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The Deductibility of the Generalized Syllogism *AAM-1*

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Abstract

Firstly, according to set theory, the validity of the generalized syllogism *AAM-1* is proved in accordance with the truth-value definitions of quantified statements. Then, on the basis of generalized quantifier theory, this paper derives the other 14 valid generalized syllogisms from the validity of generalized syllogism *AAM-1* by taking full advantage of the inner and outer negation of a generalized quantifier, the symmetry of the two Aristotelian quantifiers '*some*' and '*no'*, the subsequent weakening rule, the anti-syllogism inference rules, and other reduction operations. This research method conforms to the demand of formal transformation for natural language information in the era of mega data.

Keywords: generalized syllogism; deductibility; truth-value definition; quantifier

1. Introduction

Many natural languages use quantification expressions to describe the quantity of things (Barwise and Cooper, 1981). These expressions contain not only two standard quantifiers (that is, universal quantifier and existential quantifier) in first-order predicate logic, but also many quantifiers that cannot be expressed by these two standard quantifiers, which are called generalized quantifiers (Peters and Westerståhl, 2006). Generalized quantifier theory is a highly versatile syntactic and semantic tool that effectively reveals the semantic and inferential properties of quantified statements in natural language (Xiaojun, 2018).

Generalized quantifier theory is based on first-order logic (Hamilton, 1978), so the inference rules and axioms in the latter are still applicable in the former. It has been proved that many achievements of the generalized quantifier theory can not only formalize the logical semantics of quantified statements, but also judge and prove the validity of inference modes involving generalized quantifiers (Endrullis and Moss, 2015).

This paper mainly deals with the generalized quantifiers '*all'* and '*most'*, discusses the deductibility between the generalized syllogism *AAM-1* and other generalized syllogisms, and deduces the validity of other 14 generalized syllogisms according to that of *AAM-1*.

2. Preliminaries

In the following, let *n*, *t* and *y* be lexical variables, and *D* be their domain. The sets composed of *n*, *t* and *y* are respectively *N*, *T*, and *Y*. Let *b*, *q*, *r*, and *w* be well-formed formulas (shortened as wff). Let *Q* be a quantifier, $\neg Q$ and $Q\neg$ be its outer and inner negation, respectively. '$|N \cap T|$' represents the cardinality of the intersection of the set *N* and *T* (Halmos, 1974). '$\vdash b$' shows that the wff *b* is provable, and '$b =_{\text{def}} q$' that *b* can be defined by *q*. The others are similar.

The generalized syllogisms studied in this paper involve the following 8 propositions: *all*(*t, n*), *not all*(*t, n*), *no*(*t, n*), *some*(*t, n*), *most*(*t, n*)*, at most half of the*(*t, n*), *at least half of the*(*t, n*), *fewer than half of the*(*t, n*), which are respectively referred to as: Proposition *A*, *O*, *E*, *I*, *M*, *H*, *S*, and *F.* A non-trivial generalized syllogism includes at least one and at most three in the last four propositions (Feifei and Xiaojun, 2024). An instance of the generalized syllogism *AAM-1* in natural language is as follows:

Major premise: All cats like to eat fish.

Minor premise: All the pets in my house are cats.

Conclusion: Most of the pets in my house like to eat fish.

Let y be a cat in the domain, n be an animal that likes to eat fish, and t be a pet in my house. Then the syllogism is symbolized as '$all(y, n) \wedge all(t, y) \rightarrow most(t, n)$', which can be abbreviated as *AAM-1*. The Others are similar.

The deductive rules, definitions and facts involved in this research are shown below.

Rule 1 (Deductive Rules):

R1 (subsequent weakening): From $\vdash(b \wedge q \rightarrow r)$ and $\vdash(r \rightarrow w)$ infer $\vdash(b \wedge q \rightarrow w)$.

R2 (anti-syllogism 1): From $\vdash(b \wedge q \rightarrow r)$ infer $\vdash(\neg r \wedge b \rightarrow \neg q)$.

R3 (anti-syllogism 2): From $\vdash(b \wedge q \rightarrow r)$infer $\vdash(\neg r \wedge q \rightarrow \neg b)$.

Definition 1 (Relevant Definitions)：

D1 (outer negation): $(\neg Q)(t, n)=_{\text{def}}$ It is not that $Q(t, n)$;

D2 (inner negation): $(Q\neg)(t, n)=_{\text{def}} Q(t, D-n)$;

D3 (truth value): $all(t, n)=_{\text{def}} T \subseteq N$;

D4 (truth value): $most(t, n)$ is true if and only if $|T \cap N| > 0.5|T|$ is true.

Fact 1 (Inner Negation):

(1.1) $\vdash all(t, n) \leftrightarrow no\neg(t, n)$;

(1.2) $\vdash no(t, n) \leftrightarrow all\neg(t, n)$;

(1.3) $\vdash some(t, n) \leftrightarrow not\ all\neg(t, n)$;

(1.4) $\vdash not\ all(t, n) \leftrightarrow some\neg(t, n)$;

(1.5) $\vdash most(t, n) \leftrightarrow fewer\ than\ half\ of\ the\neg(t, n)$;

(1.6) $\vdash fewer\ than\ half\ of\ the(t, n) \leftrightarrow most\neg(t, n)$;

(1.7) $\vdash at\ least\ half\ of\ the(t, n) \leftrightarrow at\ most\ half\ of\ the\neg(t, n)$;

(1.8) $\vdash at\ most\ half\ of\ the(t, n) \leftrightarrow at\ least\ half\ of\ the\neg(t, n)$.

Fact 2 (Outer Negation):

(2.1) $\vdash \neg all(t, n) \leftrightarrow not\ all(t, n)$;

(2.2) $\vdash \neg not\ all(t, n) \leftrightarrow all(t, n)$;

(2.3) $\vdash \neg no(t, n) \leftrightarrow some(t, n)$;

(2.4) $\vdash\neg$*some*(t, n)$\leftrightarrow$*no*(t, n);

(2.5) $\vdash\neg$*most*(t, n)$\leftrightarrow$*at most half of the*(t, n);

(2.6) $\vdash\neg$*at most half of the*(t, n)$\leftrightarrow$*most*(t, n);

(2.7) $\vdash\neg$*fewer than half of the*(t, n)$\leftrightarrow$*at least half of the*(t, n);

(2.8) $\vdash\neg$*at least half of the*(t, n)$\leftrightarrow$*fewer than half of the*(t, n).

Fact 3 (Symmetry):

(3.1) $\vdash$*some*(t, n)$\leftrightarrow$*some*(n, t);

(3.2) $\vdash$*no*(t, n)$\leftrightarrow$*no*(n, t).

Fact 4 (Subordination) :

(4.1) $\vdash$*all*(t, n)$\rightarrow$*some*(t, n);

(4.2) $\vdash$*no*(t, n)$\rightarrow$*not all*(t, n);

(4.3) $\vdash$*all*(t, n)$\rightarrow$*most*(t, n);

(4.4) $\vdash$*most*(t, n)$\rightarrow$*some*(t, n);

(4.5) $\vdash$*at least half of the*(t, n)$\rightarrow$*some*(t, n);

(4.6) $\vdash$*all*(t, n)$\rightarrow$*at least half of the*(t, n);

(4.7) $\vdash$*at most half of the*(t, n)$\rightarrow$*not all*(t, n);

(4.8) $\vdash$*fewer than half of the*(t, n)$\rightarrow$*not all*(t, n).

Fact 1-4 are basic knowledge in first-order logic (Hamilton, 1978) and generalized quantifier theory (Peters and Westerståhl, 2006).

3. The Deductive Reasoning of the Generalized Syllogism *AAM-1*

The following Theorem 1 proves the validity of the generalized syllogism *AAM-1*. Theorem 2 shows that other valid generalized syllogisms can be deduced from the syllogism *AAM-1*. In other words, there are reducible relationships between/among valid generalized syllogisms.

Theorem 1(*AAM-1*): The generalized syllogism *all*(y, n)$\wedge$*all*(t, y)$\rightarrow$*most*(t, n) is valid.

Proof: Suppose that *all*(y, n) and *all*(t, y) are true, then $Y\subseteq N$ and $T\subseteq Y$ are true according to

Definition D1. It is easily obtained that $T \subseteq N$. Hence it can be concluded that *all*(t, n) is true according to Definition D1. Therefore, '*all*$(y, n) \wedge$ *all*$(t, y) \rightarrow$ *all*(t, n)' is valid. Then '*all*$(t, n) \rightarrow$ *most*(t, n)' in line with Fact (4.3). Thus, '*all*$(y, n) \wedge$ *all*$(t, y) \rightarrow$ *most*(t, n)' is valid in the light of Rule R1, just as desired.

Theorem 2: There are at least the following 14 valid generalized syllogisms deduced from *AAM-1*:

(2.1) $\vdash$*AAM-1*$\rightarrow$*AHO-2*

(2.2) $\vdash$*AAM-1*$\rightarrow$*HAO-3*

(2.3) $\vdash$*AAM-1*$\rightarrow$*EAF-1*

(2.4) $\vdash$*AAM-1*$\rightarrow$*AHO-2*$\rightarrow$*ESO-2*

(2.5) $\vdash$*AAM-1*$\rightarrow$*HAO-3*$\rightarrow$*SAI-3*

(2.6) $\vdash$*AAM-1*$\rightarrow$*HAO-3*$\rightarrow$*SAI-3*$\rightarrow$*ASI-3*

(2.7) $\vdash$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*

(2.8) $\vdash$*AAM-1*$\rightarrow$*AHO-2*$\rightarrow$*ESO-2*$\rightarrow$*ESO-1*

(2.9) $\vdash$*AAM-1*$\rightarrow$*AHO-2*$\rightarrow$*ESO-2*$\rightarrow$*ESO-1*$\rightarrow$*ASI-1*

(2.10) $\vdash$*AAM-1*$\rightarrow$*AHO-2*$\rightarrow$*ESO-2*$\rightarrow$*ESO-1*$\rightarrow$*ASI-1*$\rightarrow$*SAI-4*

(2.11) $\vdash$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*$\rightarrow$*AEF-2*

(2.12) $\vdash$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*$\rightarrow$*AEF-2*$\rightarrow$*AEF-4*

(2.13) $\vdash$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*$\rightarrow$*AEF-2*$\rightarrow$*ESO-3*

(2.14) $\vdash$*AAM-1*$\rightarrow$*EAF-1*$\rightarrow$*EAF-2*$\rightarrow$*AEF-2*$\rightarrow$*ESO-3*$\rightarrow$*ESO-4*

Proof:

[1] $\vdash$*all*$(y, n) \wedge$*all*$(t, y) \rightarrow$*most*(t, n) (i.e. *AAM-1*, Theorem 1)

[2] $\vdash \neg$*most*$(t, n) \wedge$*all*$(y, n) \rightarrow \neg$*all*(t, y) (by [1] and R2)

[3] $\vdash$*at most half of the*$(t, n) \wedge$*all*$(y, n) \rightarrow$*not all*(t, y)

(i.e. *AHO-2*, by [2], Fact (2.5) and Fact (2.1))

[4] $\vdash \neg$*most*$(t, n) \wedge$*all*$(t, y) \rightarrow \neg$*all*(y, n) (by [1] and R3)

[5] $\vdash$*at most half of the*$(t, n) \wedge$*all*$(t, y) \rightarrow$*not all*(y, n)

(i.e. *HAO-3*, by [2], Fact (2.5) and Fact (2.1))

[6] $\vdash$*no*$\neg(y, n) \wedge$*all*$(t, y) \rightarrow$*fewer than half of the*$\neg(t, n)$ (by [1], Fact (1.1) and Fact (1.5))

[7] $\vdash$*no*$(y, D-n) \wedge$*all*$(t, y) \rightarrow$*fewer than half of the*$(t, D-n)$ (i.e. *EAF-1*, by [6] and D2)

[8] $\vdash$*at least half of the*$\neg(t, n) \wedge$*no*$\neg(y, n) \rightarrow$*not all*(t, y) (by [3], Fact (1.8) and Fact (1.1))

[9] $\vdash$*at least half of the*$(t, D-n) \wedge$*no*$(y, D-n) \rightarrow$*not all*(t, y) (i.e. *ESO-2*, by [8] and D2)

[10] $\vdash$*at least half of the*$\neg(t, n) \wedge$*all*$(t, y) \rightarrow$*some*$\neg(y, n)$ (by [5], Fact (1.8) and Fact (1.4))

[11] $\vdash$*at least half of the*$(t, D-n) \wedge$*all*$(t, y) \rightarrow$*some*$(y, D-n)$ (i.e. *SAI-3*, by [10] and D2)

[12] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*all*(t, y)$\rightarrow$*some*($D{-}n$, y) (i.e. *ASI-3*, by [11] and Fact (3.1))

[13] $\vdash$*no*($D{-}{-}n$, y)$\wedge$*all*(t, y)$\rightarrow$*fewer than half of the*(t, $D{-}n$) (i.e. *EAF-2*, by [7] and Fact (3.2))

[14] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*no*($D{-}n$, y)$\rightarrow$*not all*(t, y) (i.e. *ESO-1*, by [9] and Fact (3.2))

[15] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*all*$\neg$($D{-}n$, y)$\rightarrow$*some*$\neg$(t, y) (by [14], Fact (1.2) and Fact (1.4))

[16] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*all*($D{-}n$, $D{-}y$)$\rightarrow$*some*(t, $D{-}y$) (i.e. *ASI-1*, by [15] and D2)

[17] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*all*($D{-}n$, $D{-}y$)$\rightarrow$*some*($D{-}y$, t)

(i.e. *SAI-4*, by [16] and Fact (3.1))

[18] $\vdash$*all*$\neg$($D{-}n$, y)$\wedge$*no*$\neg$(t, y)$\rightarrow$*fewer than half of the*(t, $D{-}n$) (by [13], Fact (1.1) and Fact (1.2))

[19] $\vdash$*all*($D{-}n$, $D{-}y$)$\wedge$*no*(t, $D{-}y$)$\rightarrow$*fewer than half of the*(t, $D{-}n$) (i.e. *AEF-2*, by [18] and D2)

[20] $\vdash$*all*($D{-}n$, $D{-}y$)$\wedge$*no*($D{-}y$, t)$\rightarrow$*fewer than half of the*(t, $D{-}n$)

(i.e. *AEF-4*, by [19] and Fact (3.2))

[21] $\vdash\neg$*fewer than half of the*(t, $D{-}n$)$\wedge$*no*(t, $D{-}y$)$\rightarrow\neg$*all*($D{-}n$, $D{-}y$) (by [19] and R3)

[22] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*no*(t, $D{-}y$)$\rightarrow$*not all*($D{-}n$, $D{-}y$)

(i.e. *ESO-3*, by [21], Fact (2.7) and Fact (2.1))

[23] $\vdash$*at least half of the*(t, $D{-}n$)$\wedge$*no*($D{-}y$, t)$\rightarrow$*not all*($D{-}n$, $D{-}y$)

(i.e. *ESO-4*, by [22] and Fact (3.2))

So far, 14 other valid generalized syllogisms are derived from the validity of the generalized syllogism *AAM-1* through the use of the relevant definitions and facts of the generalized quantifier theory. This process fully demonstrates the reducible relationships between/among *AAM-1* and these 14 generalized syllogisms.

5. Conclusion and Future Work

Grounded in set theory, this paper proves the validity of the generalized syllogism *AAM-1* according to the truth-value definitions of quantified statements. Specifically, some other theories and rules are fully utilized on the basis of generalized quantifier theory throughout the entire research process, which are the inner and outer negation of a generalized quantifier, the symmetry of the two Aristotelian quantifiers '*some*' and '*no*', the subsequent weakening rule, the anti-syllogism inference rules, and other reduction operations. The other 14 valid generalized syllogisms have been deduced on the foundation of above research .

This innovative research not only provides a unified mathematical research paradigm for reducible relationships between/among other kinds of syllogisms (such as Aristotelian syllogisms (Łukasiewicz, 1957; Long, 2023), Aristotelian modal syllogisms (Thomason, 1997; Johnson, 2004; Cheng, 2023), and generalized modal syllogisms (Xiaojun, 2020; Jing and

Xiaojun, 2023)), but also provides theoretical support for knowledge representation and knowledge reasoning in artificial intelligence.

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References

[1] Barwise, J., & Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy*, 4(2), 159-219.

[2] Cheng, Z. (2023). *Formal Research on Aristotelian Modal Syllogism from the Perspective of Mathematical Structuralism*, Doctoral Dissertation, Anhui University. (in Chinese)

[3] Cheng, Z. (2023). How to Deduce the Other 91 Valid Aristotelian Modal Syllogisms from the Syllogism □I□A□I-3, *Applied Science and Innovative Research*, 7(1), 46-57.

[4] Endrullis, J., & Moss, L. S. (2015). Syllogistic logic with 'most'. In V. de Paiva et al. (eds.), *Logic, Language, Information, and Computation*, 124-139.

[5] Feifei, Y., & Xiaojun, Z. (2024). Knowledge Representation and Knowledge Reasoning in Square{most} and Square{all}. *Annals of Social Sciences & Management Studies*, 10(4), 555793.

[6] Hamilton, A. G. (1978). *Logic for Mathematicians*. Cambridge: Cambridge University Press.

[7] Halmos P. R. (1974). *Naive Set Theory.* New York: Springer-Verlag.

[8] Jin, X., & Xiaojun, Z. (2023). The Reducibility of Generalized Modal Syllogisms Based on □AM◇I-1. *SCIREA Journal of Philosophy*, 3(1), 1-11.

[9] Johnson, F. (2004). Aristotle's modal syllogisms, *Handbook of the History of Logic*, I, 247-338.

[10] Long, W. (2023). Formal System of Categorical Syllogistic Logic Based on the Syllogism AEE-4. *Open Journal of Philosophy*, 13(1), 97-103.

[11] Łukasiewicz, J. (1957). *Aristotle's Syllogistic: From the Standpoint of Modern Formal*

Logic. second edition, Oxford: Clerndon Press.

[12]Malink, M. (2013). *Aristotle's Modal Syllogistic*, Cambridge, MA: Harvard University Press.

[13]Moss, L. S. (2008). Completeness theorems for syllogistic fragments. In F. Hamm and S. Kepser (Eds.), *Logics for Linguistic Structures*, Berlin: Mouton de Gruyter, 143-173.

[14]Peters, S., & Westerståhl, D. (2006). *Quantifiers in Language and Logic*, Claredon Press, Oxford.

[15]Thomason, S. K. (1997). Relational modal for the modal syllogistic. Journal of Philosophical Logic, 26, 129-141.

[16]Xiaojun, Z. (2018). Axiomatization of Aristotelian Syllogistic Logic Based on Generalized Quantifier Theory. Applied and Computational Mathematics, 7(3), 167-172.

[17]Xiaojun, Z. (2020). Screening out All Valid Aristotelian Modal Syllogisms. *Applied and Computational Mathematics,* 8(6), 95-104.

[18]Xiaojun, Z., & Baoxiang, Wu. (2021). *Study on Chinese Discourse Inference.* Beijing: People's Publishing House. (in Chinese)