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Natural Language Information Processing Based on the Valid Traditional Syllogisms EIO-4

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Abstract:

Firstly this paper proves the validity of traditional syllogism *EIO-4*, and then makes full use of relevant definitions, facts, and some inference rules. On the basis of 34 reasoning steps, the other 23 syllogisms can be inferred from the syllogism *EIO-4*. This is because Aristotelian quantifiers (that is, *all*, *no*, *some*, and *not all*) can be mutually defined. Thus, a minimalist formal axiomatic system can easily be constructed for traditional syllogistic logic. This formal research method is not only beneficial for the study of other types of syllogisms, but also for better knowledge mining and thus for better understanding of natural language.

Keywords: reducible relation; traditional syllogism; Aristotelian quantifier; symmetry

1. Introduction

It is common knowledge that syllogistic reasoning is indispensable in daily life and scientific research [1-4]. As a matter of fact, only 24 are valid out of 256 traditional syllogisms [5-6]. Łukasiewicz derived the remaining 22 valid traditional syllogisms on the basis of *AAA-1* and *AII-3* [2]. Xiaojun and Sheng used two traditional syllogisms (i.e. *AAA-1* and *EAE-1*) as the basic axioms to derive 22 valid traditional syllogisms [5].

In recent years, there have been some breakthroughs in the study of the reducible relations between/among traditional syllogisms. For example, Cheng used the valid syllogism *IAI-3* as the basic axiom to infer the other 23 syllogisms [7]. Long did the same work on the basis of the syllogism *AEE-4* [8].

Inspired by previous work, from the perspective of natural language information processing, this paper only takes the valid traditional syllogism *EIO-4* as a basic axiom to study the reducible relations between/among valid syllogisms. One can make full use of propositional logic [9], set theory [10], and generalized quantifier theory [11-12] to derive 23 remaining valid traditional syllogisms.

2. Knowledge Representation of Traditional Syllogisms

In the paper, Q represents a quantifier. $Q\neg$ and $\neg Q$ denote the inner and outer negation of Q, respectively. And d, n and z represent lexical variables. The sets composed of d, n and z are respectively D, N, and Z. And U stands for the domain of d, n and z. α, β, γ and δ are well-formed formulas (abbreviated as wff). '$\vdash\alpha$' denotes that α is provable.

A traditional syllogism is composed of three categorical propositions which have four types: 'all *d*s are *z*', 'no *d*s are *z*', 'some *d*s are *z* ', and 'not all *d*s are *z*'. They are respectively called Proposition *A*, *E*, *I* and *O*. From the mathematical structuralist perspective [13], the four propositions can be respectively expressed as the following: *all(d, z)*, *no(d, z)*, *some(d, z)*, and *not all(d, z)*.

The figures of traditional syllogisms are defined as usual. For example, the fourth figure syllogism *EIO-4* denotes 'no *d*s are *z*, and some *d*s are *z*, so not all *d*s are *z*, which can be symbolized as $no(z, n)\wedge some(n, d)\rightarrow not\ all(d, z)$.

3. Formal System of Traditional Syllogistic

This system is composed of the following: initial symbols, facts, definitions, axioms, formation and deductive rules.

3.1 Initial Symbols

(3.1.1) lexical variables: d, n, z

(3.1.2) quantifier: *some*

(3.1.3) unary negative connective: $\neg$

(3.1.4) binary implication connective: $\rightarrow$

(3.1.5) brackets: (,)

3.2 Formation Rules

(3.2.1) If Q is a quantifier, d and z are lexical variables, then $Q(d, z)$ is a wff.

(3.2.2) If α is a wff, then so is $\neg\alpha$.

(3.2.3) If α and β are wffs, then so is $\alpha\rightarrow\beta$.

(3.2.4) Only the formula obtained from the above is a wff.

3.3 Basic Axioms

(3.3.1) If α is a valid formula in first-order logic, then $\vdash\alpha$.

(3.3.2) $\vdash no(z, n)\wedge some(n, d)\rightarrow not\ all(d, z)$ (i.e. the syllogism *EIO-4*).

3.4 Deductive Rules

Rule 1: If $\vdash(\alpha\wedge\beta\rightarrow\gamma)$ and $\vdash(\gamma\rightarrow\delta)$, then $\vdash(\alpha\wedge\beta\rightarrow\delta)$.

Rule 2: If $\vdash(\alpha\wedge\beta\rightarrow\gamma)$, then $\vdash(\neg\gamma\wedge\alpha\rightarrow\neg\beta)$.

3.5 Relevant Definitions

(3.5.1) $(\alpha\wedge\beta)=_{\text{def}}\neg(\alpha\rightarrow\neg\beta)$;

(3.5.2) $(\alpha\leftrightarrow\beta)=_{\text{def}}(\alpha\rightarrow\beta)\wedge(\beta\rightarrow\alpha)$;

(3.5.3) $(Q\neg)(d, z)=_{\text{def}} Q(d, U-d)$;

(3.5.4) $(\neg Q)(d, z)=_{\text{def}}$ It is not that $Q(d, z)$;

(3.5.5) $all(d, z)=_{\text{def}} D\subseteq Z$ is true;

(3.5.6) $some(d, z)=_{\text{def}} D\cap Z\neq\varnothing$ is true;

(3.5.7) $no(d, z) =_{def} D \cap Z = \varnothing$ is true;

(3.5.8) $not\ all(d, z) =_{def} D \not\subseteq Z$ is true.

3.6 Relevant Facts

The following facts can be easily proved in generalized quantifier theory [11-12].

Fact 1 (inner negation):

(1.1) $\vdash some(d, z) \leftrightarrow not\ all\neg(d, z)$; (1.2) $\vdash not\ all(d, z) \leftrightarrow some\neg(d, z)$;

(1.3) $\vdash all(d, z) \leftrightarrow no\neg(d, z)$; (1.4) $\vdash no(d, z) \leftrightarrow all\neg(d, z)$.

Fact 2 (outer negation):

(2.1) $\vdash \neg some(d, z) \leftrightarrow no(d, z)$; (2.2) $\vdash \neg no(d, z) \leftrightarrow some(d, z)$;

(2.3) $\vdash \neg not\ all(d, z) \leftrightarrow all(d, z)$; (2.4) $\vdash \neg all(d, z) \leftrightarrow not\ all(d, z)$.

Fact 3 (symmetry):

(3.1) $\vdash some(d, z) \leftrightarrow some(z, d)$; (3.2) $\vdash no(d, z) \leftrightarrow no(z, d)$.

Fact 4 (assertoric subalternations):

(4.1) $\vdash all(d, z) \rightarrow some(d, z)$; (4.2) $\vdash no(d, z) \rightarrow not\ all(d, z)$.

4. Knowledge Reasoning Based on the Syllogism EIO-4

The proof of the validity of the syllogism *EIO-4* has been given in Theorem 1. Theorem 2 reveals the reducible relationship between syllogisms with different figures and forms. That is to say that the validity of one syllogism can be deduced from that of another syllogism. For example, '(2.1) *EIO-4*→*EIO-3*' says that the validity of *EIO-3* can be deduced from that of *EIO-4*.

Theorem 1 (*EIO-4*): The syllogism $no(z, n) \wedge some(n, d) \rightarrow not\ all(d, z)$ is valid.

Proof: Assuming that $no(z, n)$ and $some(n, d)$ are true, it follows that $Z \cap N = \varnothing$ and $N \cap D \neq \varnothing$ are true in terms of Definition (3.5.7) and (3.5.6). Then $D \not\subseteq Z$ is true. This can be proven by reductio ad absurdum. Assume that $D \not\subseteq Z$ is not true. That is, $D \subseteq Z$ is true, and $Z \cap N = \varnothing$ has been proven to be true. Thus, it follows that $N \cap D = \varnothing$ is true, which contradicts $N \cap D \neq \varnothing$. So $D \subseteq Z$ is not true. This means $D \not\subseteq Z$ is true. Then in accordance with Definition (3.5.8), *some(d,* *z)* can be obtained, just as required.

Theorem 2: The validity of the remaining 23 valid syllogisms can be deduced from that of the syllogism *EIO-4*:

(2.1) *EIO-4→EIO-3*

(2.2) *EIO-4→EIO-2*

(2.3) *EIO-4→EIO-3→EIO-1*

(2.4) *EIO-4→AEE-4*

(2.5) *EIO-4→AEE-4→AEE-2*

(2.6) *EIO-4→AEE-4→EAE-1*

(2.7) *EIO-4→AEE-4→EAE-1→EAE-2*

(2.8) *EIO-4→AEE-4→AEO-4*

(2.9) *EIO-4→AEE-4→AEE-2→AEO-2*

(2.10) *EIO-4→AEE-4→EAE-1→EAO-1*

(2.11) *EIO-4→AEE-4→EAE-1→EAE-2→EAO-2*

(2.12) *EIO-4→EIO-3→AII-3*

(2.13) *EIO-4→EIO-3→AII-3→AII-1*

(2.14) *EIO-4→EIO-3→AII-3→IAI-3*

(2.15) *EIO-4→EIO-3→AII-3→IAI-3→IAI-4*

(2.16) *EIO-4→EIO-2→AOO-2*

(2.17) *EIO-4→AEE-4→EAE-1→AAA-1*

(2.18) *EIO-4→AEE-4→EAE-1→AAA-1→AAI-1*

(2.19) *EIO-4→AEE-4→EAE-1→AAA-1→AAI-1→AAI-4*

(2.20) *EIO-4→AEE-4→EAE-1→AAA-1→OAO-3*

(2.21) *EIO-4→AEE-4→EAE-1→AAA-1→AAI-1→EAO-3*

(2.22) *EIO-4→AEE-4→EAE-1→AAA-1→AAI-1→EAO-3→EAO-4*

(2.23) *EIO-4→AEE-4→EAE-1→EAO-1→AAI-3*

Proof:

[1] ⊢*no(z, n)∧some(n, d)→not all(d, z)* (i.e. *EIO-4*, basic axiom)

[2] ⊢*no(n, z)∧some(n, d)→not all(d, z)* (i.e. *EIO-3*, by [1] and Fact (3.2))

[3] $\vdash no(z, n) \wedge some(d, n) \rightarrow not\ all(d, z)$ (i.e. *EIO-2*, by [1] and Fact (3.1))

[4] $\vdash no(n, z) \wedge some(d, n) \rightarrow not\ all(d, z)$ (i.e. *EIO-1*, by [2] and Fact (3.1))

[5] $\vdash \neg not\ all(d, z) \wedge no(z, n) \rightarrow \neg some(n, d)$ (by [1] and Rule 2)

[6] $\vdash all(d, z) \wedge no(z, n) \rightarrow no(n, d)$ (i.e. *AEE-4*, by [5], Fact (2.1) and Fact (2.3))

[7] $\vdash all(d, z) \wedge no(n, z) \rightarrow no(n, d)$ (i.e. *AEE-2*, by [6] and Fact (3.2))

[8] $\vdash all(d, z) \wedge no(z, n) \rightarrow no(d, n)$ (i.e. *EAE-1*, by [6] and Fact (3.2))

[9] $\vdash all(d, z) \wedge no(n, z) \rightarrow no(d, n)$ (i.e. *EAE-2*, by [8] and Fact (3.2))

[10] $\vdash no(n, d) \rightarrow not\ all(n, d)$ (by Fact (4.2))

[11] $\vdash all(d, z) \wedge no(z, n) \rightarrow not\ all(n, d)$ (i.e. *AEO-4*, by [6], [10] and Rule 1)

[12] $\vdash all(d, z) \wedge no(n, z) \rightarrow not\ all(n, d)$ (i.e. *AEO-2*, by [7], [10] and Rule 1)

[13] $\vdash no(d, n) \rightarrow not\ all(d, n)$ (by Fact (4.2))

[14] $\vdash all(d, z) \wedge no(z, n) \rightarrow not\ all(d, n)$ (i.e. *EAO-1*, by [8], [13] and Rule 1)

[15] $\vdash all(d, z) \wedge no(n, z) \rightarrow not\ all(d, n)$ (i.e. *EAO-2*, by [9], [14] and Rule 1)

[16] $\vdash all\neg(n, z) \wedge some(n, d) \rightarrow some\neg(d, z)$ (by [2], Fact (1.2) and Fact (1.4))

[17] $\vdash all(n, U-z) \wedge some(n, d) \rightarrow some(d, U-z)$ (i.e. *AII-3*, by [16] and Definition (3.5.3))

[18] $\vdash all(n, U-z) \wedge some(d, n) \rightarrow some(d, U-z)$ (i.e. *AII-1*, by [17] and Fact (3.1))

[19] $\vdash all(n, U-z) \wedge some(n, d) \rightarrow some(U-z, d)$ (i.e. *IAI-3,* by [17] and Fact (3.1))

[20] $\vdash all(n, U-z) \wedge some(d, n) \rightarrow some(U-z, d)$ (i.e. *IAI-4,* by [19] and Fact (3.1))

[21] $\vdash all\neg(z, n) \wedge not\ all\neg(d, n) \rightarrow not\ all(d, z)$ (by [3], Fact (1.1) and Fact (1.4))

[22] $\vdash all(z, U-n) \wedge not\ all(d, U-n) \rightarrow not\ all(d, z)$ (i.e. *AOO-2*, by [21] and Definition (3.5.3))

[23] $\vdash all(d, z) \wedge all\neg(z, n) \rightarrow all\neg(d, n)$ (by [8] and Fact (1.4))

[24] $\vdash all(d, z) \wedge all(z, U-n) \rightarrow all(d, U-n)$ (i.e. *AAA-1*, by [23] and Definition (3.5.3))

[25] $\vdash all(d, U-n) \rightarrow some(d, U-n)$ (by Fact (4.1))

[26] $\vdash all(d, z) \wedge all(z, U-n) \rightarrow some(z, U-n)$ (i.e. *AAI-1*, by [24], [25] and Rule 1)

[27] $\vdash all(d, z) \wedge all(z, U-n) \rightarrow some(U-n, z)$ (i.e. *AAI-4*, by [26] and Fact (3.1))

[28] $\vdash \neg all(d, U-n) \wedge all(d, z) \rightarrow \neg all(z, U-n)$ (by [24] and Rule 2)

[29] $\vdash not\ all(d, U-n) \wedge all(d, z) \rightarrow not\ all(z, U-n)$ (i.e. *OAO-3*, by [28] and Fact (2.4))

[30] $\vdash \neg some(d, U-n) \wedge all(d, z) \rightarrow \neg all(z, U-n)$ (by [26] and Rule 2)

[31] $\vdash no(d, U-n) \wedge all(d, z) \rightarrow not\ all(z, U-n)$ (i.e. *EAO-3*, by [30], Fact (2.1) and Fact (2.4))

[32] $\vdash no(U-n, d) \wedge all(d, z) \rightarrow not\ all(z, U-n)$ (i.e. *EAO-4*, by [31] and Fact (3.2))

[33] $\vdash\neg$*not all(d, n)*$\wedge$*all(d, z)*$\rightarrow\neg$*no(z, n)* (by [14] and Rule 2)

[34] $\vdash$*all(d, n)*$\wedge$*all(d, z)*$\rightarrow$*some(z, n)* (i.e. *AAI-3*, by [33], Fact (2.2) and Fact (2.3))

So far, one basic axiom (i.e., the syllogism *EIO-4* in this paper) can be used to deduce all the remaining 23 valid traditional syllogisms. Then, can any one of the other valid traditional syllogisms be used as a basic axiom to deduce the remaining 23 valid traditional syllogisms? This question requires further research.

5. Conclusion

Making full use of set theory, generalized quantifier theory and propositional logic, the other 23 syllogisms can be inferred from the syllogism *EIO-4*. Then a minimalist formal axiomatic system can be established for traditional syllogistic logic. In the process of deduction, one can reveal the reducible relations between/among syllogisms with the same or different figures. This is because Aristotelian quantifiers can be mutually defined, which leads to changes in the structures of research objects and the relations between structures. One or more valid syllogisms can be derived from the validity of one syllogism, which is beneficial for better knowledge mining and thus for better understanding of natural language.

In fact, the formal research method in this paper not only provides a straightforward mathematical model for the study of traditional syllogisms, but also benefits the study of other types of syllogisms (e.g., generalized syllogisms and modal syllogisms). The formal study of different types of syllogisms is beneficial for natural language information processing.

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