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Free Oscillations of Spheroids on the Elastic Spring In a Viscous Fluid

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Abstract

There is solved numerically the conjugate problem of the oscillations of the axisymmetric ellipsoids, fixed at the end of the elastic spring, in the space, filled with the incompressible and viscous fluid. There is used the non-grid method of the viscous vortex domains. There are shown the boundaries of usefulness for the simplified formulas for the calculation of the non-stationary drag force, taking into account the main stationary component, the influence of the attached mass and the influence of the history of the body motion. It is found that these formulas are more appropriate when the Reynolds numbers are of the order of 10^2 and the axisymmetric ellipsoids are more elongated.

Keywords: viscous fluid, spheroid, elastic spring, oscillations

1. Introduction

To investigate the connection of the body motion characteristics (such as speed and acceleration) and other parameters, describing the body and the environment, with the hydroaerodynamics loads there is useful to consider the model problem of the free oscillations of the fixed on an elastic spring solid bodies decaying under the influence of external viscous medium. Such systems with dissipative properties are theoretically studied in [1, 2]. In the experimental work [3] the medium viscosity has been varied by dissolving of the glycerin (as a more viscous liquid) into water as a less viscous fluid. In particular, the dependence of the damping rate of the ball's oscillations in the viscous fluid on the glycerol concentration in the solution has been studied. It is shown that due to the accounting effect of the attached weight to drag in some cases, it is possible to reproduce analytically the dependence of the drag force versus time.

In the present paper there is performed the numerical simulation based on the gridless viscous vortex domains method [4-6] of the one-dimensional oscillations of the solid sphere or spheroid (which is the axisymmetric ellipsoid) with a predetermined density fixed on the end of the linear elastic spring in an infinite space filled with an incompressible viscous medium (see. Fig. 1). In contrast to the work [7], the law of motion of the body in the present work is not defined, and there is solved the conjugate problem of hydrodynamics and dynamics. We give a comparison of the integrated results of the numerical simulations at different elongations of the ellipsoid with the results of resistance calculation based on simplified formulas, such as the formula with taking into account the attached mass and the formula, taking into account the hereditary Basse force both simultaneously with the attached mass.

2. Formulation of the problem

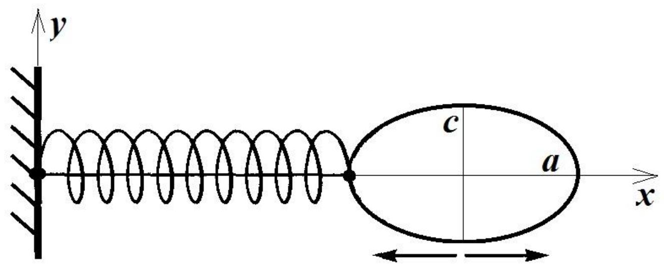

Fig. 1. Connection diagram of a spheroid with an elastic spring

One-dimensional fluctuations of the solid spheroid (axisymmetric ellipsoid) with a given constant density ρ_{body} on linear elastic spring are considered, Fig. 1. One end of the spring is fixed and the other is attached to the spheroid. Initially, the spring is stretched (initial elongation is equal to Δx), and the surrounding spheroid viscous incompressible fluid is at rest in the infinite space. The conjugate joint task of finding two scalar functions $\dot{x}_m(t),\ \Omega(t,x,y)$, satisfying the system (1), is solved numerically.

$$
\begin{aligned}
& m\ddot{x}_m \mathbf{e}_x = \mathbf{F}_a + \mathbf{F}_{ext} \\
& \mathbf{F}_a = \ddot{x}_m \mathbf{e}_x V - \pi \cdot \mathbf{e}_x \cdot \int_0^L y^2(l)\dot{\gamma}(l)dl + \frac{1}{\mathrm{Re}} \int (\mathbf{n} \times \mathbf{\Omega}) d\sigma \\
& \mathbf{F}_{ext} = -k_0 \frac{\Delta x}{L} \mathbf{e}_x \\
& \frac{d\Omega}{dt} = \frac{1}{\mathrm{Re}} \nabla^2 \Omega; \quad m = \rho_{body} \cdot V
\end{aligned} \tag{1}
$$

The boundary conditions on the adhesion of the body surface corresponding to the system (1) have a kind of connection between $\dot{x}_m$, Ω. The velocity field in the liquid is recovered from the vorticity field using the Biot-Savart formula [6]. Conjugate system (1) admits a degenerate case of the body of zero mass $m = 0$, without reducing the order of the dynamic equation, which is considered in this paper. In the case where a uniform density of ellipsoid or rigid ellipsoidal shell corresponding to zero density moves under the force of a linear elastic spring, the outer (non-hydrodynamic) force $\mathbf{F}_{ext}$ can be represented as : $\mathbf{F}_{ext} = -k_0 \frac{\Delta x}{L} \mathbf{e}_x$. Here, the dimensionless stiffness of the spring $k_0 = \frac{4k}{LU^2\rho}$,Δx - the current deviation coordinate of the center of the spheroid from the origin. Accordingly, the characteristic Reynolds number $\mathrm{Re} = \frac{UL}{\nu}$. The main dimensional scale has been adopted as L, U, ρ, where L - linear dimension of the body (as L we choose the transverse radius of the spheroid); U - the characteristic velocity of the body relative to the medium at rest at infinity; ρ - density of the medium.

3. The method of numerical solution

There is used the gridless Lagrangian method [4-6] for solving the unsteady Navier-Stokes equations - the method of viscous vortex domains (VVD). The space with nonzero vorticity is modeled by a set of small regions (vortex domains), moving relative to the fluid with the diffusion velocity. At each of the time step from the control intervals partitioning the body surface, new domains go off, modeling the flow vorticity. Circulation of each domain D remains constant. At each checkpoint of the domain R there is calculated the convective fluid velocity V and the diffusion velocity of a given domain with respect to the environment.

4. The results of calculations

As a result of calculations by this numerical method of the vortex domains at 2(a) Re = 40; 2(b) Re = 120; 2(c) Re = 300; 2(d) Re = 1200 there were obtained the dependences of the speed and hydrodynamic forces on the dimensionless time. These functions were calculated on the basis of the modeling of evolution of velocity and the pressure fields. The fig. 2 shows the variation of the instantaneous values of the velocity (V_x) for the ball and the fig. 3 – for the elongated axisymmetric ellipsoid in the process of the fluctuations of the body which is fixed at the end of the linear elastic spring with a given spring stiffness equal to 1, for a given density of a body equal to 0, and for a given initial deviation of the coordinate equal to 10 dimensionless units. At the same pictures there are presented the similar dependences on the time of the dimensionless drag force (F_x), as well as various components of this force when it calculated on the basis of simplified formulas (F_{xs} - stationary component, F_{XA} - the force taking into account the attached mass, F_{XB} - the force taking into account both the attached weight, and hereditary Basse force). From a comparison of Fig. 2(d) and Fig. 3(a), (b) it is clear that with increasing degree of elongation of spheroid from 1.0 (the ball case) to 1.2 there can be seen the improving of the quality of the approximation of the drag force dependence on the dimensionless time by taking into account the impact of the attached mass and hereditary Basse force, because with lengthening of the body the influence of the non-stationary vortices (see the figures 4, 5 and 6, showing the vortex pictures for the different values of the lengths of the body, from these pictures it can be seen that the density of the vorticity near the surface of the body is becoming lower since the relative length of this body becomes greater) on the varying with time calculated drag force (Fig. 2(c) and 2(d)) decreases. The curves at the Fig.

2(a) and 2(b) show that taking into account additionally the hereditary Basse force (curve F_{XB}) gives a slight refinement of the approximation in comparison with the formula taking into account only the attached mass (F_{XA}). From Fig. 2(b) and 2(d) it can be seen that with an increase of viscosity with a factor of 10 (Fig. 2(b)) the fluctuations of the ball are stabilized and the resistance force is more precise approximated by taking into account the attached mass (F_{XA}) and simultaneously the attached mass and the Basse force (curve F_{XB}).

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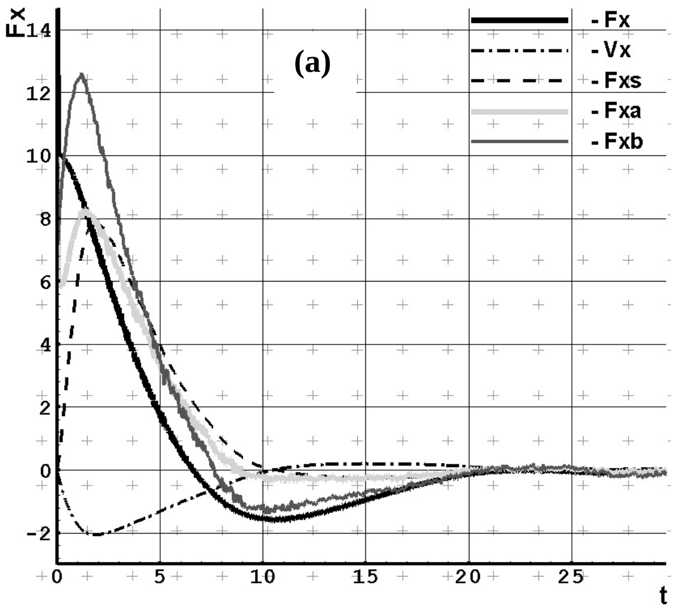

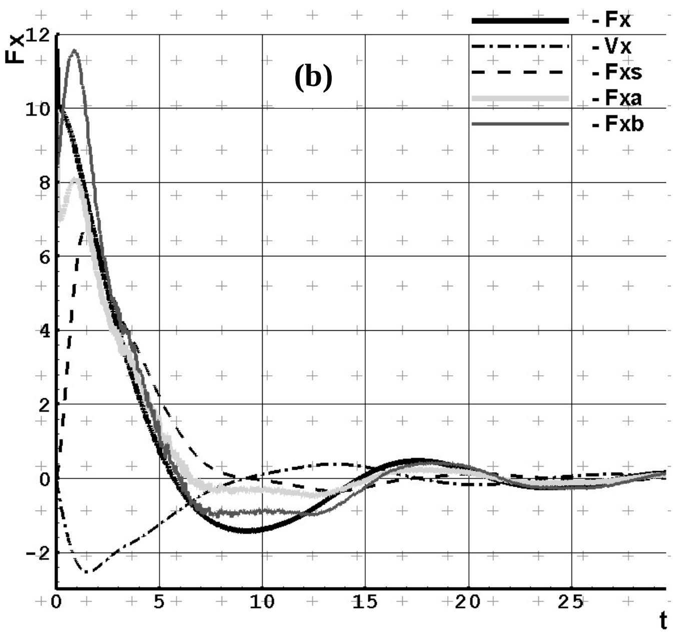

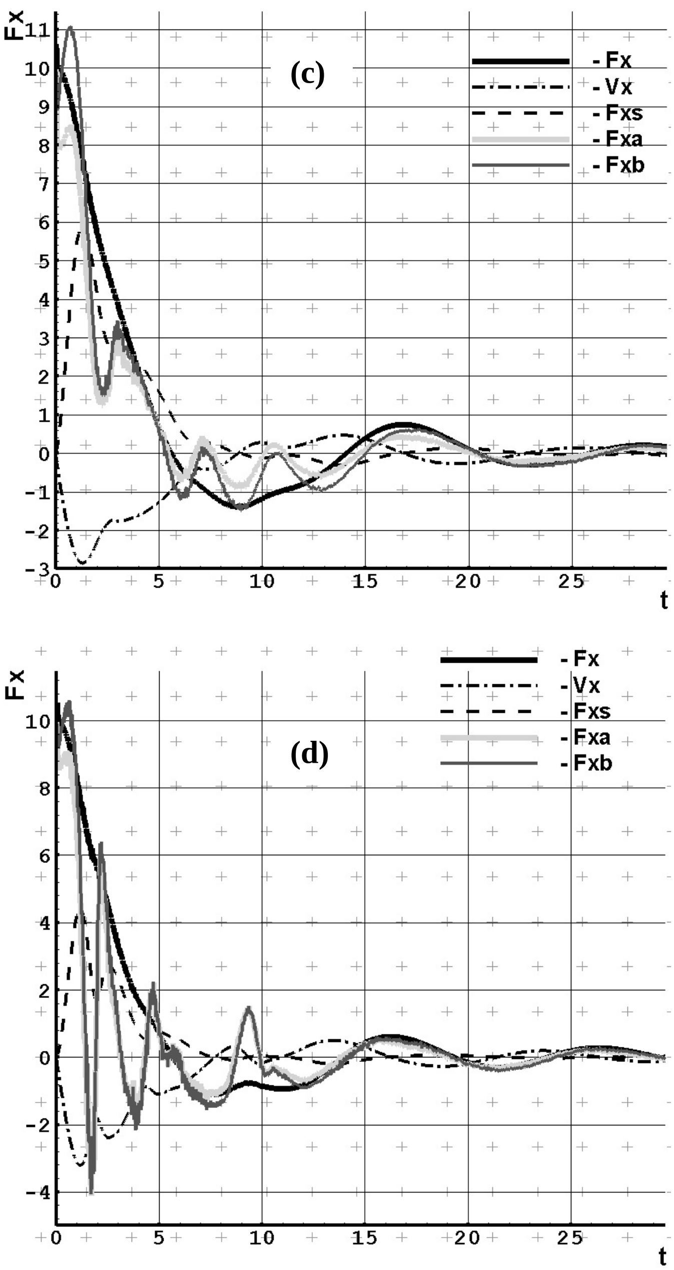

Fig. 2. Time dependences of the dimensionless speed (the V_x) and the dimensionless drag force (F_x) for the ball:

(a) Re = 40; (b) Re = 120; (c) Re = 300; (d) Re = 1200

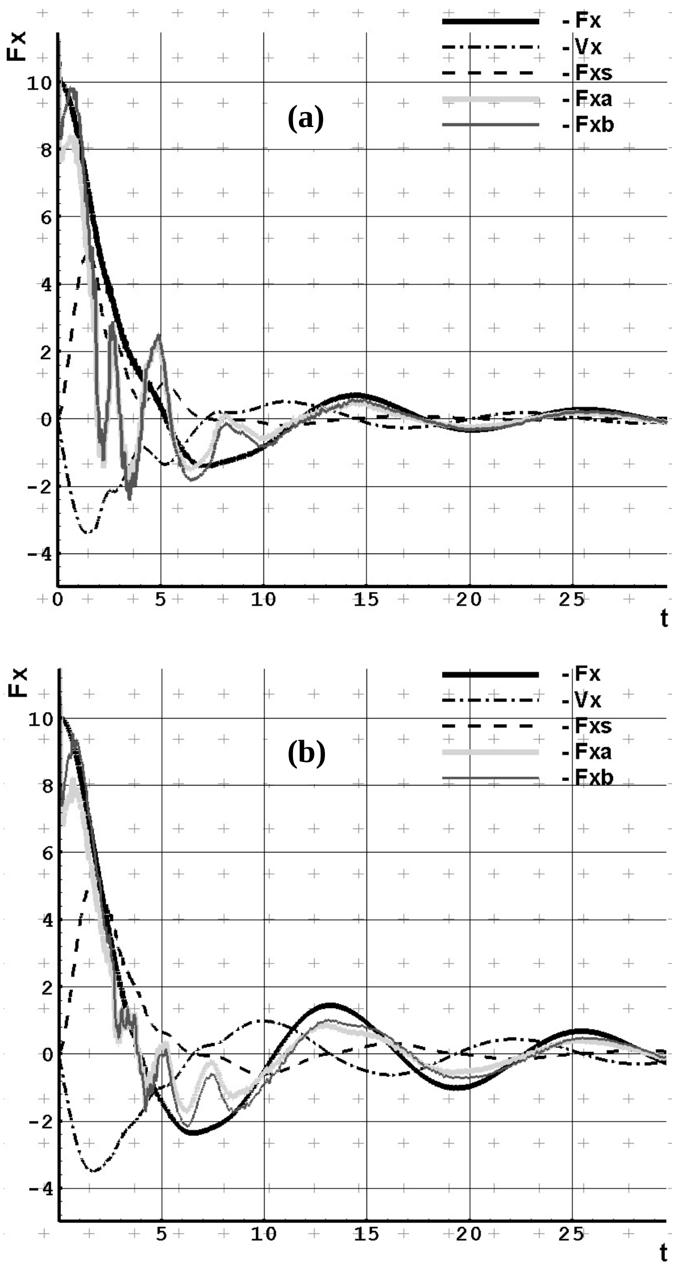

Fig. 3. Time dependences of speed (the V_x) and the dimensionless drag force (F_x)

for the spheroids with the elongations 1.1 (a) and 1.2 (b)

at Re = 1200

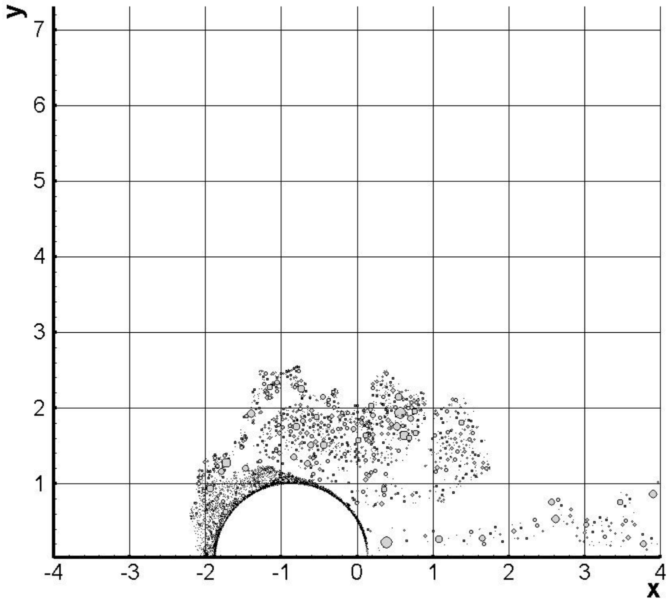

Fig. 4. Instantaneous picture of distribution of the viscous vortex domain for the spheroid with the elongation 1.0, oscillating at the elastic spring in the viscous and incompressible fluid with Re = 1200 at the time moment

t = 7.5398

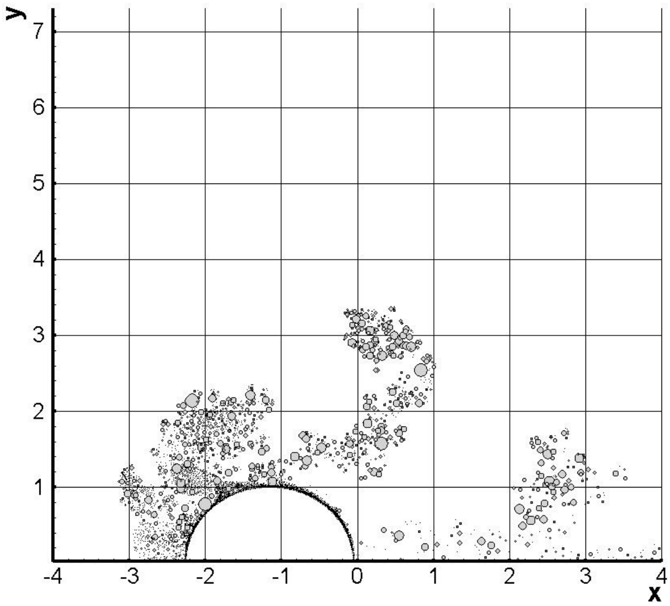

Fig. 5. Instantaneous picture of distribution of the viscous vortex domain for the spheroid with the elongation 1.1, oscillating at the elastic spring in the viscous and incompressible fluid with Re = 1200 at the time moment

t = 7.5398

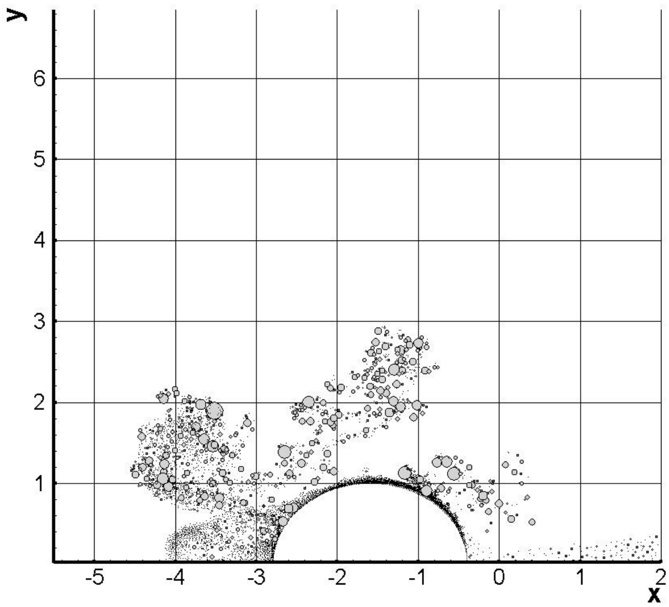

Fig. 6. Instantaneous picture of distribution of the viscous vortex domain for the spheroid with the elongation 1.2, oscillating at the elastic spring in the viscous and incompressible fluid with Re = 1200 at the time moment

t = 7.5398

5. Conclusions

These calculations performed on the basis of the method of the viscous vortex domains with the aim of modeling of the free fluctuations of the body with the zero mass shows the boundaries of usefulness of the simplified formulas for the calculation of the non-stationary drag force, taking into account the main stationary component, the influence of the attached mass and the influence of the history of the body movement. We can make the conclusion that these formulas are more effective when the Reynolds numbers are of the order of 10^2 and the axisymmetric ellipsoids are more elongated. For the ball the obtained results of the approximation are more precise when the Reynolds number is equal to 120. For the Reynolds numbers 40 and 300 the mistakes of the approximation are more significant.

References

[1] French A.P. Vibrations and Waves (New York: Norton & Company). 1971.

[2] Greiner W. Classical Mechanics. Point Particles and Relativity (New York: Springer). 2004.

[3] J. Mendoza-Arenas, E. L. D. Perico and F. Fajardo. Motion of a damped oscillating sphere as a function of the medium viscosity. European journal of physics. 31 (2010). Pp. 129–141.

[4] P.R. Andronov, M.Z. Dosaev, G.Ya. Dynnikova, Yu.D. Selyutskii, and S.D. Strekalov. Modeling of Oscillating Wind Turbine. Journal of Machinery Manufacture and Reliability, 2009, Vol. 38, No. 4, pp. 383–387.

[5] P. R. Andronov, D. A. Grigorenko, S. V. Guvernyuk, G. Ya. Dynnikova. Numerical simulation of plate autorotation in a viscous fluid flow. Fluid Dynamics, 42: 5, 719–731, 2007.

[6] G. Ya. Dynnikova. The Lagrangian approach to solving the time-dependent Navier-Stokes equations. Doklady Physics, 49: 11, 648–652, 2004.

[7] Andronov P.R., Guvernyuk S.V., Dynnikova G.Ya. Unsteady flow around and loads at a spheroid in a viscous incompressible medium. // Proceedings of the XVI All-Russian seminar on traffic control and navigation of aircraft. V. 1. Publ. SSC RAS Samara, 2013. Pp. 154-157 .