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Knowledge Reasoning for the Generalized Modal Syllogism □AM◇I-3

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Abstract

This paper first proves the validity of the generalized modal syllogism □AM◇I-3 with the non-trivial generalized quantifier '*most*' and the two trivial generalized quantifiers '*all*' and '*some*'. And then making best of relevant facts and deductive rules, this paper deduces 20 other valid generalized modal syllogisms from the syllogism □AM◇I-3. In other words, there are reducible relationships between/among the 21 valid generalized modal syllogisms. The reasons for this conclusion are as follows: (1) any quantifier in Square{*some*} can define the other three quantifiers, and so can any quantifier in Square{*most*}. (2) necessary modality (□) and possible modality (◇) can be mutually defined. This results not only provide a common mathematical paradigm for studying the validity and reducibility of different kinds of syllogisms, but also a formal method for other types of knowledge reasoning in artificial intelligence that can be used as a reference.

Keywords: generalized modal syllogisms; generalized quantifiers; reducibility

1. Introduction

Syllogism is a common and important form of reasoning in natural language, and there are many forms of syllogisms, such as Aristotelian syllogisms (Zhang, 2022; Hao, 2023; Li, 2023), generalized syllogisms (Endrullis and Moss, 2015; Hao, 2024), relational syllogisms (Ivanov and Vakarelov, 2014), Aristotelian modal syllogisms (Wei and Zhang, 2022; Zhang, 2023; Qiu, 2024), and generalized modal syllogisms (Xu and Zhang, 2023), and so on.

A generalized modal syllogism contains at most three generalized quantifiers, and at least one and at most three non-overlapping necessary modality (□) or possible modality (◇). Trivial generalized quantifiers are Aristotelian ones (i.e. *no*, *not all*, *some*, *all*). A trivial generalized modal syllogism is an Aristotelian modal syllogism. Due to the infinite number of non-trivial generalized quantifiers in natural language, there are countless generalized modal syllogisms composed of generalized quantifiers and modalities. As far as we know, there have been few works on this type of syllogism so far. Therefore, this paper focuses on knowledge reasoning for the generalized modal syllogism □AM◇I-3. Unless otherwise specified, the following syllogisms are generalized modal syllogisms.

2. Knowledge Representation

In the following, let *b*, *z* and *t* be lexical variables, *D* be the domain of lexical variable. The sets composed of *b*, *z* and *t* are respectively *B*, *Z* and *T*. '$|B\cap T|$' indicates the cardinality for the intersection of *B* and *T*. And *Q* represents a generalized quantifier, $\neg Q$ stands for its outer quantifier, and $Q\neg$ for inner quantifier. Let ω, γ, ε and δ be well-formed formulas (abbreviated as wff). '$\omega =_{def} \gamma$' shows that ω can be defined by γ. '$\vdash \omega$' means that ω is provable. '□' is necessary modality, and '◇' is possible one.

The generalized modal syllogisms discussed in this paper only involve the following eight quantifiers: Aristotelian quantifiers (namely, *not all*, *some*, *all*, *no,* which make up Square{*some*}) and the non-trivial generalized quantifiers (namely, *most*, *fewer than half of the*, *at least half of the*, *at most half of the*, which make up Square{*most*}). A statement containing any of the eight quantifiers mentioned above corresponds to one of the following eight propositions: *all*(*b, t*), *no*(*b, t*), *some*(*b, t*), *not all*(*b, t*), *most*(*b, t*), *fewer than half of the*(*b, t*), *at least half of the*(*b, t*), *at most half of the*(*b, t*), and denoted by the Proposition *A*, *E*, *I*, *O*, *F*, *S*, *M*, and *H*, respectively.

The generalized modal syllogism as the basis for reasoning in this paper is the third figure syllogism $\Box all(z, t) \wedge most(z, b) \rightarrow \Diamond some(b, t)$, which can be shortened as □AM◇I-3. An example of the syllogism □AM◇I-3 in natural language is as follows:

Major premise: All the furniture in this room are necessarily made of wood.

Minor premise: Most the furniture in this room are stools.

Conclusion: Some stools are possibly made of wood.

Let *t* be something that made of wood, *z* be the furniture in this room, and *b* be stools. Then the above example can be formalized as $\Box all(z, t) \wedge most(z, b) \rightarrow \Diamond some(b, t)$, and abbreviated as □AM◇I-3. The others are similar.

3. Formal Generalized Modal Syllogism System

The generalized modal syllogism system consists of the following several parts: primitive symbols, basic axioms, deductive rules, relevant definitions and facts.

3.1 Primitive Symbols

(1) lexical variables: *b*, *z*, *t*

(2) quantifiers: *some*, *most*

(3) modality: □

(4) operators: $\neg$, $\rightarrow$

(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *b* and *t* are lexical variables, then $Q(b, t)$ is a wff.

(2) If ω is a wff, then so are $\neg\omega$ and $\Box\omega$.

(3) If ω and δ are wffs, then so is $\omega \rightarrow \delta$.

(4) Only a formula in line with the above rules is a wff.

3.3 Basic Axioms

A1: If ω is a valid formula in classical first-order logic, then $\vdash \omega$.

A2: $\vdash \Box all(z, t) \wedge most(z, b) \rightarrow \Diamond some(b, t)$ (namely, the syllogism □AM◇I-3).

3.4 Rules of Deduction

Rule 1: From ⊢ $(\omega\wedge\delta\rightarrow\gamma)$ and ⊢ $(\gamma\rightarrow\varphi)$ infer ⊢ $(\omega\wedge\delta\rightarrow\varphi)$.

Rule 2: From ⊢ $(\omega\wedge\delta\rightarrow\gamma)$ infer ⊢ $(\neg\gamma\wedge\delta\rightarrow\neg\omega)$.

Rule 3: From ⊢ $(\omega\wedge\delta\rightarrow\gamma)$ infer ⊢ $(\neg\gamma\wedge\omega\rightarrow\neg\delta)$.

Rule 4: (antecedent strengthening): From ⊢ $(\varepsilon\rightarrow\omega)$ and ⊢ $(\omega\wedge\delta\rightarrow\gamma)$ infer ⊢ $(\varepsilon\wedge\delta\rightarrow\gamma)$.

Rule 5: (antecedent strengthening): From ⊢ $(\varepsilon\rightarrow\delta)$ and ⊢ $(\omega\wedge\delta\rightarrow\gamma)$ infer ⊢ $(\omega\wedge\varepsilon\rightarrow\varphi)$.

3.5 Relevant Definitions

D1: $(\omega\wedge\delta) =_{def} \neg(\omega\rightarrow\neg\delta)$;

D2: $(\omega\leftrightarrow\delta) =_{def} (\omega\rightarrow\delta)\wedge(\delta\rightarrow\omega)$;

D3: $Q\neg(b, t) =_{def} Q(b, D-t)$;

D4: $\neg Q(b, t) =_{def}$ It is not that $Q(b, t)$;

D5: $\Diamond Q(b, t) =_{def} \neg\Box\neg Q(b, t)$;

D6: *all*(b, t) is true iff $B\subseteq T$ is true in any real world;

D7: *some*(b, t) is true iff $B\cap T\neq\varnothing$ is true in any real world;

D8: *no*(b, t) is true iff $B\cap T=\varnothing$ is true in any real world;

D9: *not all*(b, t) is true iff $B\nsubseteq T$ is true in any real world;

D10: *most*(b, t) is true iff $|B\cap T|>0.5|B|$ is true in any real world;

D11: $\Box$ *all*(b, t) is true iff $B\subseteq T$ is true in any possible world;

D12: $\Diamond$*some*(b, t) is true iff $B\cap T\neq\varnothing$ is true in at least one possible world.

3.6 Relevant Facts

Fact 1 (Inner Negation):

(1.1) ⊢ *all*$(b, t)\leftrightarrow$*no*$\neg(b, t)$;

(1.2) ⊢ *no*$(b, t)\leftrightarrow$*all*$\neg(b, t)$;

(1.3) ⊢ *some*$(b, t)\leftrightarrow$*not all*$\neg(b, t)$;

(1.4) ⊢ *not all*$(b, t)\leftrightarrow$*some*$\neg(b, t)$;

(1.5) ⊢ *most*(*b*, *t*)↔*fewer than half of the*¬(*b*, *t*);

(1.6) ⊢ *fewer than half of the*(*b*, *t*)↔*most*¬(*b*, *t*);

(1.7) ⊢ *at least half of the*(*b*, *t*)↔*at most half of the* (*b*, *t*);

(1.8) ⊢ *at most half of the*(*b*, *t*)↔*at least half of the* (*b*, *t*).

Fact 2 (Outer Negation):

(2.1) ⊢ ¬*all*(*b*, *t*)↔*not all*(*b*, *t*);

(2.2) ⊢ ¬*not all*(*b*, *t*)↔*all*(*b*, *t*);

(2.3) ⊢ ¬*no*(*b*, *t*)↔*some*(*b*, *t*);

(2.4) ⊢ ¬*some*(*b*, *t*)↔*no*(*b*, *t*);

(2.5) ⊢ ¬*most*(*b*, *t*)↔*at most half of the*(*b*, *t*);

(2.6) ⊢ ¬*at most half of the*(*b*, *t*)↔*most*(*b*, *t*);

(2.7) ⊢ ¬*fewer than half of the*(*b*, *t*)↔*at least half of the*(*b*, *t*);

(2.8) ⊢ ¬*at least half of the*(*b*, *t*)↔*fewer than half of the*(*b*, *t*).

Fact 3 (Symmetry):

(3.1) ⊢ *some*(*b*, *t*)↔*some*(*t*, *b*);

(3.2) ⊢ *no*(*b*, *t*)↔*no*(*t*, *b*).

Fact 4 (Subordination):

(4.1) ⊢ *no(b, t)*→*not all(b, t)*;

(4.2) ⊢ *all(b, t)*→*some(b, t)*;

(4.3) ⊢ *all(b, t)*→*most(b, t)*;

(4.4) ⊢ *most(b, t)*→*some(b, t)*;

(4.5) ⊢ *all(b, t)*→*at least half of the(b, t)*;

(4.6) ⊢ *at least half of the(b, t)*→*some(b, t)*;

(4.7) ⊢ *fewer than half of the(b, t)*→*not all(b, t)*;

(4.8) ⊢ *most(b, t)*→*at least half of the(b, t)*;

(4.9) ⊢ *at most half of the(b, t)→fewer than half of the(b, t)*;

(4.10) ⊢ □*Q*(*b*, *t*)→*Q*(*b*, *t*);

(4.11) ⊢ *Q*(*b*, *t*)→◇*Q*(*b*, *t*).

Fact 5 (Dual):

(5.1) ⊢ ¬□*Q*(*b*, *t*)↔◇¬*Q*(*b*, *t*);

(5.2) ⊢ ¬◇*Q*(*b*, *t*)↔□¬*Q*(*b*, *t*).

4. The Reducibility of the Generalized Modal Syllogism □AM◇I-3

The following Theorem 1 shows the generalized modal syllogism □AM◇I-3 is valid. '□AM◇I-3→ M□A◇I-3' in Theorem 2 illustrates that the validity of the syllogism M□A◇I-3 can be deduced from that of the syllogism □AM◇I-3. That is to say that this syllogism □AM◇I-3 has reducibility and there is a reducible relationship between these two syllogisms. The others are similar.

Theorem 1 (□AM◇I-3): The generalized modal syllogism □*all*(*z*, *t*)∧*most*(*z*, *b*)→◇*some*(*b*, *t*) is valid.

Proof: Suppose that □*all*(*z*, *t*) and *most*(*z*, *b*) are true, then it is easy to deduce that '$B \subseteq T$' is true in any possible world according to Definition D1, and '$|B \cap T| > 0.5|B|$' is true in any real world in line with Definition D10. From this, it can be concluded that '$B \cap T \neq \varnothing$' is true in any real world. It is obvious that a real world is a possible world. Thus, '$B \cap T \neq \varnothing$' is true at least one possible world. It can be seen that □*all*(*z*, *t*)∧*most*(*z*, *b*)→◇*some*(*b*, *t*) is valid, just as expected.

Theorem 2: There are at least the following 20 valid generalized modal syllogisms derived from the syllogism □AM◇I-3.

(2.1) ⊢□AM◇I-3→ M□A◇I-3

(2.2) ⊢□AM◇I-3→□E□AH-1

(2.3) ⊢□AM◇I-3→□E□AH-1→□E□AH-2

(2.4) ⊢□AM◇I-3→□E□AH-1→□E□AF-1

(2.5) ⊢□AM◇I-3→□E□AH-1→□E□AF-1→□E□AF-2

(2.6) ⊢□AM◇I-3→□EM◇O-2

(2.7) ⊢□AM◇I-3→□EM◇O-2→□EM◇O-1

(2.8) ⊢□AM◇I-3→□EM◇O-3

(2.9) ⊢□AM◇I-3→□EM◇O-3→□EM◇O-4

(2.10) ⊢□AM◇I-3→ M□A◇I-3→F□A◇O-3

(2.11) ⊢□AM◇I-3→ M□A◇I-3→F□A◇O-3→H□A◇O-3

(2.12) ⊢□AM◇I-3→□E□AH-1→□A□AS-1

(2.13) ⊢□AM◇I-3→□E□AH-1→□E□AH-2→□A□EH-2

(2.14) ⊢□AM◇I-3→□E□AH-1→□E□AH-2→□A□EH-2→□A□EF-2

(2.15) ⊢□AM◇I-3→□E□AH-1→□E□AH-2→□A□EH-2→□A□EH-4

(2.16) ⊢□AM◇I-3→□E□AH-1→□E□AH-2→□A□EH-2→□A□EF-2→□A□EF-4

(2.17) ⊢□AM◇I-3→□E□AH-1→□E□AF-1→□ES◇O-4

(2.18) ⊢□AM◇I-3→□E□AH-1→□E□AF-1→□ES◇O-4→□ES◇O-1

(2.19) ⊢□AM◇I-3→□E□AH-1→□E□AF-1→S□A◇I-3

(2.20) ⊢□AM◇I-3→□E□AH-1→□E□AF-1→□A□AM-1

Proof:

[1] ⊢□*all*(*z*, *t*)∧*most*(*z*, *b*)→◇*some*(*b*, *t*) (i.e. □AM◇I-3, basic axiom A2)

[2] ⊢□*all*(*z*, *t*)∧*most*(*z*, *b*)→◇*some*(*t*, *b*) (i.e. M□A◇I-3, by [1] and Fact(3.1))

[3] ⊢¬◇*some*(*t*, *b*)∧□*all*(*z*, *t*)→¬*most*(*z*, *b*) (by [1] and Rule 2)

[4] ⊢□¬*some*(*t*, *b*)∧□*all*(*z*, *t*)→*at most half of the*(*z*, *b*) (by [3], Fact(5.2) and Fact(2.5))

[5] ⊢□*no*(*t*, *b*)∧□*all*(*z*, *t*)→*at most half of the*(*z*, *b*) (i.e.□ E□AH-1, by [4] and Fact(2.4))

[6] ⊢□*no*(*b*, *t*)∧□*all*(*z*, *t*)→*at most half of the*(*z*, *b*) (i.e. □E□AH-2, by [5] and Fact(3.2))

[7] ⊢□*no*(*t*, *b*)∧□*all*(*z*, *t*)→*fewer than half of the*(*z*, *b*) (i.e. □E□AF-1, by [5] and Fact(4.9))

[8] ⊢□*no*(*b*, *t*)∧□*all*(*z*, *t*)→*fewer than half of the*(*z*, *b*) (i.e. □E□AF-2, by [7] and Fact(3.2))

[9] ⊢¬◇*some*(*t*, *b*)∧*most*(*z*, *b*)→¬□*all*(*z*, *t*) (by [1] and Rule 3)

[10] ⊢□¬*some*(*t*, *b*)∧*most*(*z*, *b*)→◇¬*all*(*z*, *t*) (by [9], Fact(5.2) and Fact(5.1))

[11] ⊢□*no*(*t*, *b*)∧*most*(*z*, *b*)→◇*not all*(*z*, *t*) (i.e. □EM◇O-2, by [10], Fact(2.4) and Fact(2.1))

[12] $\vdash\Box no(b, t)\wedge most(z, b)\rightarrow\Diamond not\ all(z, t)$ (i.e. □EM◇O-1, by [11], Fact(2.4) and Fact(2.1))

[13] $\vdash\Box no\neg(z, t)\wedge most(z, b)\rightarrow\Diamond not\ all\neg(b, t)$ (by [1] and Fact(1.1) and Fact(1.3))

[14] $\vdash\Box no(z, D\text{-}t)\wedge most(z, b)\rightarrow\Diamond not\ all(b, D\text{-}t)$ (i.e. □EM◇O-3, by [13] and D3)

[15] $\vdash\Box no(D\text{-}t, z)\wedge most(z, b)\rightarrow\Diamond not\ all(b, D\text{-}t)$ (i.e. □EM◇O-4, by [14] and Fact(3.2))

[16] $\vdash\Box all(z, t)\wedge fewer\ than\ half\ of\ the\neg(z, b)\rightarrow\Diamond not\ all\neg(t, b)$

(by [2] and Fact(1.5) and Fact(1.3))

[17] $\vdash\Box all(z, t)\wedge fewer\ than\ half\ of\ the(z, D\text{-}b)\rightarrow\Diamond not\ all(t, D\text{-}b)$

(i.e. F□A◇O-3, by [11] and D3)

[18] $\vdash\Box all(z, t)\wedge at\ most\ half\ of\ the(z, D\text{-}b)\rightarrow\Diamond not\ all(t, D\text{-}b)$

(i.e. H□A◇O-3, by [13] and Rule 5)

[19] $\vdash\Box all\neg(t, b)\wedge\Box all(z, t)\rightarrow at\ least\ half\ of\ the\neg(z, b)$ (by [5], Fact(1.1) and Fact(1.8))

[20] $\vdash\Box all(t, D\text{-}b)\wedge\Box all(z, t)\rightarrow at\ least\ half\ of\ the(z, D\text{-}b)$ (i.e. □A□AS-1, by [19] and D3)

[21] $\vdash\Box all\neg(b, t)\wedge\Box no\neg(z, t)\rightarrow at\ most\ half\ of\ the(z, b)$ (by [6], Fact(1.2) and Fact(1.1))

[22] $\vdash\Box all(b, D\text{-}t)\wedge\Box no(z, D\text{-}t)\rightarrow at\ most\ half\ of\ the(z, b)$ (i.e. □A□EH-2, by [21] and D3)

[23] $\vdash\Box all(b, D\text{-}t)\wedge\Box no(z, D\text{-}t)\rightarrow fewer\ than\ half\ of\ the(z, b)$

(i.e. □A□EF-2, by [22] and Fact(4.9))

[24] $\vdash\Box all(b, D\text{-}t)\wedge\Box no(D\text{-}t, z)\rightarrow at\ most\ half\ of\ the(z, b)$ (i.e. □A□EH-4, by [22] and Fact(3.2))

[25] $\vdash\Box all(b, D\text{-}t)\wedge\Box no(D\text{-}t, z)\rightarrow fewer\ than\ half\ of\ the(z, b)$

(i.e. □A□EF-4, by [23] and Fact(3.2))

[26] $\vdash\neg fewer\ than\ half\ of\ the(z, b)\wedge\Box no(t, b)\rightarrow\neg\Box all(z, t)$ (by [7] and Rule 2)

[27] $\vdash at\ least\ half\ of\ the(z, b)\wedge\Box no(t, b)\rightarrow\Diamond\neg all(z, t)$ (by [26], Fact(2.7) and Fact(5.1))

[28] $\vdash at\ least\ half\ of\ the(z, b)\wedge\Box no(t, b)\rightarrow\Diamond not\ all(z, t)$ (i.e. □ES◇O-4,by [27], Fact(2.1))

[29] $\vdash at\ least\ half\ of\ the(z, b)\wedge\Box no(b, t)\rightarrow\Diamond not\ all(z, t)$ (i.e. □ES◇O-1,by [28], Fact(3.2))

[30] $\vdash\neg fewer\ than\ half\ of\ the(z, b)\wedge\Box all(z, t)\rightarrow\neg\Box no(t, b)$ (by [7] and Rule 3)

[31] $\vdash at\ least\ half\ of\ the(z, b)\wedge\Box all(z, t)\rightarrow\Diamond\neg no(t, b)$ (by [30], Fact(2.7) and Fact(5.1))

[32] $\vdash at\ least\ half\ of\ the(z, b)\wedge\Box all(z, t)\rightarrow\Diamond some(t, b)$ (i.e.S□A◇I-3, by [31] and Fact(2.3))

[33] $\vdash\Box all\neg(t, b)\wedge\Box all(z, t)\rightarrow most\neg(z, b)$ (by [7], Fact(1.2) and Fact(1.8))

[34] $\vdash\Box all(t, D\text{-}b)\wedge\Box all(z, t)\rightarrow most(z, D\text{-}b)$ (i.e. □A□AM-1,by [33] and D3)

Theorem 2 infers the above 20 valid generalized modal syllogisms from the syllogism □AM◇I-3 through 34 reasoning steps. This process indicates that there are reducible relationships between/among these 21 syllogisms. In fact, more valid generalized modal syllogisms can be obtained by repeatedly utilizing these reduction operations.

5. Conclusion and Future Work

This paper firstly proves the validity of generalized modal syllogism □AM◇I-3 by means of set theory and modal logic, and then making best of relevant facts and deductive rules, deduces 20 other valid generalized modal syllogisms from the syllogism □AM◇I-3. In other words, there are reducible relationships between/among the 21 valid generalized modal syllogisms. The reasons for this conclusion are as follows: (1) Any quantifier in Square{*some*} can define the other three quantifiers, and so can any quantifier in Square{*most*}. (2) Necessary modality (□) and possible modality (◇) can be mutually defined. This results not only provide an important mathematical paradigm for the validity and reducibility of different kinds of syllogisms, but also play an important role in knowledge reasoning in the field of artificial intelligence. Can we establish a minimalist formal axiom system for the modal syllogism fragments studied in this paper? This question deserves further exploration.

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