SCIREA Journal of Information Science
and Systems Science
ISSN: 2995-3936
http://www.scirea.org/journal/ISSS
February 3, 2024
Volume 8, Issue 1, February 2024
https://doi.org/10.54647/isss120332

# Knowledge Representation and Knowledge Reasoning Based on the Aristotelian Modal Syllogism □AE◇E-4

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## Abstract

This paper firstly formalizes Aristotelian modal syllogisms from the perspective of knowledge representation, and then uses modal logic and generalized quantifier theory to prove the validity of the Aristotelian modal syllogism □AE◇E-4. Finally, making much of some rules and facts in first-order logic and the definitions of inner negation for Aristotelian quantifiers in generalized quantifier theory, at least the other 34 valid Aristotelian modal syllogisms can be derived by the validity of the syllogism □AE◇E-4 from the perspective of knowledge reasoning in artificial intelligence. The method is not only concise and elegant, but also universal for the study of other types of syllogisms. Undoubtedly, this study benefits natural language information processing.

**Keywords:** Aristotelian modal syllogisms; validity; knowledge representation; knowledge reasoning

## 1. Introduction

The common form of reasoning in natural language and social life is syllogistic reasoning which is also one of the research topics in logic. There are many kinds of syllogisms, such as Aristotelian syllogisms ([1]), generalized syllogisms ([2]), Aristotelian modal syllogisms ([3]), and so on. This paper mainly studies the last ones. Thus, unless otherwise specified, the following syllogisms refer to Aristotelian modal syllogisms.

Aristotelian modal syllogism has been studied since the time of Aristotle himself. For example, Łukasiewicz(1957)[4], McCall(1963)[5], Thomason(1997)[6], Johnson(2004)[7], Malink(2013) [8], and Zhang (2020)[9], and so on. These studies mainly focused on their validity, and there are many inconsistencies in their research results, which have been criticized. Few studies have focused on the reducibility between such syllogisms, and this paper hopes to make a breakthrough in this regard. More specifically, it studies the reduction between the syllogism $\Box$AE$\Diamond$E-4 and other valid syllogisms. To this end, this paper first proves the validity of this syllogism. And the validity of other modal syllogisms is deduced from this syllogism by means of relevant definitions, facts and reasoning rules, so the results are consistent.

## 2. Knowledge Representation for Aristotelian modal syllogisms

In the following, let *Q* be any of the four Aristotelian quantifiers (namely, *all*, *some*, *no*, *not all*), $\neg Q$ be its outer negation quantifier and $Q\neg$ be its inner one. And let *c*, *h* and *w* be lexical variables, and *D* be their domain. The set composed of *c*, *h* and *w* is respectively *C, H*, and *W*. '$=_{def}$' means that the left can be defined by the right. Let $\phi$, $\psi$, $\delta$ and $\lambda$ be well-formed formulas (abbreviated as wff). '$\vdash\phi$' indicates that the formula $\phi$ is provable. The other cases are similar. The operators in the paper are the basic symbols in set theory ([10]) and modal logic ([11]), for instance, $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$, $\Diamond$ and $\Box$ are operators of negation, conditionality, conjunction, biconditionality, possibility and necessity, respectively.

Aristotelian syllogisms involve 4 kinds of propositions as follows: all *c*s are *w*s', 'some *c*s are *w*s', 'no *c*s are *w*s' and 'not all *c*s are *w*s', which can be respectively formalized as *all(c, w)*, *no(c, w)*, *some(c, w),* and *not all(c, w).* These four propositions are respectively called Proposition *A*, *E*, *I*, *O.* Aristotelian syllogism has four different figures, which are defined as usual ([12]).

An Aristotelian modal syllogism is obtained from an Aristotelian syllogism by adding necessary modalities (□) and/or possible ones (◇). More specifically, in addition to the four propositions mentioned above, Non-trivial modal syllogisms also involves the following eight kinds of propositions as follows: □*all(c, w)*, □*no(c, w)*, □*some(c, w),* □*not all(c, w),* ◇*all(c, w)*, ◇*no(c, w)*, ◇*some(c, w),* and ◇*not all(c, w).* And they are respectively called Proposition □A, □E, □I, □O, ◇A, ◇E, ◇I and ◇O. Then, for example, the expansion of the syllogism □AE◇E-4 is that $\Box all(w, h) \wedge no(h, c) \rightarrow \Diamond no(c, w)$. An instance of the syllogism is as follows:

Major premise: All healthy adult birds are necessarily feathered animals.

Minor premise: No feathered animals are pigs.

Conclusion: No pigs are possibly healthy adult birds.

Let *w* be the variable of a healthy adult bird, *h* be that of a feathered animal, and *c* be that of a pig. Then, this example of syllogism can be formalized as $\Box all(w, h) \wedge no(h, c) \rightarrow \Diamond no(c, w)$, which is abbreviated as □AE◇E-4. Other representations are similar to this.

## 3. Formal System of Aristotelian Modal Syllogistic

This formal system is composed of the following: initial symbols, formation rules, related definitions, basic axioms and deductive rules.

### 3.1 Initial Symbols

(1) lexical variables: *c*, *h*, *w*

(2) quantifier: *all*

(3) modality: □

(4) unary negative operator: ¬

(5) binary implication operator: →

(6) brackets: (, )

### 3.2 Formation Rules

(1) If *Q* is a quantifier, *c* and *w* are lexical variables, then *Q(c, w)* is a wff.

(2) If $\phi$ is a wff, then so are $\neg\phi$ *and* $\Box\phi$.

(3) If $\phi$ and $\psi$ are wffs, then so is $\phi\rightarrow\psi$.

(4) Only the formulas obtained by the above three rules are wffs.

**3.3 Basic Axioms**

(1) A1: if $\phi$ is a valid formula in first-order logic, then $\vdash\phi$.

(2) A2: $\vdash\Box all(w, h)\wedge no(h, c)\rightarrow\Diamond no(c, w)$ (that is, the syllogism $\Box$AE$\Diamond$E-4).

**3.4 Relevant Definitions**

Definition 1 (conjunction): $(\phi\wedge\psi)=_{def}\neg(\phi\rightarrow\neg\psi)$.

Definition 2 (biconditional): $(\phi\leftrightarrow\psi)=_{def}(\phi\rightarrow\psi)\wedge(\psi\rightarrow\phi)$.

Definition 3 (inner negation): $Q\neg(c, w)=_{def}Q(c, D-w)$.

Definition 4 (outer negation): $(\neg Q)(c, w)=_{def}$It is not that $Q(c, w)$.

Definition 5 (possibility): $\Diamond\phi=_{def}\neg\Box\neg\phi$.

Definition 6 (truth value definition):

(6.1) *all*(*c, w*) is true when and only when $C\subseteq W$ is true in any real world.

(6.2) *some*(*c, w*) is true when and only when $C\cap W\neq\varnothing$ is true in any real world.

(6.3) *no*(*c, w*) is true when and only when $C\cap W=\varnothing$ is true in any real world.

(6.4) *not all*(*c, w*) is true when and only when $C\nsubseteq W$is true in any real world.

(6.5) $\Box$*all*(*c, w*) is true when and only when $C\subseteq W$ is true in any possible world.

(6.6) $\Diamond$*all*(*c, w*) is true when and only when $C\subseteq W$ is true in at least one possible world.

(6.7) $\Box$*some*(*c, w*) is true when and only when $C\cap W\neq\varnothing$ is true in any possible world.

(6.8) $\Diamond$*some*(*c, w*) is true when and only when $C\cap W\neq\varnothing$ is true in at least one possible world.

(6.9) $\Box$*no*(*c, w*) is true when and only when $C\cap W=\varnothing$ is true in any possible world.

(6.10) $\Diamond$*no*(*c, w*) is true when and only when $C\cap W=\varnothing$ is true in at least one possible world.

(6.11) $\Box$*not all*(*c, w*) is true when and only when $C\nsubseteq W$ is true in any possible world.

(6.12) $\Diamond$*not all*(*c, w*) is true when and only when $C\nsubseteq W$ is true in at least one possible world.

### 3.5 Relevant Facts

**Fact 1** (inner negation):

(1.1) $all(c, w)=no\neg(c, w)$; (1.2) $no(c, w)=all\neg(c, w)$;

(1.3) $some(c, w)=not\ all\neg(c, w)$; (1.4) $not\ all(c, w)=some\neg(c, w)$.

**Fact 2** (outer negation):

(2.1) $\neg not\ all(c, w)=all(c, w)$; (2.2) $\neg all(c, w)=not\ all(c, w)$;

(2.3) $\neg no(c, w)=some(c, w)$; (2.4) $\neg some(c, w)=no(c, w)$.

**Fact 3 (**dual**)**: (3.1) $\neg\Box Q(c, w)=\Diamond\neg Q(c, w)$; (3.2) $\neg\Diamond Q(c, w)=\Box\neg Q(c, w)$.

**Fact 4** (a necessary proposition implies an assertoric one): $\vdash\Box Q(c, w)\rightarrow Q(c, w)$.

**Fact 5** (a necessary proposition implies a possible one): $\vdash\Box Q(c, w)\rightarrow\Diamond Q(c, w)$.

**Fact 6** (an assertoric proposition implies a possible one): $\vdash Q(c, w)\rightarrow\Diamond Q(c, w)$.

**Fact 7** (a universal proposition implies a particular one):

(7.1) $\vdash all(c, w)\rightarrow some(c, w)$; (7.2) $\vdash no(c, w)\rightarrow not\ all(c, w)$.

**Fact 8** (symmetry of *some* and *no*): (8.1) $some(c, w)\leftrightarrow some(w, c)$; (8.2) $no(c, w)\leftrightarrow no(w, c)$.

The above facts are the basic knowledge of first-order logic ([12]) or generalized quantifier theory ([13]) or modal logic ([11]), hence their proofs are omitted.

### 3.6 Inference Rules

Rule 1 (subsequent weakening): If $\vdash(\phi\wedge\psi\rightarrow\delta)$ and $\vdash(\delta\rightarrow\lambda)$, then $\vdash(\phi\wedge\psi\rightarrow\lambda)$.

Rule 2 (anti-syllogism): If $\vdash(\phi\wedge\psi\rightarrow\delta)$, then $\vdash(\neg\delta\wedge\phi\rightarrow\neg\psi)$.

Rule 3 (anti-syllogism): If $\vdash(\phi\wedge\psi\rightarrow\delta)$, then $\vdash(\neg\delta\wedge\psi\rightarrow\neg\phi)$.

## 4. Knowledge Reasoning Based on Aristotelian Modal Syllogism □AE◇E-4

In the following, Theorem 1 proves that the syllogism □AE◇E-4 is valid. '(2.1) □AE◇E-4 →□AE◇E-2' in Theorem 2 indicates that the validity of the latter is provable according to that of the former. That is to say, there is reducible relationship between them. Other cases are similar.

**Theorem 1** (□AE◇E-4): □*all*(*w, h*)∧*no*(*h, c*)→◇*no*(*c, w*) is valid.

Proof: □AE◇E-4 is the abbreviation of the fourth figure syllogism □*all*(*w, h*)∧*no*(*h, c*)→◇*no*(*c, w*). Suppose that □*all*(*w, h*) and *no*(*h, c*) are true, then $W\subseteq H$ is true at any possible world and $H\cap C=\varnothing$ is true at any real world in line with Definition (6.5) and (6.3) respectively. Because all real worlds are possible worlds. It follows that $C\cap W=\varnothing$ is true in at least one possible world. Hence ◇*no*(*c, w*) is true in the light of Definition (6.10). This proves that the syllogism □*all*(*w, h*)∧*no*(*h, c*)→◇*no*(*c, w*) is valid, just as expected.

**Theorem 2**: There are at least the following 34 valid modal syllogisms inferred from □AE◇E-4:

(2.1) □AE◇E-4→□AE◇E-2
(2.2) □AE◇E-4→□AE◇E-2→E□A◇E-2
(2.3) □AE◇E-4→E□A◇E-1
(2.4) □AE◇E-4→□AE◇O-4
(2.5) □AE◇E-4→□AE◇E-2→□AE◇O-2
(2.6) □AE◇E-4→□AE◇E-2→E□A◇E-2→E□A◇O-2
(2.7) □AE◇E-4→E□A◇E-1→E□A◇O-1
(2.8) □AE◇E-4→□I□AI-4
(2.9) □AE◇E-4→□I□AI-4→□I□AI-3
(2.10) □AE◇E-4→□I□AI-4→□I□AI-3→□A□II-3
(2.11) □AE◇E-4→□I□AI-4→□A□II-1
(2.12) □AE◇E-4→E□I◇O-4
(2.13) □AE◇E-4→E□I◇O-4→E□I◇O-2
(2.14) □AE◇E-4→E□I◇O-4→E□I◇O-2→E□I◇O-1
(2.15) □AE◇E-4→E□I◇O-4→E□I◇O-3
(2.16) □AE◇E-4→□AE◇E-2→□EA◇E-2
(2.17) □AE◇E-4→□AE◇E-2→□EA◇E-2→□EA◇E-1
(2.18) □AE◇E-4→□AE◇E-2→□EA◇E-2→□EA◇E-1→A□E◇E-4
(2.19) □AE◇E-4→□AE◇E-2→□EA◇E-2→A□E◇E-2
(2.20) □AE◇E-4→□AE◇E-2→□EA◇E-2→□EA◇O-2
(2.21) □AE◇E-4→□AE◇E-2→□EA◇E-2→□EA◇E-1→□EA◇O-1
(2.22) □AE◇E-4→□AE◇E-2→□EA◇E-2→□EA◇E-1→A□E◇E-4→A□E◇O-4
(2.23) □AE◇E-4→□AE◇E-2→□EA◇E-2→A□E◇E-2→A□E◇O-2
(2.24) □AE◇E-4→□AE◇E-2→□AE◇O-2→□A□AI-1
(2.25) □AE◇E-4→□AE◇E-2→□AE◇O-2→□A□AI-1→□A□AI-4
(2.26) □AE◇E-4→□AE◇E-2→□AE◇O-2→E□A◇O-3
(2.27) □AE◇E-4→□AE◇E-2→□AE◇O-2→E□A◇O-3→E□A◇O-4
(2.28) □AE◇E-4→□AE◇E-2→E□A◇E-2→E□A◇O-2→□A□AI-3

(2.29) □AE◇E-4→□I□AI-4→□I□AI-3→□O□AO-3

(2.30) □AE◇E-4→□I□AI-4→□I□AI-3→□A□II-3→□E□IO-3

(2.31) □AE◇E-4→□I□AI-4→□I□AI-3→□A□II-3→□E□IO-3→□E□IO-1

(2.32) □AE◇E-4→□I□AI-4→□I□AI-3→□A□II-3→□E□IO-3→□E□IO-1→□E□IO-2

(2.33) □AE◇E-4→□I□AI-4→□I□AI-3→□A□II-3→□E□IO-3→□E□IO-4

(2.34) □AE◇E-4→E□I◇O-4→E□I◇O-2→A□O◇O-2

Proof:

[1] ⊢□*all*(*w*, *h*)∧*no*(*h*, *c*)→◇*no*(*c*, *w*) (i.e.□AE◇E-4, basic axiom)

[2] ⊢□*all*(*w*, *h*)∧*no*(*c*, *h*)→◇*no*(*c*, *w*) (i.e.□AE◇E-2, by [1] and Fact 8)

[3] ⊢□*all*(*w*, *h*)∧*no*(*c*, *h*)→◇*no*(*w*, *c*) (i.e. E□A◇E-2, by [2] and Fact 8)

[4] ⊢□*all*(*w*, *h*)∧*no*(*h*, *c*)→◇*no*(*w*, *c*) (i.e. E□A◇E-1, by [1] and Fact 8)

[5] ⊢□*all*(*w*, *h*)∧*no*(*h*, *c*)→◇*not all*(*c*, *w*) (i.e.□AE◇O-4, by [1] and Fact 7)

[6] ⊢□*all*(*w*, *h*)∧*no*(*c*, *h*)→◇*not all*(*c*, *w*) (i.e.□AE◇O-2, by [2] and Fact 7)

[7] ⊢□*all*(*w*, *h*)∧*no*(*c*, *h*)→◇*not all*(*w*, *c*) (i.e. E□A◇O-2, by [3] and Fact 7)

[8] ⊢□*all*(*w*, *h*)∧*no*(*h*, *c*)→◇*not all*(*w*, *c*) (i.e. E□A◇O-1, by [4] and Fact 7)

[9] ⊢¬◇*no*(*c*, *w*)∧□*all*(*w*, *h*)→¬*no*(*h*, *c*) (by [1] and Rule 2)

[10] ⊢□¬*no*(*c*, *w*)∧□*all*(*w*, *h*)→*some*(*h*, *c*) (by [9], Fact 3 and Fact 2)

[11] ⊢□*some*(*c*, *w*)∧□*all*(*w*, *h*)→*some*(*h*, *c*) (i.e. □I□AI-4, by [10] and Fact 2)

[12] ⊢□*some*(*w*, *c*)∧□*all*(*w*, *h*)→*some*(*h*, *c*) (i.e. □I□AI-3, by [11] and Fact 8)

[13] ⊢□*some*(*w*, *c*)∧□*all*(*w*, *h*)→*some*(*c*, *h*) (i.e. □A□II-3, by [12] and Fact 8)

[14] ⊢□*some*(*c*, *w*)∧□*all*(*w*, *h*)→*some*(*c*, *h*) (i.e. □A□II-1, by [11] and Fact 8)

[15] ⊢¬◇*no*(*c*, *w*)∧*no*(*h*, *c*)→¬□*all*(*w*, *h*) (by [1] and Rule 3)

[16] ⊢□¬*no*(*c*, *w*)∧*no*(*h*, *c*)→◇¬*all*(*w*, *h*) (by [15] and Fact 3)

[17] ⊢□*some*(*c*, *w*)∧*no*(*h*, *c*)→◇*not all*(*w*, *h*) (i.e. E□I◇O-4, by [16] and Fact 2)

[18] ⊢□*some*(*w*, *c*)∧*no*(*h*, *c*)→◇*not all*(*w*, *h*) (i.e. E□I◇O-2, by [17] and Fact 8)

[19] ⊢□*some*(*w*, *c*)∧*no*(*c*, *h*)→◇*not all*(*w*, *h*) (i.e. E□I◇O-1, by [18] and Fact 8)

[20] ⊢□*some*(*c*, *w*)∧*no*(*c*, *h*)→◇*not all*(*w*, *h*) (i.e. E□I◇O-3, by [17] and Fact 8)

[21] ⊢□*no*¬(*w*, *h*)∧*all*¬(*c*, *h*)→◇*no*(*c*, *w*) (by [2] and Fact 1)

[22] ⊢□*no(w, D−h)*∧*all(c, D−h)*→◇*no*(*c*, *w*) (i.e. □EA◇E-2, by [21] and Definition 3)

[23] ⊢□*no*(*D−h*, *w*)∧*all(c, D−h)*→◇*no*(*c*, *w*) (i.e. □EA◇E-1, by [22] and Fact 8)

[24] ⊢□*no*(*D−h*, *w*)∧*all(c, D−h)*→◇*no*(*w*, *c*) (i.e. A□E◇E-4, by [23] and Fact 8)

[25] ⊢□*no(w, D−h)*∧*all(c, D−h)*→◇*no*(*w*, *c*) (i.e. A□E◇E-2, by [22] and Fact 8)

[26] ⊢□*no(w, D−h)*∧*all(c, D−h)*→◇*not all*(*c*, *w*) (i.e. □EA◇O-2, by [22] and Fact 7)

[27] ⊢□*no*(*D−h*, *w*)∧*all(c, D−h)*→◇*not all*(*c*, *w*) (i.e. □EA◇O-1, by [23] and Fact 7)

[28] ⊢□*no*(*D−h*, *w*)∧*all(c, D−h)*→◇*not all*(*w*, *c*) (i.e. A□E◇O-4, by [24] and Fact 7)

[29] ⊢□*no(w, D−h)*∧*all(c, D−h)*→◇*not all*(*w*, *c*) (i.e. A□E◇O-2, by [25] and Fact 7)

[30] ⊢¬◇*not all*(*c*, *w*)∧□*all*(*w*, *h*)→¬*no*(*c*, *h*) (by [6] and Rule 2)

[31] ⊢□¬*not all*(*c*, *w*)∧□*all*(*w*, *h*)→¬*no*(*c*, *h*) (by [30] and Fact 3)

[32] ⊢□*all*(*c*, *w*)∧□*all*(*w*, *h*)→*some*(*c*, *h*) (i.e. □A□AI-1, by [31] and Fact 2)

[33] ⊢□*all*(*c*, *w*)∧□*all*(*w*, *h*)→*some*(*h*, *c*) (i.e. □A□AI-4, by [32] and Fact 8)

[34] ⊢¬◇*not all*(*c*, *w*)∧*no*(*c*, *h*)→¬□*all*(*w*, *h*) (by [6] and Rule 3)

[35] ⊢□¬*not all*(*c*, *w*)∧*no*(*c*, *h*)→◇¬*all*(*w*, *h*) (by [34] and Fact 3)

[36] ⊢□*all*(*c*, *w*)∧*no*(*c*, *h*)→◇*not all*(*w*, *h*) (i.e. E□A◇O-3, by [35] and Fact 2)

[37] ⊢□*all*(*c*, *w*)∧*no*(*h*, *c*)→◇*not all*(*w*, *h*) (i.e. E□A◇O-4, by [36] and Fact 8)

[38] $\vdash\neg\Diamond \mathit{not\ all}(w, c)\wedge\Box \mathit{all}(w, h)\rightarrow\neg \mathit{no}(c, h)$ (by [7] and Rule 2)

[39] $\vdash\Box\neg \mathit{not\ all}(w, c)\wedge\Box \mathit{all}(w, h)\rightarrow\neg \mathit{no}(c, h)$ (by [38] and Fact 3)

[40] $\vdash\Box \mathit{all}(w, c)\wedge\Box \mathit{all}(w, h)\rightarrow \mathit{some}(c, h)$ (i.e. $\Box$A$\Box$AI-3, by [39] and Fact 2)

[41] $\vdash\Box \mathit{not\ all}\neg(w, c)\wedge\Box \mathit{all}(w, h)\rightarrow \mathit{not\ all}\neg(h, c)$ (by [12] and Fact 1)

[42] $\vdash\Box \mathit{not\ all}(w, D-c)\wedge\Box \mathit{all}(w, h)\rightarrow \mathit{not\ all}(h, D-c)$

(i.e. $\Box$O$\Box$AO-3, by [41] and Definition 3)

[43] $\vdash\Box \mathit{some}(w, c)\wedge\Box \mathit{no}\neg(w, h)\rightarrow \mathit{not\ all}\neg(c, h)$ (by [13] and Fact 1)

[44] $\vdash\Box \mathit{some}(w, c)\wedge\Box \mathit{no}(w, D-h)\rightarrow \mathit{not\ all}(c, D-h)$ (i.e. $\Box$E$\Box$IO-3, by [43] and Definition 3)

[45] $\vdash\Box \mathit{some}(c, w)\wedge\Box \mathit{no}(w, D-h)\rightarrow \mathit{not\ all}(c, D-h)$ (i.e. $\Box$E$\Box$IO-1, by [44] and Fact 8)

[46] $\vdash\Box \mathit{some}(c, w)\wedge\Box \mathit{no}(D-h, w)\rightarrow \mathit{not\ all}(c, D-h)$ (i.e. $\Box$E$\Box$IO-2, by [45] and Fact 8)

[47] $\vdash\Box \mathit{some}(w, c)\wedge\Box \mathit{no}(D-h, w)\rightarrow \mathit{not\ all}(c, D-h)$ (i.e. $\Box$E$\Box$IO-4, by [44] and Fact 8)

[48] $\vdash\Box \mathit{not\ all}\neg(w, c)\wedge \mathit{all}\neg(h, c)\rightarrow\Diamond \mathit{not\ all}(w, h)$ (by [18] and Fact 1)

[49] $\vdash\Box \mathit{not\ all}(w, D-c)\wedge \mathit{all}(h, D-c)\rightarrow\Diamond \mathit{not\ all}(w, h)$

(i.e. A$\Box$O$\Diamond$O-2, by [48] and Definition 3)

At this point, the other 34 modal syllogisms have been derived from the validity of the syllogism $\Box$AE$\Diamond$E-4. If one continues to follow similar reasoning methods, then he can deduce more valid syllogisms. Similar to Theorem 1, the validity of these syllogisms can also be proven by means of Definition 6.

## 5. Conclusion and Future Work

Using on modal logic, set theory and generalized quantifier theory, this paper firstly sought to proves the validity of the modal syllogism $\Box$AE$\Diamond$E-4. Then, with the help of relevant definitions, facts and reasoning rules, it tried to derive the other 34 valid modal syllogisms from the validity of this syllogism. The results obtained by the deductive methods are consistent. This method is not only concise and elegant, but also universally applicable for the study of other types of syllogisms.

Theorem 2 reveals the reducibility between modal syllogisms of different figures and forms. There are only 288 valid modal syllogisms out of 6656 ones ([14]). Can we use the method to select a few modal syllogisms as basic axioms and deduce all of the remaining valid modal syllogisms? If this can be achieved, a consistent axiom system can be established for modal syllogistic. In this way, we will be able to solve the long-standing unresolved problem. This still needs further in-depth research. Undoubtedly, this study benefits natural language information processing.

## Acknowledgement

This work was supported by the National Social Science Foundation of China under Grant No.22FZXB092.

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