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# A Note on Connected Six Cyclic Graphs Having Minimum Degree Distance

Nadia Khan, Fatima Ramazan and Munazza Shamas

Department of Mathematics and Statistics, The University of Lahore, Lahore Pakistan.

E-mail: nadiakhan811@gmail.com, fatimaramazan19@gmail.com, munazashms123@gmail.com

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## Abstract

Let $G_n^6$ represents the class connected 6-cyclic graphs. In this paper, first some result is derived for the characterization of class connected 6-cyclic graphs. Then we find minimum degree distance of class of connected of 6-cyclic graph.

**Keywords:** connected graph, degree distance, Six cyclic graphs

## Introduction

For any graph $G \in G_n$, $d(x, y)$ represents the shortest distance between the vertices $x, y \in V(G)$ and the maximum of $d(x, y)$ for any vertices $x, y \in V(G)$ is defined to be the diameter of $G$, denoted by $giam(G)$.

The Wiener index [8] is a well-known topological index of a molecular graph which is used to describe molecular branching and cyclicity. It's also stablishes correlation with various parameters of chemical compounds. The wiener index of a graph $G$ is defined as

$$\sum_{x \in V(G)} \sum_{x \in V(G)} d(x, y).$$

Dobrynin and Kotchetova [1] and Gutman [2] introduced a new graph invariant that is more sensitive than the Wiener index. It is defined in the following way: given any $G \in G_n$ the degree distance of a vertex $x \in V(G)$ is defined by

$$D^{'}(x) = d(x)D(x)$$

where $d(x)$ is the degree of $x$ and $D(x) = \sum_{x, y \in V(G)} d(x, y)$. The degree distance of a graph $G$ is defined as:

$$D^{'}(G) = \sum_{x \in V(G)} D^{'}(x) = \sum_{x \in V(G)} d(x)D(x) = \frac{1}{2} \sum_{x, y \in V(G)} d(x, y) \, (d(x) = d(y)).$$

In this paper, we determine all the extremal 5-cyclic graphs achieving the minimum degree distance.

In section 2, we list some known results needed in this paper. In section 3, we characterize extremal 5-cyclic graphs achieving the minimum degree distance.

## Some Lemmas

In this section, we characterize connected six cyclic graphs by their degree sequence in the following lemma.

**Lemma 1.** Let $n \geq 6$. The degrees of the vertices of a graph $G \in G_n^6$ are the integers $n - 1 \geq d_1 \geq d_2 \geq \cdots \geq d_n \geq 1$, iff

(i) $\sum_{i=1}^{n} d_i = 2n + 10$

(ii) $d_i \geq 2$, for at least six indices.

***Proof:*** $\Longrightarrow$ Let $G \in G_n^5$. Then by the definition of 5-cyclic graph condition (i) and (ii) is verified.

$\Leftarrow$ For $n = 6$, we have $\sum_{i=1}^{6} d_i = 22$. If $d_6 \geq 4$ then $d_1 + d_2 + d_3 + d_4 + d_5 \geq 24$ a contradiction. So $d_6 < 4$ . If $d_6 = 3$ then $\sum_{i=1}^{n} d_i = 19$ , which implies that $(d_1, d_2, d_3, d_4, d_5, d_6) = (5, 5, 3, 3, 3, 3)$, or $(5, 4, 4, 3, 3, 3)$ or $(4, 4, 4, 4, 3, 3)$ represented by graphs $H_1$ or $H_2$ or $H_3$ , respectively in fig 1. If $d_6 < 3$ , then $d_6 = 2$ and $(d_1, d_2, d_3, d_4, d_5, d_6) = (5,4, 4, 4, 3, 2)$, or $(5, 5, 4, 3, 3, 2)$ represented by graphs $H_4$ or $H_5$, respectively in fig 1. Let $n \geq 7$ and assume that the result is true for all $k \leq n$.

**Case 1.** If $d_n > 1$, then $d_n = 2$. Otherwise, $d_1 + d_2 + \cdots + d_n > 3n > 2n + 10$. If $d_n = 3$ for n=7, 8, 9 or 10, we do not have a graphical sequence. For $d_n = 2$, and sufficiently large *n*, by solving the equation, $d_1 + d_2 + \cdots + d_{n-1} = 2n + 8$. We have the following possibilities

**Subcase 1.1.** If $d_1 = 12, d_2 = d_3 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_1$ in fig.2.

**Subcase 1.2.** If $d_1 = 11, d_2 = 3, d_3 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_2$ in fig.2.

**Subcase 1.3.** If $d_1 = 10, d_2 = d_3 = 3, d_4 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_3$ or $G_4$ in fig.2.

**Subcase 1.4.** If $d_1 = 9, d_2 = d_3 = d_4 = 4, d_5 = d_6 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_5$ or $G_6$ in fig.2.

**Subcase 1.5.** If $d_1 = 8, d_2 = d_3 = d_4 = d_5 = 3, d_6 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_7$ or $G_8$ in fig.2.

**Subcase 1.6.** If $d_1 = 7, d_2 = 4, d_3 = d_4 = d_5 = 3, d_6 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_9$ in fig.2.

**Subcase 1.7.** If $d_1 = 6, d_2 = d_3 = 4, d_4 = d_5 = 3, d_6 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_{10}$ or $G_{11}$ in fig.2.

**Subcase 1.8.** If $d_1 = 6, d_2 = 4, d_3 = d_4 = d_5 = d_6 = 3, d_7 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_{12}$ in fig.2.

**Subcase 1.9.** If $d_1 = d_2 = 6, d_3 = d_4 = 4, d_5 = d_6 = \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_{13}$ in fig.2.

**Subcase 1.10.** If $d_1 = 6, d_2 = 5, d_3 = 4, d_4 = 3, d_5 = d_6 \cdots = d_n = 2$ , then the unique resultant graph is isomorphic to $G_{14}$ in fig.2.

**Subcase 1.11.** If $d_1 = 6, d_2 = d_3 = d_4 = d_5 = d_6 = d_7 = 3, d_8 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{15}$ in fig.2.

**Subcase 1.12.** If $d_1 = 5,\ d_2 = d_3 = d_4 = d_5 = d_6 = d_7 = 3, d_8 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{16}$ in fig.2.

**Subcase 1.13.** If $d_1 = d_2 = 5,\ d_3 = d_4 = 4,\ d_5 = d_6 = 3,\ d_7 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{17}$ in fig.2.

**Subcase 1.14.** If $d_1 = d_2 = 5,\ d_3 = 4,\ d_4 = d_5 = 3,\ d_6 = d_7 = d_8 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{18}$ in fig.2.

**Subcase 1.15.** If $d_1 = d_2 = d_3 = d_4 = d_5 = 4,\ d_6 = d_7 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{19}$ in fig.2.

**Subcase 1.16.** If $d_1 = d_2 = d_3 = d_4 = d_5 = d_6 = d_7 = d_8 = d_9 = d_{10} = 3,\ d_{11} = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{20}$ in fig.2.

**Subcase 1.17.** If $d_1 = 5,\ d_2 = 4, d_3 = d_4 = d_5 = d_6 = d_7 = 3,\ d_8 = \cdots = d_n = 2$, then the unique resultant graph is isomorphic to $G_{21}$ in fig.2.

**Case 2.** If $d_n = 1$, we consider the following two subcases:

**Subcase 2.1.** If $d_1 = n - 1$, and for sufficiently large *n*, we have the following possibilities,

**Subcase 2.1.1.** If $d_1 = n - 1,\ d_2 = d_3 = d_4 = 3,\ d_5 = \cdots = d_{10} = 2,\ d_{11} = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_1$ in fig.3.

**Subcase 2.1.2.** If $d_1 = n - 1, d_2 = \cdots = d_7 = 3,\ d_{11} = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_2$ in fig.3.

**Subcase 2.1.3.** If $d_1 = n - 1, d_2 = 5,\ d_3 = 3,\ d_4 = d_5 = d_6 = d_7 = 2, d_8 = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_3$ in fig.3.

**Subcase 2.1.4.** If $d_1 = n - 1, d_2 = 5, d_3 = 3,\ d_4 = d_5 = d_6 = 3,\ d_7 = d_8 = 2, d_9 = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_4$ in fig.3.

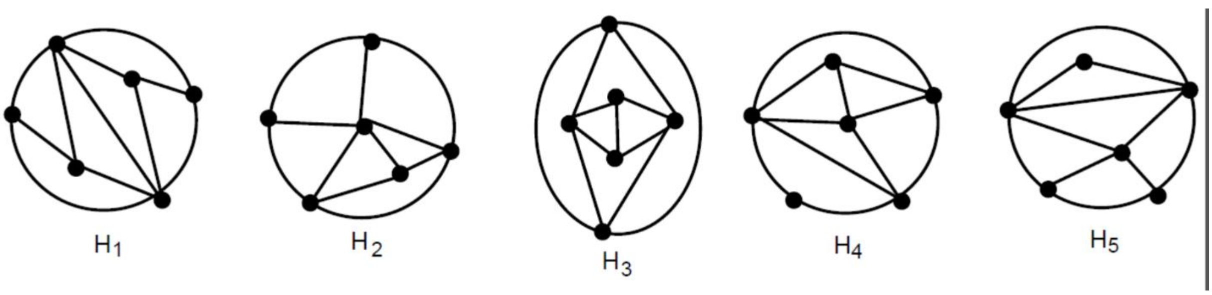

**Fig. 1**

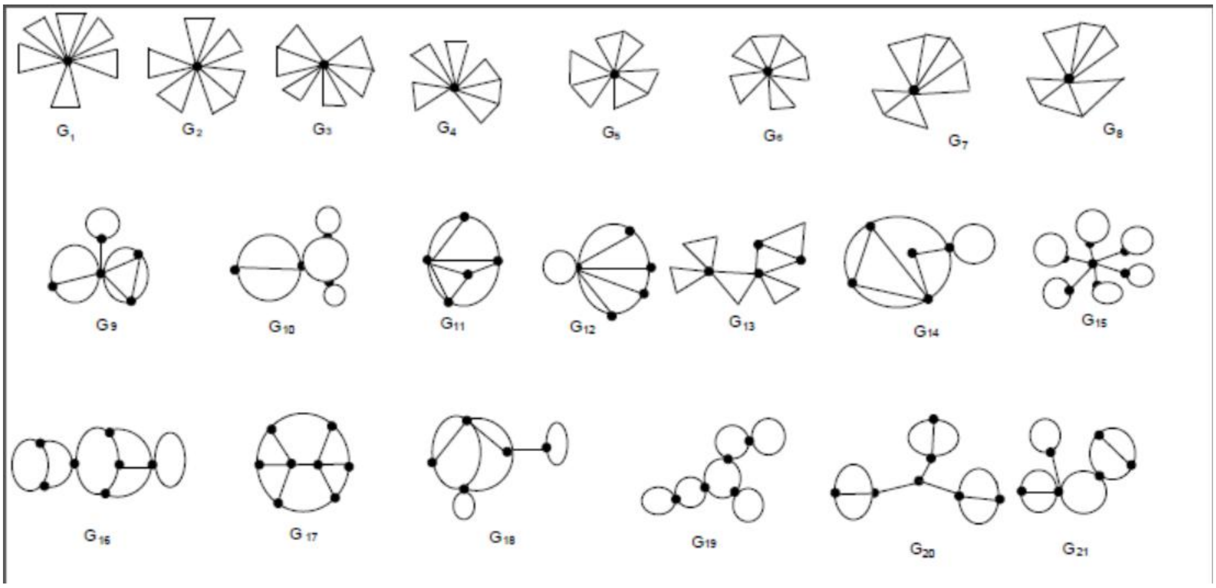


**Fig. 2**

**Subcase 2.1.5.** If $d_1 = n-1, d_2 = \cdots = d_5 = 3,\ d_6 = \cdots = d_9 = 2,\ d_{10} = \cdots = d_n = 1$ , then the unique resultant graph is isomorphic to $A_5$ in fig.3.

**Subcase 2.1.6.** If $d_1 = n-1, d_2 = 4,\ d_3 = d_4 = d_5 = 3, d_6 = d_7 = 2,\ d_8 = d_9 = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_6$ in fig.3.

**Subcase 2.1.7.** If $d_1 = n-1, d_2 = d_3 = d_4 = d_5 = d_6 = d_7 = 3,\ d_8 = \cdots d_{19} = 2,\ d_{20} = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A$ in fig.3.

**Subcase 2.1.8.** If $d_1 = n-1, d_2 = 7,\ d_3 = d_4 = d_5 = d_6 = d_7 = d_8 = 2,\ d_9 \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A$ in fig.3.

**Subcase 2.1.9.** If $d_1 = n-1, d_2 = 4, d_3 = d_4 = 3,\ d_5 = d_6 = d_7 = d_8 = 2,\ d_9 = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_9$ in fig.3.

**Subcase 2.1.9.** If $d_1 = n-1, d_2 = d_3 = 4, d_4 = 3,\ d_5 = d_6 = d_7 = 2,\ d_8 = \cdots = d_n = 1$, then the unique resultant graph is isomorphic to $A_{10}$ in fig.3.

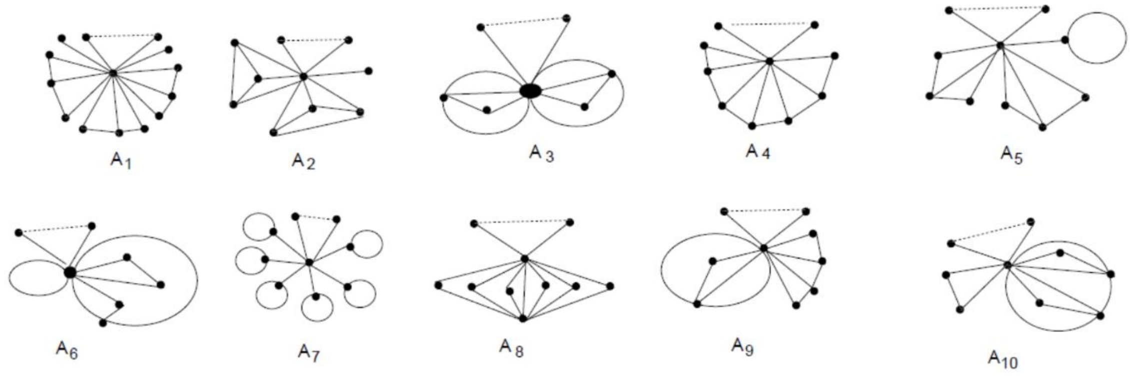


**Fig 3.**

**Subcase 2.2.** Suppose $d_1 \leq n-2$ and $d_n = 1$. For each $1 \leq i \leq n-1$ if $d_i \leq 2$ then $\sum_i d_i \leq 2n-1$, which is a contradiction. We can find a maximal index $j$, $1 \leq j \leq n-1$, such that $d_j \geq 3$, and $d_{j+1} \leq 2$ and $d_1 \geq d_2 \geq \cdots \geq d_{j-1} \geq d_j \geq \cdots \geq d_n \geq 1$. At least six members of the sequence $d_1, \cdots, d_{j-1}, d_j - 1, \cdots, d_{n-1}$ are greater than 2, for which $d_1 \leq n-2$ and $d_1 + d_2 + \cdots + d_n = 2(n-1) + 10 = 2n+8$. By induction hypothesis there exist $G \in G_n^6$ having this degree sequence. By adding a new vertex, joined by an edge with the vertex having degree $d_{j-1}$, we obtain a graph having six cycles with the degree sequence $d_1 \geq d_2 \geq \cdots \geq d_n = 1$. This completes the proof of Lemma 1.

If for any vertex $v \in V(G), d(v) = k$, then $D(v) \geq 2n-k-2$, and if for all $v \in V(G)$ , $, d(v,y) \leq 2$ then $D(v) = 2n-k-2$. Consequently, $D^{'}(G) = \sum_{v \in V(G)} d(v)D(v) \geq \frac{1}{2} \sum_{k=1}^{n-1} kx_k (2n-k-2)$, where $x_i$ denotes the number of vertices of degree $i, 1 \leq i \leq n-1$. By denoting as in [2], $F(x_1, x_2, \cdots, x_{n-1}) = \sum_{k=1}^{n-1} kx_k (2n-k-2)$. We will find the minimum of $F(x_1, x_2, \cdots, x_{n-1})$ over all-natural numbers $x_1, x_2, \cdots, x_{n-1} \geq 0$ satisfying the conditions in above lemma. We have the following corollary:

**Corollary:** Let $n \geq 6$. The integers $x_1, x_2, \cdots, x_{n-1} \geq 0$ are the multiplicities of the degrees of a graph $G \in G_n^6$ iff

(i) $\sum_{i=1}^{n-1} x_i = n$

(ii) $\sum_{i=1}^{n-1} ix_i = 2n+10$

(iii) $x_1 \leq n-6$

Let the set of vectors $x_1, x_2, \cdots, x_{n-1}$, where $x_1, \cdots, x_{n-1}$ are non-negative integers be denoted by $\Delta$ satisfying the conditions (i)-(iii) of corollary. Let us define transformations $T_1$ and $T_2$ $m \geq 2, p > 0, m+p \leq n-2, x_m \geq 1, x_p \geq 1$, by

$T_1(x_1, \cdots, x_{n-1}) = (x_1^{'}, \cdots, x_{n-1}^{'}) = ( x_1, \cdots, x_{m-1}+1, x_m - 1, \cdots, x_{m+p} - 1, x_{m+p+1} + 1, \cdots, x_{n-1})$ and

$T_2(x_1, \cdots, x_{n-1}) = (x_1^{'}, \cdots, x_{n-1}^{'}) =( x_1, \cdots, x_{m-1}+1, x_m - 2, x_{m+1} + 1, \cdots, x_{n-1})$ we have $x_i = x_i^{'}$ for $i \neq \{\mathrm{m}-1, \mathrm{m}, \mathrm{m}+\mathrm{p}, \mathrm{m}+\mathrm{p}+1\}$.

**Lemma 2.** Let $(x_1, \cdots, x_{n-1}) \in \Delta$ then

(a). $T_1(x_1, \cdots, x_{n-1}) \in \Delta$ if $m \neq 2$ and $x_1 \neq n-7$, moreover $F(T_1(x_1, \cdots, x_{n-1})) < F(x_1, \cdots, x_{n-1})$

(b). $T_2(x_1, \cdots, x_{n-1}) \in \Delta$ if $m \neq 2$ and $x_1 \neq n-7$, moreover $F(T_2(x_1, \cdots, x_{n-1})) < F(x_1, \cdots, x_{n-1})$

***Proof:*** (a). As $\sum_{i=1}^{n-1} x_i = \sum_{i=1}^{n-1} x_i^{'}$ and $\sum_{i=1}^{n-1} i x_i = \sum_{i=1}^{n-1} i x_i^{'} = 2n+6$. If $(x_1, \cdots, x_{n-1}) \in \Delta$, $m = 2$ and $n-6$ then $x_1^{'} > n-6$ a contradiction. Also $F(x_1, \cdots, x_{n-1}) - F(T_1(x_1, \cdots, x_{n-1})) = 2p+2 > 0$.

Similarly, (b) also hold. This completes the proof of lemma 2.

## Main result

**Theorem:** Let $G \in G_n^6$,

(a). If $n = 6$ then $minD^{'}(G) = 132$, and the unique extremal graph is isomorphic to $H_2$ as shown in fig.1.

(b). If $n \geq 7$ then $minD^{'}(G) = 3n^2 + 17n - 80$ then all the extremal graphs are isomorphic to the graphs $F_1$ and $F_2$ in fig.5.

***Proof.*** In order to find min $F(x_1, \cdots, x_{n-1})$ where $(x_1, \cdots, x_{n-1}) \in \Delta$.

Firstly, let $n = 6$ the resultant graphs are given in Fig 1, and $D^{'}(H_2) = 132$ is the minimum value.

Secondly, let us consider $n = 7$. Then all graphs $G \in G_7^6$ are $C_i$'s where $1 \leq i \leq 11$. Here

$D'(C_1) = D'(C_7) = 202,\ D'(C_2) = 190,\ D'(C_3) = 200, D'(C_4) = D'(C_6) = 198,\ D'(C_5) = 194,\ D'(C_8) = 188\ ,\ D'(C_9) = D'(C_{10}) = 198$ and $D'(C_{11}) = 186,$ where $C_i$ are shown in Fig.4.

Finally, $n \geq 8$. If $x_{n-1} \geq 2$,consider two different vertices $u, v \in V(G)$ such that $d(u) = d(v) = n - 1$. As $n \geq 8$, we can choose at least eight different vertices distinct from $u, v$ which are adjacent to $u, v$ . Thus, we have at least seven cycles, which contradicts the hypothesis, Therefore $x_{n-1} \leq 1$.

Now we analyze the possible values of $x_1$,$x_2, \cdots, x_{n-2}$ If there exist $8 < i, j < n - 2$ such that $x_i \geq 1$ and $x_j \geq 1$ then by applying the transformation $T_1$ for the position *i* and *j*, we have a new vector $(x'_1,\cdots, x'_{n-1}) \in \Delta$ for which $F(x'_1,\cdots, x'_{n-1}) < F(x_1, \cdots, x_{n-1})$. Similarly, if there exist $6 < i, j < n - 2$ such that $x_i \geq 2$ then we obtain a new degree sequence in $\Delta$ for which $F(x'_1,\cdots, x'_{n-1}) < F(x_1, \cdots, x_{n-1})$. by applying the transformation $T_2$. Now we consider two cases:

**Case 1.** Suppose that there exist distinct indices *i, k* with $8 < i,\ k < n - 2$ such that $x_i = 1$ and $x_k = 0$. In this case, if $x_7 \geq 1$ then by applying the transformation $T_1$ for position *7* and *i*, we obtain smaller a smaller value of $F$, Suppose that $x_7 = 0$. Since $x_{n-1} \in \{0, 1\}$, we will analyze the two cases separately.

**(a)** In this case $x_{n-1} = x_i = 1$, where $7 < i$ and $x_5 = 0$. We can consider different vertices u,v, w,x ,y ,p,q $\in$ V(G) such that $d(u) = n - 1, d(v) = i \geq 7$ then w,x,y,p,q are all adjacent to u and v respectively. Meanwhile, u and v are adjacent too, so we have found six cycles which contradicts the hypothesis.

**(b)** If $x_{n-1} = 0$ then $x_5 = 0$ and $x_i = 1$, $(7 < i,n - 2)$ and $\Delta$ is characterized by the equations $x_1 + x_2 + x_3 + x_4 = n - 1$ and $x_1 + 2x_2 + 3x_3 + 4x_4 = 2n + 8 - i$ which implies that $x_2 + 2x_3 = n + 8 - i$ by solving for $x_2$ and $x_3$ and then by applying the transformation for position 2 and *i* or 3 and *i* or 4 and *i*, we obtain smaller value of *F*.

**Case 2.** Suppose that $x_8, \cdots, x_{n-1} = 0,$ hold and the degree sequence is $(x_1,\ x_2, x_3,\ x_4, x_5, x_6, x_7, 0,\ \cdots, 0,\ x_n)$. As $x_{n-1} \in \{0, 1\}$, so we have to analyze two cases:

**($a'$).** If $x_{n-1} = 0$, then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 + 6x_7 = n + 10$. This equation does not hold. If all $x_2, x_3,\ x_4$ , $x_5$ , $x_6$ and $x_7$ are not greater than 2, then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 + 6x_7 \leq 42$ which contradicts the hypothesis $n \geq 8$. If one of them is greater than two, then by using $T_2$ for the corresponding position, we obtain a smaller value of $F$.

$(\boldsymbol{b}')$. If $x_{n-1} = 1$, then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 + 6x_7 = 12$. If $x_7 \geq 3$, then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 + 6x_7 \geq 18$ which is not possible. So $x_7 \leq 2$, if $x_7 = 2$ then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 = 0$, which implies that $x_2 = x_3 = x_4 = x_5 = x_6 = 0$ and $x_1 = n - 3$ which is a contradiction as $x_1 \leq n - 6$. So $x_7 \neq 2$. Thus either $x_7 = 0$ or $x_7 = 1$.

If $x_7 = 1$, then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 = 6$, the possible solutions which gives the graphical degree sequence and which satisfy all the conditions of corollary 1 is $x_2 = 6$, $x_3 = x_4 = x_5 = x_6 = 0$. The degree sequence $(\text{n} - 8, 6, 0, 0, 0, 0, 1, 0,\cdots, 0, 1)$ and $t_2(\text{n} - 8, 6,0, 0,0, 0, 1, 0,\cdots, 0, 1)$ is not graphical. Thus $F(\text{n} - 8, 6,0, 0,0, 0, 1, 0,\cdots, 0, 1) = 3n^2 + 17n - 86$.

Next consider if $x_7 = 0$, then then $x_2 + 2x_3 + 3x_4 + 4x_5 + 5x_6 = 12$, all possible solution of $x_2, x_3, x_4, x_5$ and $x_6$ which follows the above corollary and construct the twenty dour degree sequences which are graphical. The sequences are
$(\text{n} - 6,0,3,2,0,\cdots,0,1)$, $(\text{n} - 6,0,4,0,1, 0,\cdots,0,1)$, $(n - 7, 3, 0, 3,0, \cdots,0,1)$,
$(n - 7, 0, 6, 0, \cdots,0, 1)$, $(\text{n} - 7, 2, 3, 0, 1, 0, \cdots,, 0, 1)$, $(\text{n} - 7, 1, 4, 1, 0, \cdots,0, 1)$, $(\text{n} - 7,3, 1, 1, 1, \cdots,,0,1)$,
$(n - 8, 4, 1, 2, 0, \cdots,0,1)$, $(\text{n} - 8, 3, 3, 1, 0, \cdots,0,1)$, $(\text{n} - 8, 2, 5, 0, \cdots,0, 1)$, $(\text{n} - 8, 5, 1, 0, 0, 1, 0,\cdots,0,1)$,
$(n - 7, 3, 2, 0, 0, 1, 0, \cdots,0, 1)$, $(\text{n} - 9,5, 2, 1, 0, \cdots,0,1)$, $(\text{n} - 9, 6, 1, 0, 1, 0,\cdots,0, 1)$,$(\text{n} - 9,6,0,2,0\cdots,0,1)$,
$(n - 9, 7, 0, 0, 0, 1, \cdots,0,1)$, $(\text{n} - 10, 8, 0, 0, 1, 0,\cdots,0,1)$, $(\text{n} - 10, 7,1,1,0,\cdots,0,1)$,$(\text{n} - 10,6,3,0,\cdots,,0,1)$,
$(n - 11, 8, 2, 0, \cdots,,0, 1)$, $(\text{n} - 11, 9, 0, 1, 0, \cdots,,0,1)$, $(\text{n} - 12, 10, 1, 0,\cdots,0,1)$, $(\text{n} - 13, 12, 0,\cdots,0, 1)$.

By applying transformations $T_1$ and $T_2$ these degree sequences are transformed to either the sequence $(\text{n} - 7,3,2,0,0,1, 0, \cdots,0,1)$ or $(\text{n} - 6,1,2,1,1,0,\cdots,0,1)$ represented by graphs $F_1$ and $F_2$ respectively. But $F(\text{n} - 7,3,2,0,0,1, 0, \cdots,0,1) = 3n^2 + 17n - 80 < F(\text{n} - 6,1,2,1,1,0,\cdots,0,1) = 3n^2 + 17n - 78$. Hence for $n \geq 7$, $minD'(G) = 3n^2 + 17n - 80$.

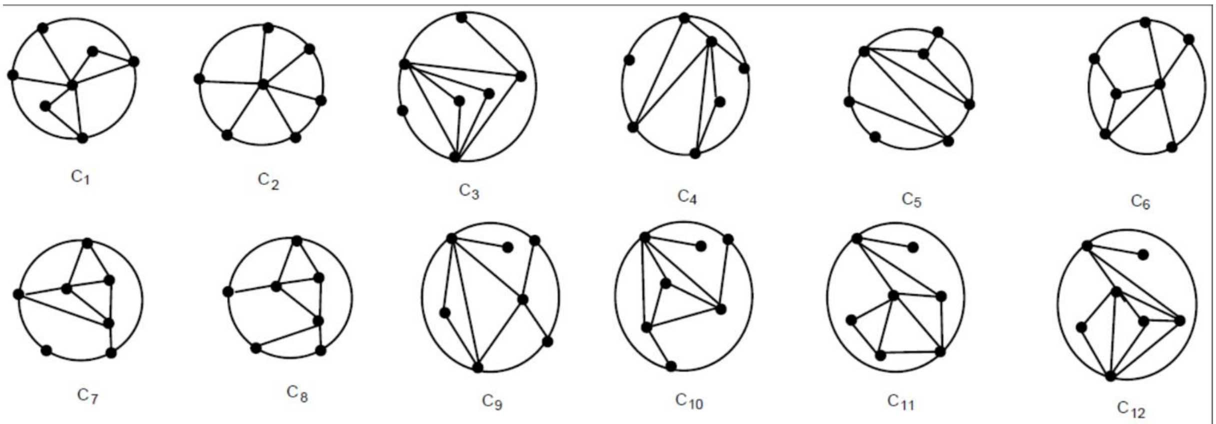


**Fig. 4**

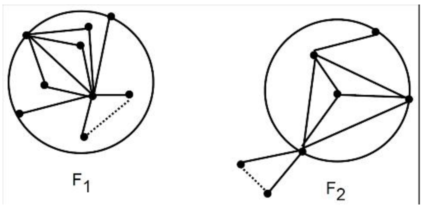


**Fig. 5**

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