SCIREA Journal of Information Science
and Systems Science
ISSN: 2995-3936
http://www.scirea.org/journal/ISSS
October 21, 2024
Volume 8, Issue 5, October 2024
https://doi.org/10.54647/isss120367

The Deductibility of Categorical Syllogisms Based on the Syllogism *EIO-3* from the Perspective of Dialectics

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Abstract

This paper firstly formalizes categorical syllogisms with the help of set theory, and then conducts specific formal reasoning for them by taking advantage of generalized quantifier theory and first-order logic, and derives the remaining 23 valid syllogisms from mere *EIO-3* as a basic axiom. The deductibility between different syllogisms and the non-uniqueness of their deductive sequences again exemplify and highlight the dialectical materialist worldview that 'things are universally connected'. This knowledge reasoning pattern is not only beneficial for the in-depth development of other types of syllogistic, but also for knowledge mining in computer science.

Keywords: Categorical Syllogisms, Deductibility, Categorical Syllogisms, Knowledge Reasoning

1. Introduction

Since Aristotle, syllogism reasoning has played an undeniable role in promoting logical reasoning. In the natural language there are many types of syllogisms, such as categorical syllogisms (Patzig, 1969; Moss, 2008; Hao, 2023; Qiu, 2024), generalized syllogisms

(Endrullis and Moss, 2005; Murinová and Novák, 2012; Hao, 2024), Aristotelian modal syllogisms (Johnson, 2004; Malink, 2013), and generalized modal syllogisms (Xu and Zhang, 2023; Wang and Yuan, 2024). Among them, categorical syllogistic is the most widely used and studied. This paper focuses on studying this type of syllogisms, and the syllogisms in the following refer to categorical syllogisms.

There are studies on categorical syllogistic from different perspectives, such as Moss(2008), Zhang(2022), Wei (2023), Wang and Zhang(2024). Inspired by previous works, this paper focuses on the reduction between the syllogism *EIO-3* and the remaining valid ones.

2. Symbolization of Syllogisms

In the following, let Q be any one of Aristotelian quantifiers (that is, *all*, *some*, *no*, *not all*). The outer and inner negative quantifiers of Q are respectively denoted as $\neg Q$, $Q\neg$ (Peters & Westerståhl (2006)). And let b, h, and z be lexical variables. The sets formed of b, h, and z are respectively B, H, and Z. And D be the domain of them. '$=_{def}$' means that the left can be defined by the right. '$\vdash$' means that a proposition can be proved. Categorical syllogisms consist of the following four kinds of categorical propositions: A, E, I, and O. Proposition A is an abbreviation for 'all *b*s are *z*s' which is formalized as *all(b, z)*. Proposition E for 'no *b*s are *z*s' formalized as *no(b, z)*. Proposition I for 'some *b*s are *z*s' as *somel(b, z)*. Proposition O for 'not all *s*s are *m*s' as *not all(b, z)*.

The definitions of figures in syllogisms are as usual (Chen, 2020). For example, the expansion of the syllogism *EIO-3* is '*no(h, z)*$\rightarrow$(*some(h, b)*$\rightarrow$*not all(b, z))*', where b, h, and z are lexical variables. The others are similar.

3. Formal System of Categorical Syllogistic

The primitive symbols, formation rules, related definitions and facts, and axioms involved in this paper are as follows:

3.1 Primitive Symbols

(1) lexical variables: b, h, z

(2) operators: $\neg$, $\rightarrow$

(3) quantifier: *all*

(4) brackets: (,)

3.2 Formation Rules

(1) If Q is a quantifier, b and z are lexical variables, then $Q(b, z)$ is a well-formed formula.

(2) If p and q are well-formed formulas, then so are $\neg p$ and $p \rightarrow q$.

(3) Only the formulas obtained through (1) and (2) are well-formed formulas.

3.3 Related Definitions

Definition 1 (outer negative quantifier): $\neg Q(b, z) =_{def}$ It is not that $Q(b, z)$.

Definition 2 (inner negative quantifier): $Q\neg(b, z) =_{def} Q(b, D-z)$.

Definition 3 (truth value definition of *no*): $no(b, z) =_{def} B \cap Z = \emptyset$.

Definition 4 (truth value definition of *some*): $some(b, z) =_{def} B \cap Z \neq \emptyset$.

Definition 5 (truth value definition of *not all*): $not\ all(b, z) =_{def} B \not\subseteq Z$.

3.4 Basic Axioms

(1) A1: If p is a valid formula in propositional logic, then $\vdash p$.

(2) A2: $\vdash no(z, b) \wedge some(z, h) \rightarrow not\ all(h, b)$ (that is, the syllogism *EIO-3*).

3.5 Inference Rules

In the following rules, p, q, r and s are well-formed formulas.

Rule 1 (Subsequent weakening): From $\vdash(p \rightarrow (q \rightarrow r))$ and $\vdash(r \rightarrow s)$ infer $\vdash(p \rightarrow (q \rightarrow s))$.

Rule 2 (anti-syllogism): From $\vdash(p \rightarrow (q \rightarrow r)$ infer $\vdash(\neg r \rightarrow (p \rightarrow \neg q))$.

Rule 3 (anti-syllogism): From $\vdash(p \rightarrow (q \rightarrow r)$ infer $\vdash(\neg r \rightarrow (q \rightarrow \neg p))$.

3.6 Relevant Facts

The following facts are the fundamental knowledge of first-order logic(Hamilton, 1978)).

Fact 1 (inner negation):

(1.1) $\vdash all(b, z) \leftrightarrow no\neg(b, z)$; (1.2) $\vdash no(b, z) \leftrightarrow all\neg(b, z)$;

(1.3) $\vdash some(b, z) \leftrightarrow not\ all\neg(b, z)$; (1.4) $\vdash not\ all(b, z) \leftrightarrow some\neg(b, z)$.

Fact 2 (outer negation):

(2.1) ⊢¬*not all(b, z)*↔*all(b, z)*; (2.2) ⊢¬*all(b, z)*↔*not all(b, z)*;

(2.3) ⊢¬*no(b, z)*↔*some(b, z)*; (2.4) ⊢¬*some(b, z)*↔*no(b, z)*.

Fact 3 (symmetry of *some* and *no*):

(3.1) ⊢*some(b, z)*↔*some(z, b)*; (3.2) ⊢*no(b, z)*↔*no(z, b).*

Fact 4 (assertoric subalternations):

(4.1) ⊢*no(b, z)*→*not all(b, z)*; (4.2) ⊢*all(b, z)*→*some(b, z)*.

4. Knowledge Reasoning Based on the Syllogisms *EIO-3*

The following Theorem 1 proves the validity of the syllogism *EIO-3*. The clause '(1) *EIO-3*→*EIO-4*' in Theorem 2 means that the validity of the syllogism *EIO-4* from that of the syllogism *EIO-3*. Then one can say that there is a deducible relation between these two syllogisms. The others are similar.

Theorem 1 (the validity of the syllogism *EIO-3*): The syllogism *no(z, b)→(some(z, h)→not all(h, b))* is valid.

Proof: Suppose that *no(z, b)* and *some(z, h)* are true, then *no(z, b)*$=_{def}$ $Z\cap B=\varnothing$ and *some(z, h)* $=_{def}$ $Z\cap H\neq\varnothing$ are true by means of Definition 3 and 4 , respectively. It can be seen that $Z\cap B=\varnothing$ and $Z\cap H\neq\varnothing$ are true. It follows that $H\not\subseteq B$. Otherwise, Suppose that $H\subseteq B$. And $Z\cap B=\varnothing$, then, $Z\cap H=\varnothing$ holds. However, this contradicts the previous conclusion $Z\cap H\neq\varnothing$. Thus, $H\not\subseteq B$ holds. Therefore, *some(h, b)* is true according to Definition 5, just as desired.

Theorem 2: The remaining 23 valid syllogisms can be only derived from the syllogism *EIO-3*. According to inference steps, the deducible relationships between the syllogisms *EIO-3* and the remaining valid ones as follows:

(1) *EIO-3→EIO-4*
(2) *EIO-3→EIO-1*
(3) *EIO-3→EIO-1→EIO-2*
(4) *EIO-3→AEE-2*
(5) *EIO-3→AEE-2→AEE-4*
(6) *EIO-3→AEE-2→EAE-2*
(7) *EIO-3→AEE-2→EAE-2→EAE-1*

(8) *EIO-3→AEE-2→EAE-2→EAE-1→EAO-1*

(9) *EIO-3→AEE-2→EAE-2→EAE-1→EAO-1→EAO-2*

(10) *EIO-3→AEE-2→AEE-4→AEO-4*

(11) *EIO-3→AEE-2→AII-1*

(12) *EIO-3→AEE-2→AII–1→AII-3*

(13) *EIO-3→AEE-2→AII-1→IAI-4*

(14) *EIO-3→AEE-2→AII-1→IAI-3*

(15) *EIO-3→EIO-1→EIO-2→AOO-2*

(16) *EIO-3→AEE-2→AII-1→IAI-3→OAO-3*

(17) *EIO-3→AEE-2→AEO-2*

(18) *EIO-3→AEE-2→AEO-2→AAI-1*

(19) *EIO-3→AEE-2→AEE-4→AEO-4→AAI-4*

(20) *EIO-3→AEE-2→EAE-2→EAE-1→EAO-1→AAI-3*

(21) *EIO-3→AEE-2→EAE-2→EAE-1→EAO-1→AAI-3→EAO-3*

(22) *EIO-3→AEE-2→EAE-2→EAE-1→EAO-1→AAI-3→EAO-3→EAO-4*

(23) *EIO-3→AEE-2→EAE-2→EAE-1→AAA-1*

Proof:

[1] ⊢*no(z, b)→(some(z, h)→not all(h, b))* (i.e. *EIO-3*, basic axiom A2)

[2] ⊢*no(z, b)↔no(b, z))* (by Fact (3.2))

[3] ⊢*no(b, z)→(some(z, h)→not all(h, b))* (i.e. *EIO-4*, by [1] and [2])

[4] ⊢*no(z, b)→(some(h, z)→not all(h, b))* (i.e. *EIO-1*, by [1] and Fact (3.1))

[5] ⊢*no(b, z)→(some(h, z)→not all(h, b))* (i.e. *EIO-2*, by [4] and Fact (3.2))

[6] ⊢*¬not all(h, b)→(no(z, b)→¬some(z, h))* (by [1] and Rule 2)

[7] ⊢*all(h, b)→(no(z, b)→no(z, h))* (i.e. *AEE-2*, by [6], Fact (2.1) and (2.4))

[8] ⊢*all(h, b)→(no(b, z)→no(z, h))* (i.e. *AEE-4*, by [7] and Fact (3.2))

[9] ⊢*all(h, b)→(no(z, b)→no(h, z))* (i.e. *EAE-2*, by [7] and Fact (3.2))

[10] ⊢*all(h, b)→(no(b, z)→no(h, z))* (i.e. *EAE-1*, by [9] and Fact (3.2))

[11] ⊢*all(h, b)→(no(b, z)→not all(h, z))* (i.e. *EAO-1*, by [10] and Fact (4.1))

[12] ⊢*all(h, b)→(no(z, b)→not all(h, z))* (i.e. *EAO-2*, by [11] and Fact (3.2))

[13] ⊢*all(h, b)→(no(b, z)→not all(z, h))* (i.e. *AEO-4*, by [8] and Fact (4.1))

[14] ⊢*¬no(z, h)→(all(h, b)→¬no(z, b))* (by [7] and Rule 2)

[15] ⊢*some(z, h)→(all(h, b)→some(z, b))* (i.e. *AII-1*, by [14] and Fact (2.3))

[16] ⊢*some(h, z)→(all(h, b)→some(z, b))* (i.e. *AII-3*, by [15] and Fact (3.1))

[17] ⊢*some(z, h)→(all(h, b)→some(b, z))* (i.e. *IAI-4*, by [15] and Fact (3.1))

[18] ⊢*some(h, z)→(all(h, b)→some(b, z))* (i.e. *IAI-3*, by [15] and Fact (3.1))

[19] ⊢*all¬(b, z)→(not all¬(h, z)→not all(h, b))* (i.e. by [5], Fact (1.2) and (1.3))

[20] $\vdash$*all(b, D−z)→(not all(h, D−z)→not all(h, b))* (i.e.*AOO-2*, by [19] and Definition 2)

[21] $\vdash$*not all¬(h, z)→(all(h, b)→not all¬(b, z))* (by [18] and Fact (1.3))

[22] $\vdash$*not all(h, D−z)→(all(h, b)→not all(b, D−z))* (i.e.*OAO-3*, by [21] and Definition 2)

[23] $\vdash$*all(h, b)→(no(z, b)→not all(z, h))* (i.e. *AEO-2*, by [7] and Fact (4.1))

[24] $\vdash$*¬not all(z, h)→(all(h, b)→¬no(z, b))* (by [23] and Rule 2)

[25] $\vdash$*all(z, h)→(all(h, b)→some(z, b))* (i.e. *AAI-1*, by [24], Fact (2.1) and Fact (2.3))

[26] $\vdash$*¬not all(z, h)→(all(h, b)→¬no(b, z))* (by [13] and Rule 2)

[27] $\vdash$*all(z, h)→(all(h, b)→some(b, z))* (i.e. *AAI-4*, by [26], Fact (2.1) and Fact (2.3))

[28] $\vdash$*¬not all(h, z)→(all(h, b)→¬no(b, z))* (by [11] and Rule 2)

[29] $\vdash$*all(h, z)→(all(h, b)→some(b, z))* (i.e. *AAI-3*, by [28], Fact (2.1) and (2.3))

[30] $\vdash$*no¬(h, z)→(all(h, b)→not all¬(b, z))* (by [29], Fact (1.1) and (1.3))

[31] $\vdash$*no(h, D−z)→(all(h, b)→not all(b, D−z))* (i.e. *EAO-3*, by [30] and Definition 2)

[32] $\vdash$*no(D−z, h)→(all(h, b)→not all(s, D−z))* (i.e. *EAO-4*, by [31] and Fact (3.2))

[33] $\vdash$*all(h, b)→(all¬(b, z)→all¬(h, z))* (by [10] and Fact (1.2))

[34] $\vdash$*all(h, b)→(all(b, D−z)→all(h, D−z))* (i.e. *AAA-1*, by [33] and Definition 2)

So far, the other 23 valid syllogisms are deduced from the syllogism *EIO-3* on the basis of 34 reasoning steps.

5. Conclusion

This paper firstly formalizes categorical syllogisms with the help of set theory, and then conducts specific formal reasoning for them by taking advantage of generalized quantifier theory and first-order logic, and derives the remaining 23 valid syllogisms from *EIO-3* as a basic axiom. In fact, the proof path from one syllogism to another is non-unique. The deductibility between different syllogisms and the non-uniqueness of their deductive sequences again exemplify and highlight the dialectical materialist worldview that ‘things are universally connected’. This knowledge reasoning pattern is not only beneficial for the in-depth development of other types of syllogistic, but also for knowledge mining in computer science.

Acknowledgements

This work was supported by the Key Project of Philosophy and Social Sciences Planning in Anhui Province under Grant No.AHSKD2023D023.

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