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A Simulation Model of Repairable Parallel Systems Reliability

Yuriy Zhernovyi
Faculty of Mechanics and Mathematics, Ivan Franko National University of Lviv, Ukraine
Email: yuriy.zhernovyy@lnu.edu.ua

Abstract

We consider repairable parallel system that consists of m identical units. At any moment of time, a unit can be in one of the two states: either operational or failed. Suppose, the number n of repair facilities is restricted ($n \leq m$), so failed units can form a queue for recovering. Assuming that the distributions of the time to failure (X) and the repair time (Y) for each unit are known, our task is to determine the reliability indices of the system. One of the methods for assessing system reliability is the simulation method. In this approach, the model replicates the operation of a real system, emulating the functioning process of the actual system over time. In many cases, simulation becomes the most effective and, often, the only practical method for determining the reliability of repairable systems. Using a GPSS World simulation model, we studied the dependencies of system's reliability indicators on the following parameters: the coefficient of variation of time to failure and repair time of units, and the ratio $\rho = E(Y)/E(X)$. By utilizing the expressions for average transition times between states of the Markov birth-death process, we derive formulas for the mean values of the system's time to failure and time between failures in the case of exponential distributions of random variables X and Y. We validated the simulation models by comparing the results with

those obtained by an analytical method. We determined the simulation time required to obtain results corresponding to the stationary process.

Keywords: *reliability, parallel systems, repairable systems, simulation model, GPSS World*

1. Introduction

By definition, a parallel system is factually a single operating unit with a group of identical redundant units, which are independent in sense of failing. Parallel redundancy is an important commonly used method for enhancing system reliability. However, such systems can also operate independently, as seen in instances such as the engine systems of four-engine aircraft or the parallel switching of diodes in powerful rectifiers. There are many industrial systems in which parallel operation of components is required not only to improve their performance but also to share the working stress. The system of electrical transformers can be sited as good example of such a situation in which transformers having same polarity and voltage ratio are connected in parallel to meet the total load requirement to provide continuous power supply to essential services. A two-unit parallel system with one repairman has been one of the traditional models in the literature of reliability theory [1].

The determination of the distribution of the time to failure of a complex system based on the known reliability characteristics of its components is a very important task in the theory and practice of reliability. This problem has been studied by many authors [2, 3], but it is still far from its final practical solution. Suffice it to say that at present there is no effective algorithm for calculating the probability of failure-free operation of a duplicated repairable system with a continuously operating reserve and arbitrary distribution of time to failure and repair time of components. To be fair, it should be noted that there are simple analytical models that describe the functioning of technical systems with arbitrary distributions of time to failure and repair time of their components. Unfortunately, these models prove to be very difficult to implement on a computer. They involve systems of integral or partial differential equations with respect to functions featuring a substantial number of variables.

One of the methods for studying queuing systems and determining the reliability of systems is the simulation method. In this approach, the model simulates the operation of a real system, reproducing the process of functioning of a real system over time. In many cases, simulation becomes the most effective and, often, the only practical method for determining the reliability of repairable systems. In this paper, we use the GPSS World simulation system [4, 5].

GPSS (General-Purpose Simulation System) is a general process-oriented simulation software environment. GPSS World is a Microsoft Windows application designed to run on various Windows operating systems.

The main contributions of this paper are as follows.

We develop a GPSS World simulation model to assess the reliability of repairable parallel systems. We explore scenarios involving both exponential and non-exponential distributions for the lifetimes and repair times of units. Our investigation considers the impact of coefficients of variation in these distributions, along with the number of repair channels, the repair intensity and the failure rate of the units, on the system availability coefficient and the mean values of random variables such as time to failure, time between failures, system downtime, and the number of failed units.

Using the constructed simulation model, we have the opportunity to obtain not only the mean values of the random variables that characterize the reliability of the system but also the distributions of these random variables, as well as their graphical representations.

2. The Simulation Model

2.1. Basic Definitions and Assumptions

Consider a repairable parallel system that consists of m identical units. At any moment of time, a unit can be in one of the two states: either operational or failed. Suppose, the number n of repair facilities is restricted ($n \leq m$), so failed units can form a queue for recovering. Assuming that the distributions of the time to failure (X) and the repair time (Y) for each unit are known, our task is to determine the reliability indices of the system.

Let us denote states by natural numbers 0, 1, 2,..., where the number of a state corresponds to the number Z of failed units. If the distributions of random variables X and Y are exponential, then the state graphs have the form shown in Fig. 1.

$$\boxed{0} \underset{\mu}{\overset{m\lambda}{\rightleftarrows}} \boxed{1} \underset{2\mu}{\overset{(m-1)\lambda}{\rightleftarrows}} \boxed{2} \underset{3\mu}{\overset{(m-2)\lambda}{\rightleftarrows}} \cdots \underset{n\mu}{\overset{(m-n+1)\lambda}{\rightleftarrows}} \boxed{n} \underset{n\mu}{\overset{(m-n)\lambda}{\rightleftarrows}} \cdots \underset{n\mu}{\overset{2\lambda}{\rightleftarrows}} \boxed{m-1} \underset{n\mu}{\overset{\lambda}{\rightleftarrows}} \boxed{m}$$

Figure 1. Transition graph for the repairable parallel system

Here, λ and μ are the parameters of exponential distributions of the time to failure X and the repair time Y, respectively. For this system, the time to failure X_S and the time between failures X_{SB} do not coincide, because the time interval X_S begins at the moment of transition from state 1 to state 0, and the time X_{SB} begins at the moment of transition from state m to state $m-1$. Both states end simultaneously at the moment of transition from the state $m-1$ to the state m. The downtime X_{SD} is the time spent in a group of states $(0,\ldots,m-1)$.

We denote as $E(T)$ the mean of the random variables T. Let us denote by p_k the stationary probability that there are k failed units. The system stationary availability coefficient is determined by the formula

$$K = 1 - p_m.$$

For exponential distributions of random variables X and Y, $E(X_{SD}) = 1/(n\mu),$ and the stationary distribution of random variable Z is known [6]:

$$p_k = p_0 \tilde{p}_k, \quad 1 \le k \le m; \quad \tilde{p}_k = C_m^k \rho^k, \quad 1 \le k \le n; \quad C_m^k = \frac{m!}{k!(m-k)!};$$

$$\tilde{p}_k = \frac{m!\rho^k}{n!(m-k)!n^{k-n}}, \quad n+1 \le k \le m; \quad p_0 = \frac{1}{1+\sum_{k=1}^{m} \tilde{p}_k}; \quad \rho = \frac{\lambda}{\mu}.$$

By utilizing the expressions for average transition times between states of the Markov birth-death process, as obtained in the paper [7], we derive the following formulas:

$$E(X_S) = \frac{1}{m\mu\rho} + \frac{1}{m!\mu}\sum_{k=2}^{n}(k-1)!(m-k)!\left(\rho^{-k} + \sum_{s=0}^{k-2}\frac{m!\,\rho^{s+1-k}}{(m-s-1)!(s+1)!}\right) +$$
$$+\frac{(n-1)!}{m!\mu}\sum_{k=n+1}^{m}(m-k)!n^{k-n}\left(\rho^{-k} + \sum_{s=0}^{n-1}\frac{m!\,\rho^{s+1-k}}{(m-s-1)!(s+1)!} + \sum_{s=n}^{k-2}\frac{m!\,\rho^{s+1-k}}{n!(m-s-1)!n^{s+1-n}}\right), \quad 1 \le n \le m;$$

$$E(X_{SB}) = \frac{(n-1)!}{m!\mu}n^{m-n}\left(\rho^{-m} + \sum_{s=0}^{n-1}\frac{m!\rho^{s+1-m}}{(m-s-1)!(s+1)!} + \sum_{s=n}^{m-2}\frac{m!\rho^{s+1-m}}{n!(m-s-1)!n^{s+1-n}}\right), \quad 1 \le n < m;$$

$$E(X_{SB}) = \frac{1}{m\mu\rho^m}\left(1 + \sum_{s=0}^{m-2}\frac{m!\,\rho^{s+1}}{(m-s-1)!(s+1)!}\right), \quad n = m.$$

In our simulation model, we can consider any distributions of random variables X and Y, but we will only examine examples with exponential, uniform and gamma distributions of these random variables.

Let us denote for the random variable T the probability density function, variance and coefficient of variation as $f_T(t)$, $D(T)$, and V, respectively, then for the gamma distribution, we have

$$f_T(t) = \frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{t}{\beta}}, \quad \alpha > 0, \quad \beta > 0, \quad t \geq 0, \quad \Gamma(t) = \int_0^{\infty} x^{t-1} e^{-x} dx;$$

$$E(T) = \alpha\beta, \qquad D(T) = \alpha\beta^2, \qquad V = \frac{\sqrt{D(T)}}{E(T)} = \frac{1}{\sqrt{\alpha}}.$$

For the gamma distribution, fixing $E(T)$ and varying the parameters α and β, we can consider distributions with different values of the coefficient of variation V.

2.2. A General Simulation Model

Below we provide a GPSS simulation model in the case when $m = 4$, $n = 2$, $E(X) = 0.5$, and $E(Y) = 1$. We consider exponential, uniform and gamma distributions of random variables X and Y, with the flexibility to easily adjust the parameters and types of these distributions. The simulation time $t_{\text{mod}} = 5 \cdot 10^5$.

```
*****Definition of parameters and tables
Sys STORAGE 2 ; We specify the number of repair facilities here
Tmod EQU 500000 ; simulation time
em EQU 4 ; the number of units
Time TABLE MP$LIFE,0.5,0.5,15 ; distribution of the time to failure
Btime TABLE MP$BLIFE,0.5,0.5,15 ; distribution of the time between failures
Dtime TABLE MP$DLIFE,0.5,0.5,15 ; distribution of the downtime
Dis TABLE (S$Sys+Q1) 0,1,5 ; distribution of the number of failed units
*****Tabulation of Z
GENERATE 1
TABULATE Dis
*****Calculation of the availability coefficient K(t)=N$LT0/(N$LT0+N$LT1)
TEST E (S$Sys+Q1),em,LT0
LT1 TERMINATE
LT0 TERMINATE
*****Main segment of the model
GENERATE ,,,em ; At the initial time all units are put into operation
EL1 ADVANCE (Exponential(1,0,0.5)) ; random lifetime (exponential)
;EL1 ADVANCE (Uniform(1,0,1)) ; uniform distribution on the interval [0,1]
;EL1 ADVANCE (Gamma(1,0,2,0.25)) ; gamma distribution (V=2)
;EL1 ADVANCE (Gamma(1,0,8,1/16)) ; gamma distribution (V=4)
TEST E (S$Sys+Q1),(em-1),E1 ; condition for start of downtime
LOGIC S KEY ; the logic switch is set to the "on" state (for start of
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;downtime and completion of time between failures)
SPLIT 1,E1
MARK DLIFE ; record the start of downtime
GATE LR KEY ; skip when the downtime ends
TABULATE Dtime ; tabulation of downtime
TERMINATE
E1 QUEUE 1
ENTER Sys
DEPART 1
ADVANCE (Exponential(1,0,1)) ; random repair time (exponential)
;ADVANCE (Uniform(1,0,2)) ; uniform distribution on the interval [0,2]
;ADVANCE (Gamma(1,0,4,0.25)) ; gamma distribution (V=2)
;ADVANCE (Gamma(1,0,16,1/16)) ; gamma distribution (V=4)
TEST E (S$Sys+Q1),em,GG ; condition for end of downtime
LEAVE Sys
SPLIT 1,EL1 ; exit from the "m" state
LOGIC S TKEY ; the logic switch is set to the "on" state (for the
;possibility of the start of time to failure)
MARK BLIFE ; record the start of time between failures
LOGIC R KEY ; the logic switch is set to the "off" state (for completion of
;downtime)
GATE LS KEY ; skip when the time between failures ends
TABULATE Btime ; tabulation of time between failures
TERMINATE
GG LEAVE Sys
SPLIT 1,EL1
TEST E S$Sys,0,TER ; transition to the "0" state
GATE LS TKEY,TER ; skip, since there is a first transition to the "0" state
;after leaving "m" state
LOGIC R TKEY ; the first transition to the "0" state is fixed
MARK LIFE ; record the start of time to failure
GATE LS KEY ; skip when downtime begins
TABULATE Time ; tabulation of time to failure
TER TERMINATE
*****Completion of the simulation
GENERATE Tmod
SAVEVALUE K,(N$LT0/(N$LT0+N$LT1)) ; calculation of K
TERMINATE 1
START 1
```

The simulation model is constructed as a sequence of blocks, referred to as block segments. Block segments generally start with a GENERATE block that inserts transactions into the simulation model and ends with a TERMINATE block that removes transactions from the simulation model. Such a block segment specifies a process, i.e., a life cycle, for transactions.

We begin the model by setting the parameters and tables of distributions for the random variables that we aim to obtain through the simulation. The STORAGE command defines a Storage Entity named Sys with a total capacity of 2 units. By changing the capacity, we can set a desired value for the number of repair facilities. The Tables are used to calculate the distributions for several random variables, including the system time to failure (X_S), the system time between failures (X_{SB}), the system downtime (X_{SD}), and the number of failed units (Z).

To obtain the distributions of the random variable Z, we use the GENERATE block, which creates transactions through each unit of model time and directs them to the TABULATE block associated with the table of the distribution of this random variable (Dis TABLE).

The main segment forms the basis of the model and is designed to simulate the process of failure-free operation and the repair of units. The last segment sets a simulation time, saves a value of the stationary availability coefficient K, and stops the simulation process.

The first block of the main segment of the model is the GENERATE block. At the initial time, all units are put into operation as a result of the action of this block. Transactions (units) are delayed for a random failure-free time using the ADVANCE block.

The main segment of the model consists of the combination of blocks ENTER and LEAVE provides the operation of a Storage Entity. The Storage Entity models the operation of a multi-server repair system. The QUEUE and DEPART blocks are used to update the statistics associated with a queue. Transactions (units) are delayed for a random repair time using the ADVANCE block.

The TEST blocks are used to check the conditions for the system's start and end of downtime. The logic switches Key and Tkey, controlled by the LOGIC blocks, are used to capture the moments of system start and end of downtime, as well as the moment of transition to the "0" state. The GATE blocks check the state of the logic switches.

This model offers flexibility to easily change not only the number of units and repair facilities but also the probability distributions of the random variables X and Y, which are defined in the ADVANCE blocks.

2.3. Checking the Simulation Models and Choice the Simulation Time

Let us use analytical results from paragraph 2.1 to test the constructed simulation model and to choose the optimal value of the simulation time. We assume that $m = 4$, $n = 2$, $\lambda = 2$, $\mu = 1$, and consider exponential distributions of random variables X and Y.

The GPSS World uses random number generators to sample random numbers for ADVANCE blocks. We can select which random number generator number is to be used as the source of the random number. The results obtained for different values of the random number generator may differ slightly from each other. In this work, we use the number of the random number generator, which is equal to 1.

We evaluate the accuracy of the results using the relative error for different values of the simulation time (see Table 1). The simulation time on the interval $(10^5, 10^6)$ appears to be the optimal choice for further calculations due to its high accuracy and quick implementation time.

Table 1. Relative Error for Different Values of Simulation Time

t_{mod}	Relative error in percentage when calculating values:				
	$E(Z)$	K	$E(X_S)$	$E(X_{SB})$	$E(X_{SD})$
10^4	0.370	0.941	2.000	0.444	0.600
10^5	0.140	0.304	1.000	0.385	0.600
10^6	0.041	0.144	0.067	0.030	0.000

3. Comparing System Reliability Indicators across Various Time-to-Failure and Repair Time Distributions of Units

3.1. Changing the Coefficient of Variation

We consider the case when $m = 4, n = 2,$ and the random variables X and Y have the same distribution with means $E(X)$=0.5 and $E(Y)$=1. If $t_{\mathrm{mod}} = 5 \cdot 10^5$, then we get the results presented in Table 2.

Table 2. Influence of Coefficient of Variation in Distributions *X* and *Y* on Reliability Indicators

Distributions of X and Y	K	$E(X_S)$	$E(X_{SB})$	$E(X_{SD})$	$E(Z)$
Uniform (V<1)	0.673	0.871	0.801	0.390	3.026
Exponential (V=1)	0.628	1.500	0.844	0.500	3.039
Gamma, V=2	0.558	3.803	1.092	0.868	3.061
Gamma, V=4	0.525	11.857	1.777	2.285	3.077

We can get the distributions of those random variables for which tables are given in a simulation model. GPSS World tools make it possible to obtain graphic representations of distribution tables in the form of histogram.

The histograms presented in Figures 1 to 6 clearly demonstrate the impact of the distribution type and the coefficient of variation of the random variables X and Y on the distribution shape of the considered random variables, which are indicators of system reliability.

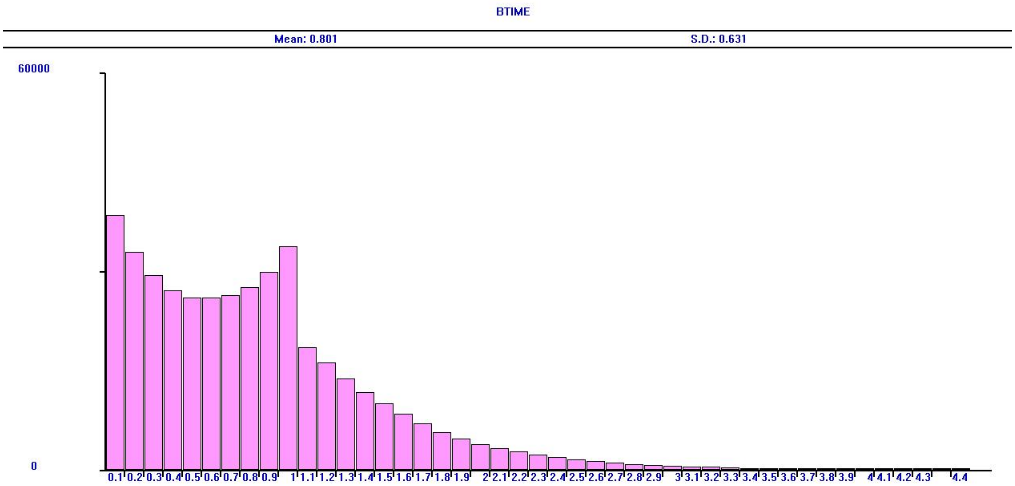

Figure 1. Distribution of time between failures for the system with uniform distributions of X and Y

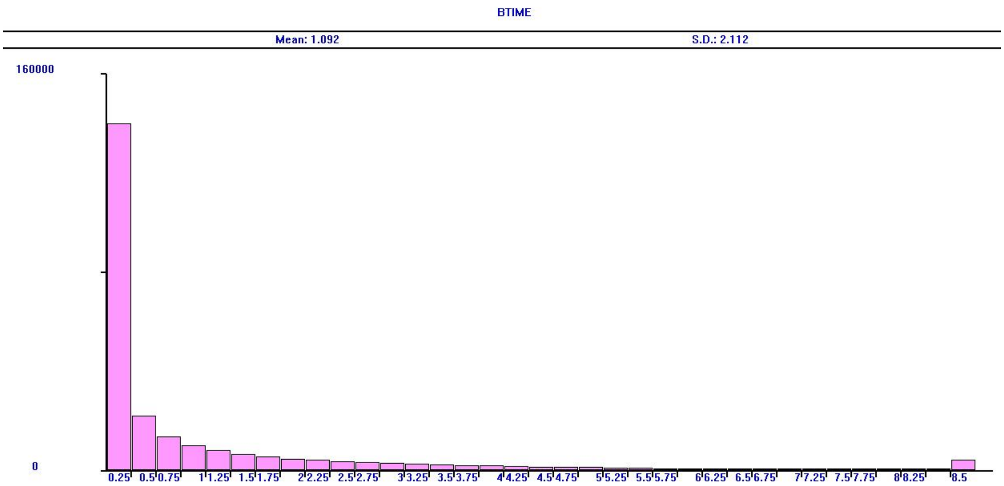

Figure 2. Distribution of time between failures for the system with gamma distributions (V=2) of X and Y

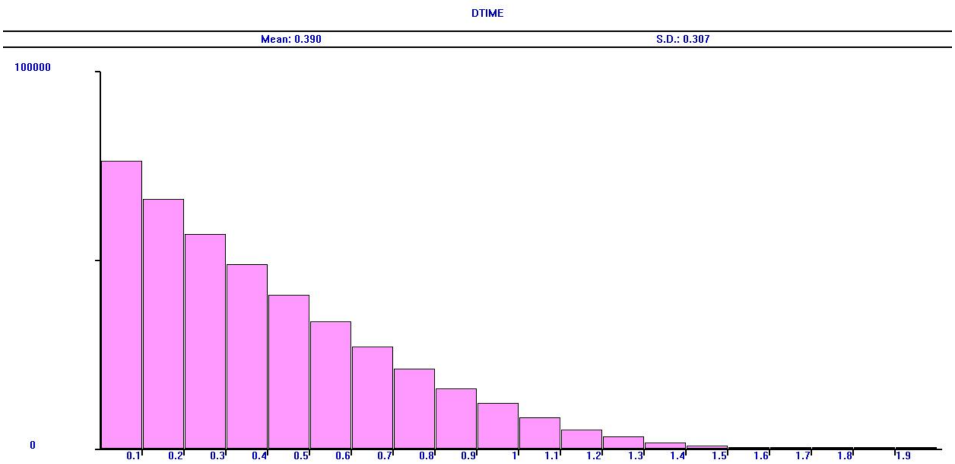

Figure 3. Distribution of downtime for the system with uniform distributions of *X* and *Y*

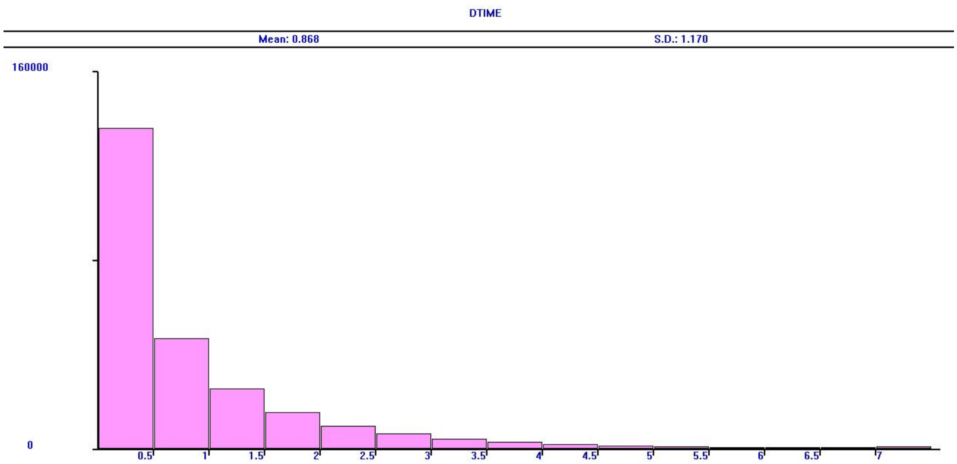

Figure 4. Distribution of downtime for the system with gamma distributions (*V*=2) of *X* and *Y*

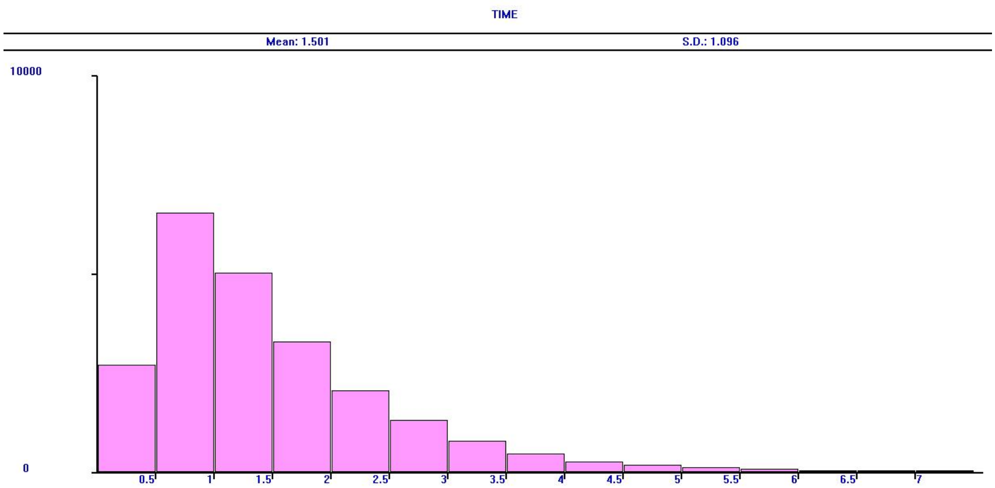

Figure 5. Distribution of time to failure for the system with exponential distributions of *X* and *Y*

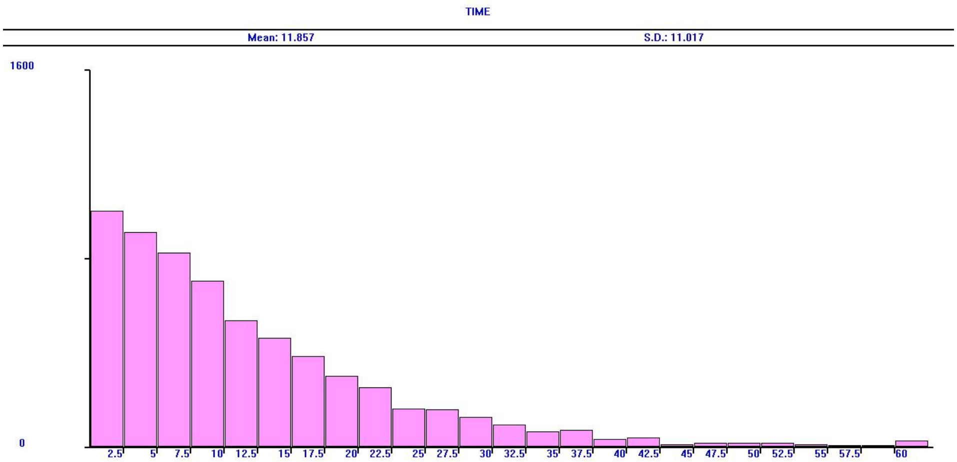

Figure 6. Distribution of time to failure for the system with gamma distributions (*V*=4) of *X* and *Y*

Graphs illustrating the dependencies of *K*, $E(X_S)$, $E(X_{SB})$, $E(X_{SD})$, and $E(Z)$ on the coefficient of variation (*V*) are presented in Figures 7 to 11 for *m*=4 and varying numbers of repair facilities.

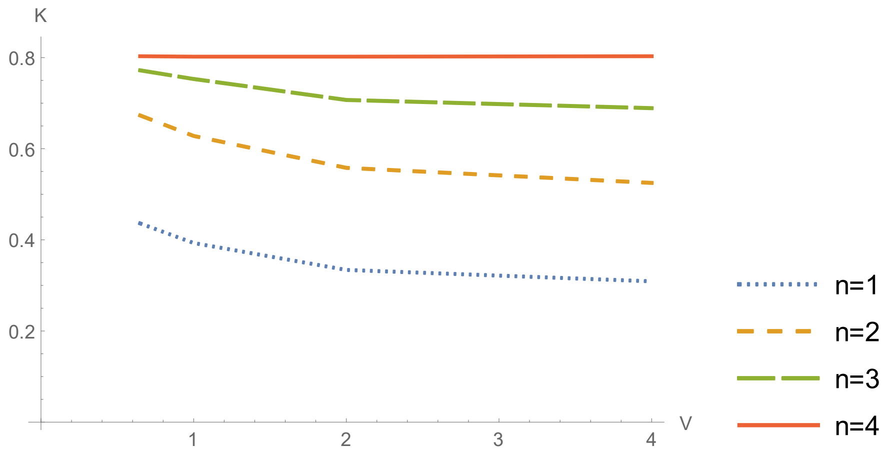

Figure 7. Dependencies of the availability coefficient on the coefficient of variation of *X* and *Y* for different numbers of repair facilities

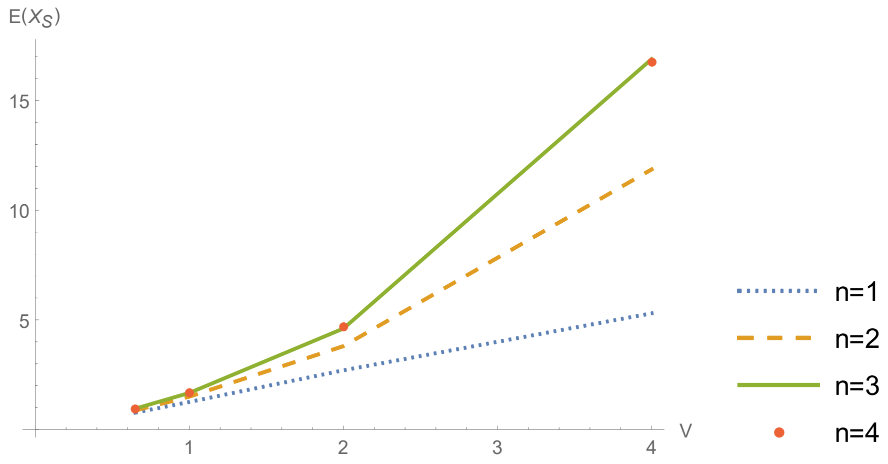

Figure 8. Dependencies of $E(X_S)$ on the coefficient of variation of X and Y for different numbers of repair facilities

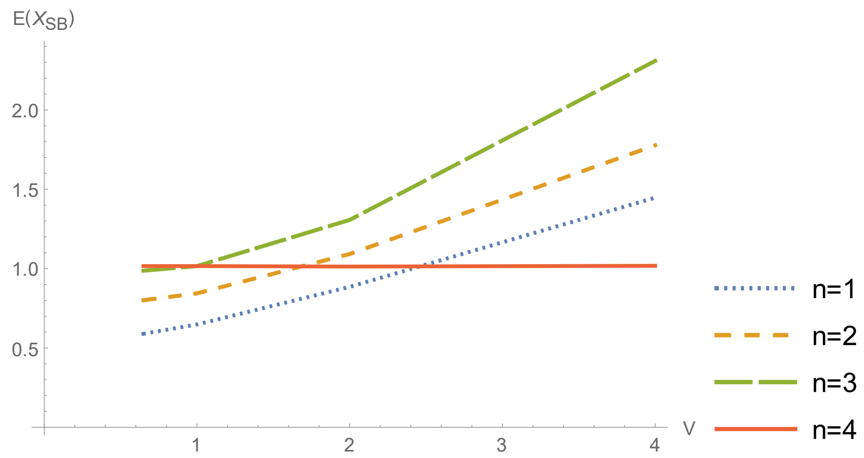

Figure 9. Dependencies of $E(X_{SB})$ on the coefficient of variation of X and Y for different numbers of repair facilities

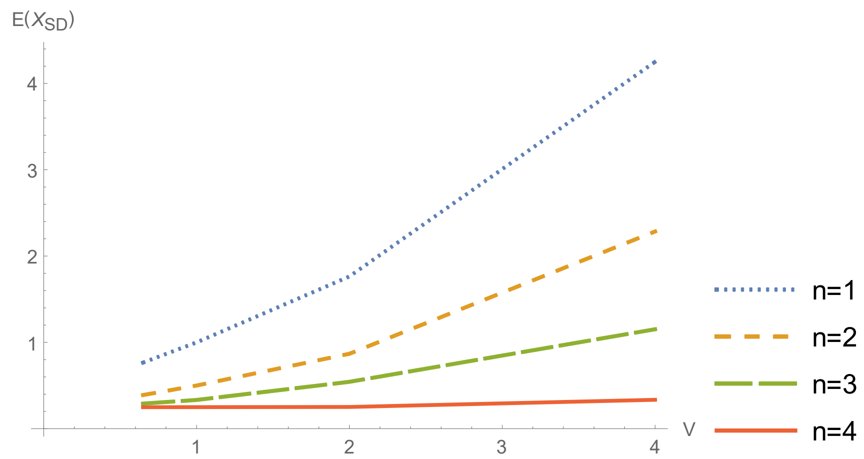

Figure 10. Dependencies of $E(X_{SD})$ on the coefficient of variation of X and Y for different numbers of repair facilities

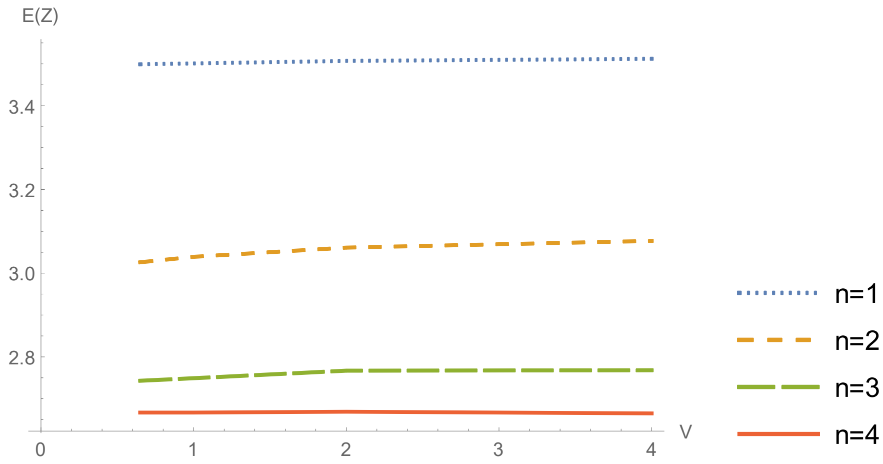

Figure 11. Dependencies of $E(Z)$ on the coefficient of variation of X and Y for different numbers of repair facilities

Analyzing the constructed graphs, we conclude that the system stationary availability coefficient (K) decreases with an increase in the coefficient of variation of X and Y. However, the value of K increases with an increase in the number of repair facilities. The average values of the random variables X_S, X_{SB}, and X_{SD} increase with an increase in V, and the increase in the number of repair facilities leads to an increase in $E(X_S)$ and $E(X_{SB})$ and a decrease in $E(X_{SD})$. The average number of failed units ($E(Z)$) practically does not depend on V and decreases with an increase in the number of repair facilities.

3.2. Changing the ratio $E(Y)/E(X)$

Graphs illustrating the dependencies of K, $E(X_S)$, $E(X_{SB})$, and $E(X_{SD})$ on the ratio $\rho=E(Y)/E(X)$ are presented in Figures 12 to 15 for m=4, n=2, $E(X)$=0.5 and varying values of the coefficient of variation of X and Y.

Analyzing the constructed graphs, we conclude that the system stationary availability coefficient (K) decreases with an increase in the ratio $\rho=E(Y)/E(X)$. The average values of the random variables X_S, X_{SB}, and X_{SD} increase with an increase in ρ. and the increase in the number of repair facilities leads to an increase in $E(X_S)$ and $E(X_{SB})$ and a decrease in $E(X_{SD})$. The nature of the dependencies of reliability indicators on the coefficients of variation X and Y, established in paragraph 3.1, remains unchanged.

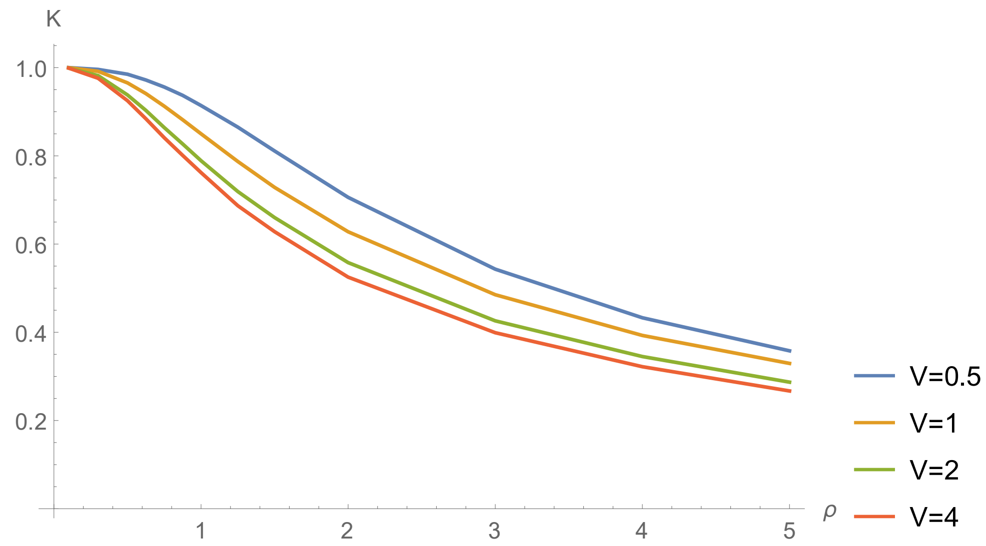

Figure 12. Dependencies of the availability coefficient on the ratio *E*(*Y*)/*E*(*X*) for different values of the coefficient of variation of *X* and *Y*

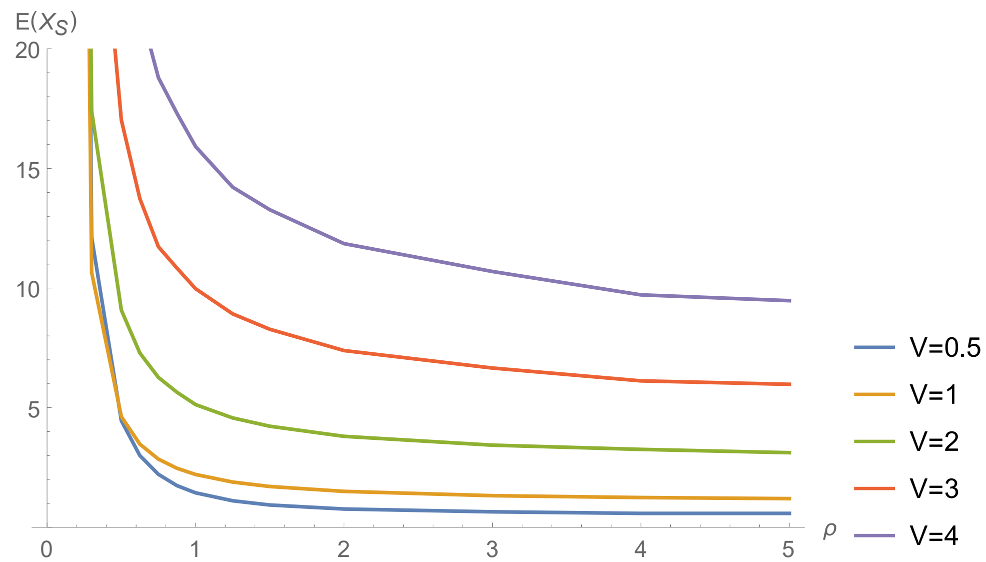

Figure 13. Dependencies of $E(X_S)$ on the ratio *E*(*Y*)/*E*(*X*) for different values of the coefficient of variation of *X* and *Y*

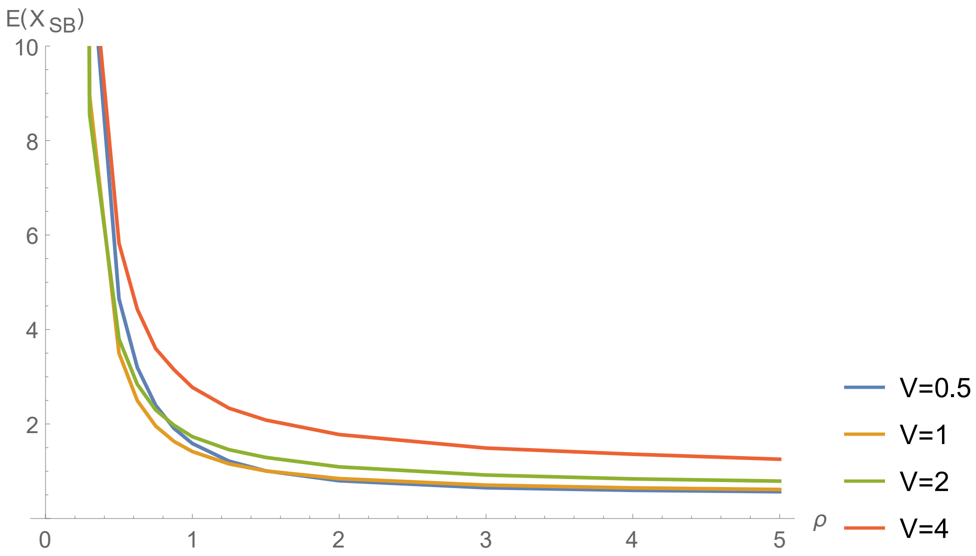

Figure 14. Dependencies of $E(X_{SB})$ on the ratio $E(Y)/E(X)$ for different values of the coefficient of variation of X and Y

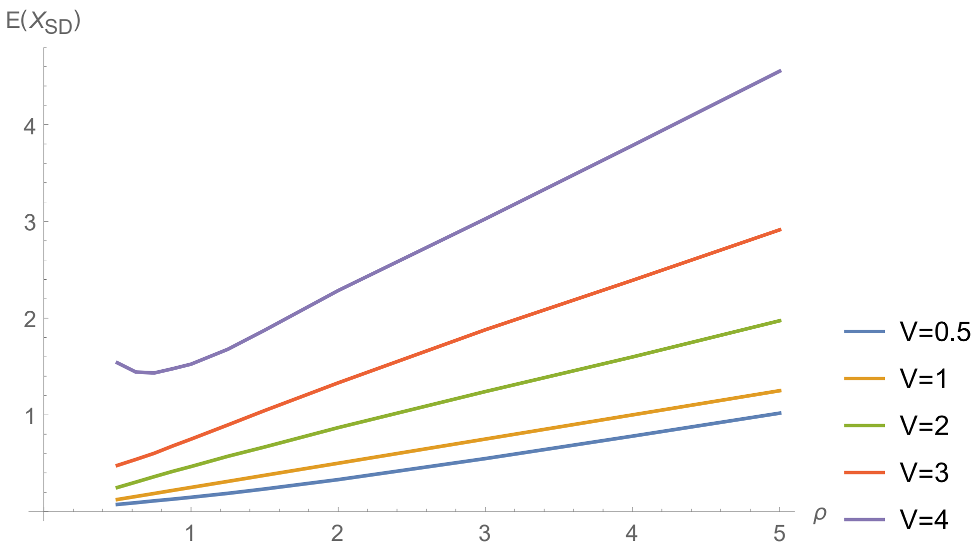

Figure 15. Dependencies of $E(X_{SD})$ on the ratio $E(Y)/E(X)$ for different values of the coefficient of variation of X and Y

4. Conclusion

A simulation model has been developed to assess the reliability of a repairable parallel system. The constructed simulation model provides a fundamental capability to predict the impact of each input parameter on the system's reliability indicators, considering arbitrary distributions of time to failure and repair time of units. The calculations showed good agreement between our simulation results and the results of the analytical model for the case of exponential distributions of time to failure and repair time of units.

The obtained results indicate a significant dependence of the system's reliability indicators on the values of the coefficient of variation of time to failure and repair time of units. Consequently, incorporating the practical application of analytical models becomes challenging, as these models were derived exclusively for exponential distributions.

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