the theorem for $E_c(resp.\delta - \beta_c) - T_2 - Space$ and the other are similar.

Let $y_1,\ y_2\ (y_1 \neq y_2) \in Y.$ since f is bijective, so there exist $\varkappa_1,\ \varkappa_2\ (\varkappa_1 \neq \varkappa_2) \in X.$ such that $f(\varkappa_1) = y_1\ and\ f(\varkappa_2) = y_2$. Since X is T_2 , then there exist two disjoint open sets $\mathcal{U}\ and\ \mathcal{V}\ of\ X\ (s.t)\ \varkappa_1 \in \mathcal{U}\ \&\ \varkappa_2 \in \mathcal{V}.$ Since f is E_c(*resp.* δ-β_c)-open mapping, then $f(\mathcal{U})\ \&\ f(\mathcal{V})$ are E_c(*resp.* δ-β_c)-open sets of Y with $y_1 \in f(\mathcal{U})\ \&\ y_2 \in f(\mathcal{V})$. therefore Y is $E_c(resp.\delta - \beta_c) - T_2 - Space.$

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Conclusion

Compactness is the generalization to topological spaces of the property of closed and bounded subsets of the real line. The notions of compactness are useful and fundamental notions not only of general topology but also of other advanced branches of mathematics. Many researchers have investigated the basic properties of compactness". The productivity of these notions of compactness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness have been introduced and investigated. The class of generalized closed sets has an important role in general topology, especially its suggestion of new separation axioms which are useful in digital topology. "The investigation on generalization of closed set has lead to significant contribution to the theory of separation axioms. In this work we introduced and study new types of spaces called E_c-(*resp.* δ-β_c)-compact spaces". Several characterizations and fundamental properties concerning of these forms of spaces are obtained. Moreover, some new types of separation axioms in topological spaces called E_c-(*resp.* δ-β_c)- separation axioms via E_c *(resp.* δ-β_c*)*-open sets are studied.

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