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Knowledge Mining Based on the Valid Generalized Syllogism MMI-3 with the Quantifier *'Most'*

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Abstract

This paper firstly presents knowledge representations of generalized syllogisms, and then uses relevant facts and reasoning rules to conduct knowledge reasoning on the basis of the generalized syllogism *MMI-3* with the quantifier *'most'*. The main conclusion is that there are at least the other 25 valid generalized syllogisms that can be deduced from the validity of this syllogism. The paper achieves the initial goal of knowledge mining for this generalized syllogism logical fragment.

Keywords: generalized syllogisms; knowledge representation; knowledge reasoning; knowledge mining

1. Introduction

Syllogistic reasoning is a common form of reasoning in scientific language and natural language (Hui, 2023). Since Aristotle, scholars have studied various types of syllogisms. For example, Aristotelian syllogisms (Łukasiewicz, 1957; Moss, 2008; Yijiang, 2023), Aristotelian modal syllogisms (Thomson, 1997; Johnson, 2004; Malink, 2013; Long and Xiaojun, 2023), generalized syllogisms (Moss, 2010; Endrullis and Moss, 2015; Liheng, 2024), and generalized modal syllogisms (Jing and Xiaojun, 2023).

Generalized syllogisms commonly exist in everyday reasoning, and there are few studies on them by domestic and foreign scholars. Therefore, this paper focuses on their study. Generalized syllogism reasoning is not only a common form of reasoning in human thinking, but also one of the important contents of knowledge mining in artificial intelligence.

2. Knowledge Representation for Generalized Syllogisms

Let *g*, *c* and *p* be lexical variables, and *G*, *C* and *P* be the set of *g*, *c*, and *p*, respectively, and *U* be the domain of lexical variables. Let θ, δ, π, and ψ be well-formed formulas (i.e. wff). Let *Q* be a quantifier, and $\neg Q$ and $Q\neg$ be its outer and inner negative quantifiers, respectively. '$|G|$'as usual indicates the cardinality of the set *G*. '$\vdash\theta$' represents that the wff θ is provable, and '$\vDash\theta$' that the wff θ is valid. '$\theta =_{def} \delta$' means that one can use δ to define θ. The other operators (such as $\neg$, $\rightarrow$, $\wedge$, $\leftrightarrow$) are common symbols in mathematical logic (Hamilton, 1978).

This paper studies generalized syllogisms involving the quantifiers *all* and *most*, as well as their three kinds of negative (i.e. inner, outer and dual) quantifiers, that is, *all*, *some*, *no*, *not all, most, at least half of, fewer than half of* and *at most half of.* Thus, the propositions we consider in these syllogisms are as follows: *all(g, c)*, *some(g, c)*, *no(g, c)*, *not all(g, c)*, *most(g, c)*, *at least half of(g, c)*, *at most half of(g, c)* and *fewer than half of(g, c)*. They are abbreviated as Proposition *A*, *I*, *E*, *O*, *M*, *S*, *H*, and *F,* respectively. A non-trivial generalized syllogism includes at least one of the last four. For example, the third figure syllogism *most(c,* *p)* $\wedge$ *most(c, g)* $\to$ *some*(*g, p*) is a non-trivial generalized syllogism, which is abbreviated as *MMI-3.* A natural language example of *MMI-3* is as follows:

Major premise: Most flowers wither in winter.

Minor premise: Most flowers bear fruits.

Conclusion: Some things that bear fruits wither in winter.

Let *c* be a lexical variable that represents flowers, *p* be a lexical variable denoting things that wither in winter, and *g* be a lexical variable denoting things that bear fruits. Then the above example can be represented as *most(c, p)*$\wedge$*most(c, g)*$\to$*some*(*g, p*).

3. Generalized Syllogism System with the Quantifier '*Most*'

To formalize a logical system, it is necessary to provide its syntax and semantics.

3.1 Syntax

The syntax of a logical system includes initial symbols, formation rules, basic axioms and deductive rules.

3.1.1 Primitive Symbols

(1) lexical variables: *g*, *c*, *p*

(2) quantifiers: *all, most*

(3) operators: $\neg$, $\to$

(4) brackets: (,)

3.1.2 Formation Rules

(1) If *Q* is a quantifier, *g* and *c* are lexical variables, then *Q(g, c)* is a wff.

(2) If θ is a wff, then so is $\neg\theta$.

(3) If θ and δ are wffs, then so is $\theta\to\delta$.

Only the formulas constructed in line with the above rules are wffs.

3.1.3 Basic Axioms

A1: all tautologies in propositional logic are axioms.

A2: *most(c, p)*$\wedge$*most(c, g)*$\rightarrow$*some*(*g, p*). (that is, the syllogism *MMI-3*).

3.1.4 Deductive Rules

Rule 1 (subsequent weakening): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ and $\vdash(\delta\rightarrow\gamma)$ infer $\vdash(\pi\wedge\psi\rightarrow\gamma)$.

Rule 2 (anti-syllogism): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ infer $\vdash(\neg\delta\wedge\pi\rightarrow\neg\psi)$.

Rule 3 (anti-syllogism): From $\vdash(\pi\wedge\psi\rightarrow\delta)$ infer $\vdash(\neg\delta\wedge\psi\rightarrow\neg\pi)$.

The above rules are basic rules of first-order logic (Hamilton, 1978).

3.2 Semantics

Let $\mathfrak{M}=(U, \Delta)$ be a model, in which $U\neq\varnothing$, and Δ be an interpretation, where

$\Delta(g)=G$, $G\subseteq U$ and $G\neq\varnothing$.

$\Delta(c)=C$, $C\subseteq U$ and $C\neq\varnothing$.

$\Delta(p)=P$, $P\subseteq U$ and $P\neq\varnothing$.

$\Delta(u-d)= U-\Delta(d)$, in which *d is g, c or p.*

If a wff θ is true in $\mathfrak{M}$ under an interpretation Δ, one can say that $\mathfrak{M}, \Delta\vDash\theta$.

(S1) $\mathfrak{M}, \Delta\vDash$*all(g, c),* if and only if, $\Delta(g)\subseteq\Delta(c)$, that is $G\subseteq C$.

(S2) $\mathfrak{M}, \Delta\vDash$*not all(g, c),* if and only if, $G\not\subseteq C$.

(S3) $\mathfrak{M}, \Delta\vDash$*no(g, c),* if and only if, $G\cap C=\varnothing$.

(S4) $\mathfrak{M}, \Delta\vDash$*some(g, c),* if and only if, $G\cap C\neq\varnothing$.

(S5) $\mathfrak{M}, \Delta\vDash$*most(g, c),* if and only if, $|G\cap C|>0.5|G|$.

(S6) $\mathfrak{M}, \Delta\vDash$*at most half of(g, c),* if and only if, $|G\cap C|\leq0.5|G|$.

(S7) $\mathfrak{M}, \Delta\vDash$*fewer than half of(g, c),* if and only if, $|G\cap C|<0.5|G|$.

(S8) $\mathfrak{M}, \Delta\vDash$ *at least half of(g, c),* if and only if, $|G\cap C|\geq0.5|G|$.

If θ is true under all interpretations in a model $\mathfrak{M}$, one can say that θ is valid in $\mathfrak{M}$, that is $\mathfrak{M}\vDash\theta$. If θ is valid in all models, one can say that θ is valid, $\vDash\theta$.

3.3 Relevant Definitions

D1 (conjunction): $(\theta\wedge\delta)=_{def}\neg(\theta\rightarrow\neg\delta)$

D2 (biconditional): $(\theta\leftrightarrow\delta) =_{def} (\theta\rightarrow\delta)\wedge(\delta\rightarrow\theta)$

D3 (inner negation): $Q\neg(g, c)=_{def} Q(g, u-c)$. More specifically,

(D3.1) *no(g, c)*$=_{def}$ *all¬(g, c)*;

(D3.2) *all(g, c)*$=_{def}$ *no¬(g, c)*;

(D3.3) *some(g, c)*$=_{def}$ *not all¬(g, c)*;

(D3.4) *not all(g, c)*$=_{def}$ *some¬(g, c)*;

(D3.5) *most(g, c)*$=_{def}$ *fewer than half of the¬(g, c)*;

(D3.6) *fewer than half of(g, c)* $=_{def}$ *most¬(g, c)*;

(D3.7) *at least half of the(g, c)*$=_{def}$ *at most half of the¬(g, c)*;

(D3.8) *at most half of the(g, c)*$=_{def}$ *at least half of the¬(g, c)*.

D4 (outer negation): (¬*Q*)*(g, c)*$=_{def}$ It is not that *Q(g, c)*. To be more specific,

(D4.1) ¬*all(g, c)*$=_{def}$ *not all(g, c)*;

(D4.2) ¬*not all(g, c)*$=_{def}$ *all(g, c)*;

(D4.3) ¬*some(g, c)*$=_{def}$ *no(g, c)*;

(D4.4) ¬*no(g, c)*$=_{def}$ *some(g, c)*;

(D4.5) ¬*most(g, c)*$=_{def}$ *at most half of the(g, c)*;

(D4.6) ¬*at most half of the(g, c)*$=_{def}$ *most(g, c)*.

(D4.7) ¬*fewer than half of(g, c)* $=_{def}$ *at least half of the(g, c)*;

(D4.8) ¬*at least half of the(g, c)*$=_{def}$ *fewer than half of(g, c)*;

3.4 Relevant Facts

Fact 1: $\vdash$*some(g, c)*$\leftrightarrow$*some(c, g)*;

Fact 2: $\vdash$*no(g, c)*$\leftrightarrow$*no(c, g)*;

Fact 3: $\vdash$*all(g, c)*$\rightarrow$*some(g, c)*;

Fact 4: $\vdash$*no(g, c)*$\rightarrow$*not all(g, c)*;

Fact 5: $\vdash$*all(g, c)*$\rightarrow$*most(g, c)*;

Fact 6: $\vdash$*all(g, c)*$\rightarrow$*at least half of the(g, c)*;

Fact 7: $\vdash$*most(g, c)*$\rightarrow$*some(g, c)*;

Fact 8: $\vdash$*at least half of the(g, c)*$\rightarrow$*some(g, c)*;

Fact 9: $\vdash$*fewer than half of the(g, c)*$\rightarrow$*not all(g, c)*;

Fact 10: $\vdash$*at most half of the(g, c)*$\rightarrow$*not all(g, c)*.

The above facts can be easily proven by generalized quantifier theory (Peters and Westerståhl, 2006).

4. Knowledge Reasoning for Generalized Syllogisms

The following theorem 1 proves the generalized syllogism *MMI-3* is valid. Theorem 2 illustrates that generalized syllogisms after implication symbol (i.e.$\rightarrow$) can be deduced from *MMI-3*.

Theorem 1(*MMI-3*): The generalized syllogism *most(c, p)*$\wedge$*most(c, g)*$\rightarrow$*some*(*g, p*) is valid.

Proof: For any $\mathfrak{M}$, Δ, suppose that *most(c, p)* and *most(g, c)* are true, then $|C\cap P|>0.5|C|$ and $|C\cap G|>0.5|C|$ according to (S5) in the above semantic part. Hence it can be concluded that $G\cap P\neq\emptyset$. If not, then $G\cap P=\emptyset$, while $|C\cap P|>0.5|C|$, It follows that $|C\cap G|\leq0.5|C|$, which contradicts $|C\cap G|>0.5|C|$. Therefore, $G\cap P\neq\emptyset$, so *some*(*g, p*) is true. It means that *MMI-3* is valid.

Theorem 2: There are at least the following 25 generalized syllogisms inferred from *MMI-3*:

(1) *MMI-3→EMH-2*

(2) *MMI-3→EMH-2→EMO-2*

(3) *MI-3→EMH-2→EMO-2→EMO-1*

(4) *MMI-3→EMH-1*

(5) *MMI-3→FMO-3*

(6) *MMI-3→EMH-2→AFH-2*

(7) *MMI-3→EMH-2→AFH-2→AFO-2*

(8) *MMI-3→EMH-2→EMO-2→MAI-3*

(9) *MMI-3→EMH-2→EMO-2→MAI-3→AMI-3*

(10) *MMI-3→EMH-2→EMO-2→EAH-1*

(11) *MMI-3→EMH-2→EMO-2→EAH-1→EAH-2*

(12) *MMI-3→EMH-2→EMO-2→EAH-1→EAH-2→EAO-2*

(13) *MMI-3→EMH-2→EMO-2→EAH-1→EAH-2→EAO-2→EAO-1*

(14) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1*

(15) *MMI-3→EMH-2→EMO-2→EMO-1→MAI-4*

(16) *MMI-3→EMH-1→ AMS-1*

(17) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1→AEH-2*

(18) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1→AEH-2→AEO-2*

(19) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1→AEH-2→AEO-2→AEO-4*

(20) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1→EMO-3*

(21) *MMI-3→EMH-2→EMO-2→EMO-1→AMI-1→EMO-3→EMO-4*

(22) *MMI-3→EMH-2→AFH-2→AFO-2→FAO-3*

(23) *MMI-3→EMH-2→AFH-2→AFO-2→AAS-1*

(24) *MMI-3→EMH-2→AFH-2→AFO-2→AAS-1→AAI-1*

(25) *MMI-3→EMH-2→AFH-2→AFO-2→AAS-1→AAI-1→AAI-4*

Proof:

[1] ⊢*most(c, p)*∧*most(c, g)*→*some*(*g, p*) (i.e. *MMI-3*, Axiom A2)

[2] ⊢*most(c, p)*∧¬*some*(*g, p*)→ ¬*most(c, g)* (by [1] and Rule 2)

[3] ⊢*most(c, p)*∧*no*(*g, p*)→ *at most half of(c, g)* (i.e. *EMH-2*, by [2] and (D4.3) and (D4.5))

[4] ⊢*most(c, p)*∧*no*(*g, p*)→ *not all(c, g)* (i.e. *EMO-2*, by [3] and Fact 10)

[5] ⊢*most(c, p)*∧*no*(*p, g*)→ *not all(c, g)* (i.e. *EMO-1*, by [4] and Fact 1)

[6] ⊢¬*some*(*g, p*)∧*most(c, g)*→ ¬*most(c, p)* (by [1] and Rule 3)

[7] ⊢*no*(*g, p*)∧*most(c, g)*→ *at most half of(c, p)* (i.e. *EMH-1*, by [2] and (D4.3) and (D4.5))

[8] ⊢*few than half of*¬*(c, p)*∧*most(c, g)*→*not all*¬(*g, p*) (by [1] and (D3.5) and (D3.4))

[9] ⊢*few than half of(c, u*−*p*)∧*most(c, g)*→*not all*(*g, u*−*p*) (i.e. *FMO-3*, by [8] and (D3))

[10] ⊢*few than half of*¬*(c, p)*∧ *all*¬(*g, p*) → *at most half of(c, g)* (by [3] and (D3.5) and (D3.1))

[11] ⊢*few than half of(c, u*−*p*)∧*all*(*g, u*−*p*)→ *at most half of(c, g)* (i.e. *AFH-2*, by [10] and (D3))

[12] ⊢*few than half of(c, u*−*p*)∧*all*(*g, u*−*p*)→ *not all(c, g)* (i.e. *AFO-2*, by [11] and Fact 10)

[13] ⊢*most(c, p)*∧¬*not all(c, g)*→ ¬*no*(*g, p*) (by [4] and Rule 3)

[14] ⊢*most(c, p)*∧*all(c, g)*→ *some*(*g, p*) (i.e. *MAI-3*, by [13], (D4.2) and (D4.4))

[15] ⊢*most(c, p)*∧*all(c, g)*→ *some*(*p, g*) (i.e. *AMI-3*, by [14] and Fact 1)

[16] ⊢¬*not all(c, g)*∧*no*(*g, p*)→ ¬*most(c, p)* (by [4] and Rule 2)

[17] ⊢*all(c, g)*∧*no*(*g, p*)→ *at most half of(c, p)* (i.e. *EAH-1*, by [16], (D4.2) and (D4.5))

[18] ⊢*all(c, g)*∧*no*(*p, g*)→ *at most half of(c, p)* (i.e. *EAH-2*, by [17] and Fact 2)

[19] ⊢*all(c, g)*∧*no*(*p, g*)→ *not all(c, p)* (i.e. *EAO-2*, by [18] and Fact 10)

[20] ⊢*all(c, g)*∧ *no*(*g, p*)→ *not all(c, p)* (i.e. *EAO-1*, by [19] and Fact 2)

[21] ⊢*most(c, p)*∧ *all*¬(*p, g*)→ *some*¬*(c, g)* (by [5], (D3.2) and (D3.4))

[22] ⊢*most(c, p)*∧ *all*(*p, u*−*g*)→ *some(c, u*−*g*) (i.e. *AMI-1*, by [21] and (D3))

[23] $\vdash most(c, p) \wedge all(p, u-g) \rightarrow some(u-g, c)$ (i.e. *MAI-4*, by [22], by Fact 1)

[24] $\vdash all\neg(g, p) \wedge most(c, g) \rightarrow at\ least\ half\ of\neg(c, p)$ (by [7], (D3.1) and (D3.8))

[25] $\vdash all(g, u-p) \wedge most(c, g) \rightarrow at\ least\ half\ of(c, u-p)$ (i.e. *AMS-1*, by [24] and (D3))

[26] $\vdash \neg some(c, u-g) \wedge all(p, u-g) \rightarrow \neg most(c, p)$ (by [22] and Rule 2)

[27] $\vdash no(c, u-g) \wedge all(p, u-g) \rightarrow at\ most\ half\ of(c, p)$ (i.e. *AEH-2*, by [26], (D4.3) and (D4.5))

[28] $\vdash no(c, u-g) \wedge all(p, u-g) \rightarrow not\ all(c, p)$ (i.e. *AEO-2*, by [27] and Fact 10)

[29] $\vdash no(u-g, c) \wedge all(p, u-g) \rightarrow not\ all(c, p)$ (i.e. *AEO-4*, by [28] and Fact 2)

[30] $\vdash most(c, p) \wedge \neg some(c, u-g) \rightarrow \neg all(p, u-g)$ (by [22] and Rule 3)

[31] $\vdash most(c, p) \wedge no(c, u-g) \rightarrow not\ all(p, u-g)$ (i.e. *EMO-3*, by [30], (D4.3) and (D4.1))

[32] $\vdash most(c, p) \wedge no(u-g, c) \rightarrow not\ all(p, u-g)$ (i.e. *EMO-4*, by [31] and Fact 2)

[33] $\vdash few\ than\ half\ of(c, u-p) \wedge \neg not\ all(c, g) \rightarrow \neg all(g, u-p)$ (by [12] and Rule 2)

[34] $\vdash few\ than\ half\ of(c, u-p) \wedge all(c, g) \rightarrow not\ all(g, u-p)$ (i.e.*FAO-3*, by [33], (D4.2)and (D4.1))

[35] $\vdash \neg not\ all(c, g) \wedge all(g, u-p) \rightarrow \neg few\ than\ half\ of(c, u-p)$ (by [12] and Rule 3)

[36] $\vdash all(c, g) \wedge all(g, u-p) \rightarrow at\ least\ half\ of(c, u-p)$ (i.e. *AAS-1*, by [35], (D4.2) and (D4.7))

[37] $\vdash all(c, g) \wedge all(g, u-p) \rightarrow some\ (c, u-p)$ (i.e. *AAI-1*, by [36] and Fact 8)

[38] $\vdash all(c, g) \wedge all(g, u-p) \rightarrow some\ (u-p, c)$ (i.e. *AAI-4*, by [37] and Fact 1)

If one continues with similar deductions, more valid generalized syllogisms can be deduced from the generalized syllogism *MMI-3*.

5. Conclusion and Future Work

This paper firstly presents knowledge representations of generalized syllogisms, and then uses relevant facts and reasoning rules to conduct knowledge reasoning on the basis of the generalized syllogism *MMI-3* with the quantifier *'most'*. The main conclusion is that there are at least the other 25 valid generalized syllogisms that can be derived from the validity of this syllogism. The paper achieves the initial goal of knowledge mining for this generalized syllogism logical fragment.

There are many generalized syllogisms in natural language. Which syllogisms are valid? What are the correlations between these valid syllogisms, and these questions need further discussion.

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