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The Deductibility of the Aristotelian Modal Syllogism □IAI-4 from the Perspective of Natural Language Information Processing

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Abstract

Aristotelian modal syllogisms characterize the semantic and reasoning properties of Aristotelian quantifiers and modalities. In order to give a consistent explanation for Aristotelian modal syllogisms, this paper reveals the reduction between modal syllogisms on the basis of generalized quantifier theory, set theory, first-order logic, and modern modal logic. To be more specific, this paper firstly proves the validity of the modal syllogism □*IAI-4* based on the truth value definitions of modal categorical propositions, and secondly deduces the other 32 valid modal syllogisms from this syllogism based on related definitions, facts and inference rules. That is to say that there is reducibility among modal syllogisms with different figures and forms. This formal study not only conforms with the needs for formal transformation of all kinds of information in the era of artificial intelligence, but also provides other types of syllogisms with unified mathematical paradigm.

Key words: Aristotelian modal syllogisms; Aristotelian quantifiers; possible worlds; symmetry

1. Introduction

There are various types of syllogisms in natural language, such as Aristotelian syllogisms (Patzig, 1969; Long, 2023; Hui, 2023), Aristotelian modal syllogisms (Łukasiewicz, 1957; Cheng, 2023), and generalized syllogisms (Xiaojun and Baoxiang, 2021), and so on. This paper mainly discusses Aristotelian modal syllogisms.

Aristotelian modal syllogisms have been studied by many scholars, for example, Xiaojun (2020a, 2020b) and Cheng (2023) provide a formal study of Aristotelian modal syllogisms from the perspective of modern logic. Protin (2022) proposes a new deductive system to explain the validity of Aristotelian modal syllogisms. However, many scholars agree that the existing research can't give a consistent explanation of Aristotelian modal syllogisms.

This paper attempts to provide a consistent explanation of Aristotelian modal syllogisms. To this end, on the basis of relevant definitions, facts, and reasoning rules, this paper first proves the validity of the modal syllogism □*IAI-4*, and then deduces other 32 valid syllogisms from the modal syllogism □*IAI-4*.

2. Relevant Basic Knowledge

Aristotelian syllogisms characterize the semantic and reasoning properties of the following Aristotelian quantifiers: *all*, *no*, *some*, and *not all*. An Aristotelian modal syllogism can be obtained by adding at least one possible modality (◇) or necessary modality (□) to an Aristotelian syllogism. Aristotelian modal syllogisms describe the semantic and reasoning properties of Aristotelian quantifiers and modalities.

In this paper, *w*, *v* and *z* represents the lexical variables of categorical propositions, which are elements in the set *W*, *V* and *Z*, respectively. Let *m*, *n*, *s* and *t* be propositional variables. The symbol $=_{def}$ means that left can be defined by right.

Aristotelian syllogisms consists of the following: Propositions *A*, *E*, *I* and *O* (Cheng and Xiaojun, 2023). Proposition *A* represents 'all *w*s are *v*s', which is denoted as *all*(*w*, *v*). Proposition *E* represents 'no *w*s are *v*s', denoted as *no*(*w*, *v*). Proposition *I* 'some *w*s are *v*s', as *some*(w, v). The proposition 'not all ws are vs', as *not all*(w, v).

Definition 1 (truth value of categorical propositions):

(1.1) $all(w, v)=_{def}W\subseteq V$; (1.2) $some(w, v)=_{def}W\cap V\neq\varnothing$;

(1.3) $no(w, v)=_{def}W\cap V=\varnothing$; (1.4) $not\ all(w, v)=_{def}W\not\subseteq V$.

Definition 2 (truth values of modal propositions):

(2.1) $\Box m$ is true, when and only when m is true in any possible world ω;

(2.2) $\Diamond m$ is true, when and only when there is at least one possible world ω in which m is true.

Definition 3 (inner negation): $Q\neg(w, v)=_{def}Q(w, D-v)$.

Definition 4 (outer negation): $\neg Q(w, v)=_{def}$It is not that $Q(w, v)$.

According to modal logic (Chagrov and Zakharyaschev, 1997) and generalized quantifier theory (Peters and Westerståhl, 2006), the following facts are provable:

Fact 1 (a necessary proposition implies an assertion one):

(1.1) $\vdash\Box all(w, v)\rightarrow all(w, v)$, abbreviated as: $\Box A\rightarrow A$;

(1.2) $\vdash\Box no(w, v)\rightarrow no(w, v)$, abbreviated as: $\Box E\rightarrow E$;

(1.3) $\vdash\Box some(w, v)\rightarrow some(w, v)$, abbreviated as: $\Box I\rightarrow I$;

(1.4) $\vdash\Box not\ all(w, v)\rightarrow not\ all(w, v)$, abbreviated as: $\Box O\rightarrow O$.

Fact 2 (an universal proposition implies a particular one):

(2.1) $\vdash all(w, v)\rightarrow some(w, v)$, abbreviated as: $A\rightarrow I$;

(2.2) $\vdash no(w, v)\rightarrow not\ all(w, v)$, abbreviated as: $E\rightarrow O$;

(2.3) $\vdash\Box all(w, v)\rightarrow\Box some(w, v)$, abbreviated as: $\Box A\rightarrow\Box I$;

(2.4) $\vdash\Box no(w, v)\rightarrow\Box not\ all(w, v)$, abbreviated as: $\Box E\rightarrow\Box O$;

(2.5) $\vdash\Diamond all(w, v)\rightarrow\Diamond some(w, v)$, abbreviated as: $\Diamond A\rightarrow\Diamond I$;

(2.6) $\vdash\Diamond no(w, v)\rightarrow\Diamond not\ all(w, v)$, abbreviated as: $\Diamond E\rightarrow\Diamond O$.

Fact 3 (symmetry):

(3.1) $\vdash some(w, v)\leftrightarrow some(v, w)$; (3.2) $\vdash\Box some(w, v)\leftrightarrow\Box some(v, w)$;

(3.3) $\vdash\Diamond some(w, v)\leftrightarrow\Diamond some(v, w)$; (3.4) $\vdash no(w, v)\leftrightarrow no(v, w)$;

(3.5) $\vdash\Box no(w, v)\leftrightarrow\Box no(v, w)$; (3.6) $\vdash\Diamond no(w, v)\leftrightarrow\Diamond no(v, w)$.

Fact 4 (inner negation):

(4.1) $\vdash all(w, v)\leftrightarrow no\neg(w, v)$; (4.2) $\vdash no(w, v)\leftrightarrow all\neg(w, v)$;

(4.3) $\vdash some(w, v)\leftrightarrow not\ all\neg(w, v)$; (4.4) $\vdash not\ all(w, v)\leftrightarrow some\neg(w, v)$.

Fact 5 (outer negation):

(5.1) $\vdash\neg not\ all(w, v)\leftrightarrow all(w, v)$; (5.2) $\vdash\neg all(w, v)\leftrightarrow not\ all(w, v)$;

(5.3) $\vdash\neg no(w, v)\leftrightarrow some(w, v)$; (5.4) $\vdash\neg some(w, v)\leftrightarrow no(w, v)$.

Fact 6 (dual): (6.1) $\vdash\neg\Box Q(w, v)\leftrightarrow\Diamond\neg Q(w, v)$; (6.2) $\vdash\neg\Diamond Q(w, v)\leftrightarrow\Box\neg Q(w, v)$.

Rule 1 (subsequent weakening): If $\vdash(m\wedge n\rightarrow s)$ and $\vdash(s\rightarrow t)$, then $\vdash(m\wedge n\rightarrow t)$.

Rule 2 (anti-syllogism): If $\vdash(m\wedge n\rightarrow s)$, then $\vdash(\neg s\wedge m\rightarrow\neg n)$ or $\vdash(\neg s\wedge n\rightarrow\neg m)$.

3. The Validity of the Syllogism □*IAI-4*

In order to discuss the reducibility of modal syllogisms based on the syllogism □*IAI-4*, it is important to prove the validity of the syllogism □*IAI-4*.

Theorem 1 (□*IAI-4*): $\Box some(v, z)\wedge all(z, w)\rightarrow some(w, v)$ is valid.

Proof: The modal syllogism $\Box some(v, z)\wedge all(z, w)\rightarrow some(w, v)$ can be abbreviated as □*IAI-4*. Suppose that $\Box some(v, z)$ and $all(z, w)$ are true, then $some(v, z)$ is true in any possible world according to the Definition (2.1). Due to that any real world is a possible world, $some(v, z)$ is true in any real world, thus $V\cap Z\neq\varnothing$ is true in line with Definition (1.2). And $all(z, w)$ is true in real world, then $Z\subseteq W$ is true according to the Definition (1.1). Now it follows $W\cap V\neq\varnothing$ is true in any real world. Hence $some(w, v)$ is true according to the Definition (1.2). The above proves that the syllogism $\Box some(v, z)\wedge all(z, w)\rightarrow some(w, v)$ is valid.

4. The Other 32 Modal Syllogisms Derived from □*IAI-4*

According to Theorem 1, syllogism □*IAI-4* is valid. '(1) □*IAI-4*→□*IAI-3*' in Theorem 2 means that the syllogism □*IAI-3* can be derived from the syllogism □*IAI-4*. That is to say that there is reducibility between these two syllogisms. The others are similar.

Theorem 2: The following 32 valid modal syllogisms can be inferred from □*IAI-4*:

(1) □*IAI-4*→□*IAI-3*

(2) □*IAI-4*→*A*□*II-1*

(3) □*IAI-4*→*A*□*II-3*

(4) □*IAI-4*→*E*□*IO-4*

(5) □*IAI-4*→*AE*◇*E-4*

(6) □*IAI-4*→□*IAI-3*→*E*□*IO-2*

(7) □*IAI-4*→□*IAI-3*→*EA*◇*E-1*

(8) □*IAI-4*→□*IAI-3*→□*OAO-3*

(9) □*IAI-4*→*A*□*II-1*→*E*□*IO-3*

(10) □*IAI-4*→*A*□*II-1*→*AE*◇*E-2*

(11) □*IAI-4*→*A*□*II-1*→*E*□*IO-1*

(12) □*IAI-4*→*A*□*II-3*→*EA*◇*E-2*

(13) □*IAI-4*→*AE*◇*E-4*→*AE*◇*O-4*

(14) □*IAI-4*→□*IAI-3*→*E*□*IO-2*→*A*□*OO-2*

(15) □*IAI-4*→□*IAI-3*→*EA*◇*E-1*→*EA*◇*O-1*

(16) □*IAI-4*→□*IAI-3*→*EA*◇*E-1*→*AA*◇*A-1*

(17) □*IAI-4*→*A*□*II-1*→*AE*◇*E-2*→*AE*◇*O-2*

(18) □*IAI-4*→*A*□*II-3*→*EA*◇*E-2*→*EA*◇*O-2*

(19) □*IAI-4*→*AE*◇*E-4*→*AE*◇*O-4*→*E*□*AO-4*

(20) □*IAI-4*→*AE*◇*E-4*→*AE*◇*O-4*→□*AAI-4*

(21) □*IAI-4*→□*IAI-3*→*EA*◇*E-1*→*EA*◇*O-1*→*E*□*AO-2*

(22) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow \Box AAI\text{-}3$

(23) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow AA\Diamond I\text{-}1$

(24) $\Box IAI\text{-}4 \rightarrow A\Box II\text{-}1 \rightarrow AE\Diamond E\text{-}2 \rightarrow AE\Diamond O\text{-}2 \rightarrow E\Box AO\text{-}3$

(25) $\Box IAI\text{-}4 \rightarrow A\Box II\text{-}1 \rightarrow AE\Diamond E\text{-}2 \rightarrow AE\Diamond O\text{-}2 \rightarrow A\Box AI\text{-}1$

(26) $\Box IAI\text{-}4 \rightarrow A\Box II\text{-}3 \rightarrow EA\Diamond E\text{-}2 \rightarrow EA\Diamond O\text{-}2 \rightarrow E\Box AO\text{-}1$

(27) $\Box IAI\text{-}4 \rightarrow A\Box II\text{-}3 \rightarrow EA\Diamond E\text{-}2 \rightarrow EA\Diamond O\text{-}2 \rightarrow A\Box AI\text{-}3$

(28) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow E\Box AO\text{-}2 \rightarrow A\Box EO\text{-}2$

(29) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow \Box AAI\text{-}3 \rightarrow \Box EAO\text{-}3$

(30) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow AA\Diamond I\text{-}1 \rightarrow AA\Diamond I\text{-}4$

(31) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow E\Box AO\text{-}2 \rightarrow A\Box EO\text{-}2 \rightarrow A\Box EO\text{-}4$

(32) $\Box IAI\text{-}4 \rightarrow \Box IAI\text{-}3 \rightarrow EA\Diamond E\text{-}1 \rightarrow EA\Diamond O\text{-}1 \rightarrow \Box AAI\text{-}3 \rightarrow \Box EAO\text{-}3 \rightarrow \Box EAO\text{-}4$

Proof:

[1] $\vdash \Box some(v, z) \wedge all(z, w) \rightarrow some(w, v)$ (i.e. $\Box IAI\text{-}4$, Theorem 1)

[2] $\vdash \Box some(z, v) \wedge all(z, w) \rightarrow some(w, v)$ (i.e. $\Box IAI\text{-}3$, by [1] and Fact (3.2))

[3] $\vdash \Box some(v, z) \wedge all(z, w) \rightarrow some(v, w)$ (i.e. $A\Box II\text{-}1$, by [1] and Fact (3.1))

[4] $\vdash \Box some(z, v) \wedge all(z, w) \rightarrow some(v, w)$ (i.e. $A\Box II\text{-}3$, by [1], Fact (3.1) and Fact (3.2))

[5] $\vdash \neg some(w, v) \wedge \Box some(v, z) \rightarrow \neg all(z, w)$ (by [1] and Rule 2)

[6] $\vdash no(w, v) \wedge \Box some(v, z) \rightarrow not\ all(z, w)$ (i.e. $E\Box IO\text{-}4$, by [5], Fact (5.2) and Fact (5.4))

[7] $\vdash \neg some(w, v) \wedge all(z, w) \rightarrow \neg \Box some(v, z)$ (by [1] and Rule 2)

[8] $\vdash no(w, v) \wedge all(z, w) \rightarrow \Diamond \neg some(v, z)$ (by [7], Fact (5.4) and Fact (6.1))

[9] $\vdash no(w, v) \wedge all(z, w) \rightarrow \Diamond no(v, z)$ (i.e. $AE\Diamond E\text{-}4$, by [8] and Fact (5.4))

[10] $\vdash \neg some(w, v) \wedge \Box some(z, v) \rightarrow \neg all(z, w)$ (by [2] and Rule 2)

[11] $\vdash no(w, v) \wedge \Box some(z, v) \rightarrow not\ all(z, w)$ (i.e. $E\Box IO\text{-}2$, by [10], Fact (5.2) and Fact (5.4))

[12] $\vdash \neg some(w, v) \wedge all(z, w) \rightarrow \neg \Box some(z, v)$ (by [2] and Rule 2)

[13] $\vdash no(w, v) \wedge all(z, w) \rightarrow \Diamond \neg some(z, v)$ (by [12], Fact (5.4) and Fact (6.1))

[14] $\vdash no(w, v)\wedge all(z, w)\rightarrow\Diamond no(z, v)$ (i.e. *EA◇E-1*, by [13] and Fact (5.4))

[15] $\vdash\Box not\ all\neg(z, v)\wedge all(z, w)\rightarrow not\ all\neg(w, v)$ (by [2] and Fact (4.3))

[16] $\vdash\Box not\ all(z, D-v)\wedge all(z, w)\rightarrow not\ all(w, D-v)$ (i.e. *□OAO-3*, by [15] and Definition 3)

[17] $\vdash\neg some(v, w)\wedge\Box some(v, z)\rightarrow\neg all(z, w)$ (by [3] and Rule 2)

[18] $\vdash no(v, w)\wedge\Box some(v, z)\rightarrow not\ all(z, w)$ (i.e. *E□IO-3*, by [17], Fact (5.2) and Fact (5.4))

[19] $\vdash\neg some(v, w)\wedge all(z, w)\rightarrow\neg\Box some(v, z)$ (by [3] and Rule 2)

[20] $\vdash\neg some(v, w)\wedge all(z, w)\rightarrow\Diamond\neg some(v, z)$ (by [19] and Fact (6.1))

[21] $\vdash no(v, w)\wedge all(z, w)\rightarrow\Diamond no(v, z)$ (i.e. *AE◇E-2*, by [20] and Fact (5.4))

[22] $\vdash\Box some(v, z)\wedge no\neg(z, w)\rightarrow not\ all\neg(v, w)$ (by [3], Fact (4.1) and Fact (4.3))

[23] $\vdash\Box some(v, z)\wedge no(z, D-w)\rightarrow not\ all(v, D-w)$ (i.e. *E□IO-1*, by [22] and Definition 3)

[24] $\vdash\neg some(v, w)\wedge all(z, w)\rightarrow\neg\Box some(z, v)$ (by [4] and Rule 2)

[25] $\vdash\neg some(v, w)\wedge all(z, w)\rightarrow\Diamond\neg some(z, v)$ (by [24] and Fact (6.1))

[26] $\vdash no(v, w)\wedge all(z, w)\rightarrow\Diamond no(z, v)$ (i.e. *EA◇E-2*, by [25] and Fact (5.4))

[27] $\vdash no(w, v)\wedge all(z, w)\rightarrow\Diamond not\ all(v, z)$ (i.e. *AE◇O-4*, by [9], Fact (2.6) and Rule 1)

[28] $\vdash all\neg(w, v)\wedge\Box not\ all\neg(z, v)\rightarrow not\ all(z, w)$ (by [11], Fact (4.2) and Fact (4.3))

[29] $\vdash all(w, D-v)\wedge\Box not\ all(z, D-v)\rightarrow not\ all(z, w)$ (i.e. *A□OO-2*, by [28] and Definition 3)

[30] $\vdash no(w, v)\wedge all(z, w)\rightarrow\Diamond not\ all(z, v)$ (i.e. *EA◇O-1*, by [14], Fact (2.6) and Rule1)

[31] $\vdash all\neg(w, v)\wedge all(z, w)\rightarrow\Diamond all\neg(z, v)$ (by [14] and Fact (4.2))

[32] $\vdash all(w, D-v)\wedge all(z, w)\rightarrow\Diamond all(z, D-v)$ (i.e. *AA◇A-1*, by [31] and Definition 3)

[33] $\vdash no(v, w)\wedge all(z, w)\rightarrow\Diamond not\ all(v, z)$ (i.e. *AE◇O-2*, by [21], Fact (2.6) and Rule 1)

[34] $\vdash no(v, w)\wedge all(z, w)\rightarrow\Diamond not\ all(z, v)$ (i.e. *EA◇O-2*, by [26], Fact (2.6) and Rule 1)

[35] $\vdash\neg\Diamond not\ all(v, z)\wedge no(w, v)\rightarrow\neg all(z, w)$ (by [27] and Rule 2)

[36] $\vdash\Box\neg not\ all(v, z)\wedge no(w, v)\rightarrow\neg all(z, w)$ (by [35] and Fact (6.2))

[37] $\vdash\Box all(v, z)\wedge no(w, v)\rightarrow not\ all(z, w)$ (i.e. *E□AO-4*, by [36], Fact (5.1) and Fact (5.2))

[38] $\vdash\neg\Diamond not\ all(v, z)\wedge all(z, w)\rightarrow\neg no(w, v)$ (by [27] and Rule 2)

[39] $\vdash\Box\neg not\ all(v, z)\wedge all(z, w)\rightarrow\neg no(w, v)$ (by [38] and Fact (6.2))

[40] $\vdash\Box all(v, z)\wedge all(z, w)\rightarrow some(w, v)$ (i.e. $\Box AAI$-4, by [39], Fact (5.1) and Fact (5.3))

[41] $\vdash\neg\Diamond not\ all(z, v)\wedge no(w, v)\rightarrow\neg all(z, w)$ (by [30] and Rule 2)

[42] $\vdash\Box\neg not\ all(z, v)\wedge no(w, v)\rightarrow\neg all(z, w)$ (by [41] and Fact (6.2))

[43] $\vdash\Box all(z, v)\wedge no(w, v)\rightarrow not\ all(z, w)$ (i.e. $E\Box AO$-2, by [42], Fact (5.1) and Fact (5.2))

[44] $\vdash\neg\Diamond not\ all(z, v)\wedge all(z, w)\rightarrow\neg no(w, v)$ (by [30] and Rule 2)

[45] $\vdash\Box\neg not\ all(z, v)\wedge all(z, w)\rightarrow\neg no(w, v)$ (by [44] and Fact (6.2))

[46] $\vdash\Box all(z, v)\wedge all(z, w)\rightarrow some(w, v)$ (i.e. $\Box AAI$-3, by [45], Fact (5.1) and Fact (5.3))

[47] $\vdash all\neg(w, v)\wedge all(z, w)\rightarrow\Diamond some\neg(z, v)$ (by [30], Fact (4.2) and Fact (4.4))

[48] $\vdash all(w, D-v)\wedge all(z, w)\rightarrow\Diamond some(z, D-v)$ (i.e. $AA\Diamond I$-1, by [47] and Definition 3)

[49] $\vdash\neg\Diamond not\ all(v, z)\wedge no(v, w)\rightarrow\neg all(z, w)$ (by [33] and Rule 2)

[50] $\vdash\Box\neg not\ all(v, z)\wedge no(v, w)\rightarrow\neg all(z, w)$ (by [49] and Fact (6.2))

[51] $\vdash\Box all(v, z)\wedge no(v, w)\rightarrow not\ all(z, w)$ (i.e. $E\Box AO$-3, by [50], Fact (5.1) and Fact (5.2))

[52] $\vdash\neg\Diamond not\ all(v, z)\wedge all(z, w)\rightarrow\neg no(v, w)$ (by [33] and Rule 2)

[53] $\vdash\Box\neg not\ all(v, z)\wedge all(z, w)\rightarrow\neg no(v, w)$ (by [52] and Fact (6.2))

[54] $\vdash\Box all(v, z)\wedge all(z, w)\rightarrow some(v, w)$ (i.e. $A\Box AI$-1, by [53], Fact (5.1) and Fact (5.3))

[55] $\vdash\neg\Diamond not\ all(z, v)\wedge no(v, w)\rightarrow\neg all(z, w)$ (by [34] and Rule 2)

[56] $\vdash\Box\neg not\ all(z, v)\wedge no(v, w)\rightarrow\neg all(z, w)$ (by [55] and Fact (6.2))

[57] $\vdash\Box all(z, v)\wedge no(v, w)\rightarrow not\ all(z, w)$ (i.e. $E\Box AO$-1, by [56], Fact (5.1) and Fact (5.2))

[58] $\vdash\neg\Diamond not\ all(z, v)\wedge all(z, w)\rightarrow\neg no(v, w)$ (by [34] and Rule 2)

[59] $\vdash\Box\neg not\ all(z, v)\wedge all(z, w)\rightarrow\neg no(v, w)$ (by [58] and Fact (6.2))

[60] $\vdash\Box all(z, v)\wedge all(z, w)\rightarrow some(v, w)$ (i.e. $A\Box AI$-3, by [59], Fact (5.1) and Fact (5.3))

[61] $\vdash\Box no\neg(z, v)\wedge all\neg(w, v)\rightarrow not\ all(z, w)$ (by [43], Fact (4.1) and Fact (4.2))

[62] $\vdash\Box no(z, D-v)\wedge all(w, D-v)\rightarrow not\ all(z, w)$ (i.e. $A\Box EO$-2, by [61] and Definition 3)

[63] $\vdash\Box no\neg(z, v)\wedge all(z, w)\rightarrow not\ all\neg(w, v)$ (by [46], Fact (4.1) and Fact (4.3))

[64] $\vdash\Box no(z, D-v)\wedge all(z, w)\rightarrow not\ all(w, D-v)$ (i.e. $\Box EAO$-3, by [63] and Definition 3)

[65] $\vdash all(w, D-v)\wedge all(z, w)\rightarrow\Diamond some(D-v, z)$ (i.e. $AA\Diamond I$-4, by [48] and Fact (3.3))

[66] $\vdash\Box no(D-v, z)\wedge all(w, D-v)\rightarrow not\ all(z, w)$ (i.e. $A\Box EO$-4, by [62] and Fact (3.5))

[67] $\vdash\Box no(D-v, z)\wedge all(z, w)\rightarrow not\ all(w, D-v)$ (i.e. $\Box EAO$-4, by [64] and Fact (3.5))

At this point, the other 32 modal syllogisms have been deduced from the syllogism $\Box IAI$-4.

5. Conclusion

To sum up, in order to give a consistent explanation for Aristotelian modal syllogisms, this paper shows the reduction between modal syllogisms on the basis of generalized quantifier theory, set theory, first-order logic, and modern modal logic. To be more specific, this paper firstly proves the validity of the modal syllogism $\Box IAI$-4 based on the truth value definitions of modal categorical propositions, and secondly deduces the other 32 valid modal syllogisms from this syllogism based on related definitions, facts and inference rules. In fact, one can derive more valid syllogisms when he continues to derive by means of similar reasoning steps. In other words, there is reducibility among modal syllogisms with different figures and forms.

For future study, can we consider using this research method to discuss the validity and reducibility of other types of syllogisms, such as generalized syllogisms and generalized modal syllogisms?

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