



Parameterization of prompt neutron multiplicity for first-chance actinide photofission

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Abstract

A parametric description of the dependence of the average number of prompt neutrons on the mass of fragments $\nu(A)$ from the actinides (^{235}U and ^{238}U) photofission has been developed for the range of energies of the first chance of fission (from thresholds of (γ, f) reaction to thresholds of (γ, nf) reactions). The proposed method gives a possibility to describe the dependence of "saw-tooth" behavior of prompt neutron yield from fission fragments, using several free parameters dependent on the energy and composition of nucleons, and to predict $\nu(A)$ for other isotopes of actinides. The phenomenological Wahl method was applied to estimate the number of neutrons emitted by corresponding fission fragments of atomic mass A . The total averaged number of prompt neutrons needs to construct the parametrizing function, which reasonably reproduces the characteristic features of its behavior and parameterization of an average number of prompt neutrons. To verify the proposed parametrization, according to the established fact of the equivalence of the formation of identical compound nuclei caused by reactions stimulated by photons and neutrons, calculations were carried out for the compound nuclei $^{234}\text{U}^*$, $^{236}\text{U}^*$, $^{238}\text{Np}^*$, $^{240}\text{Pu}^*$, formed as a result of neutron fission reactions for which the experimental data is

present. The results of the calculations obtained by the parametric formula of the dependence of the neutron yield on the fragment mass for the indicated fissile nuclei are consistent with experimental data within the error limits. It should be noted that the dependence values of the prompt neutron yield on the light and heavy fragments of photofission for the specified actinides, obtained as a result of calculations using the parametric formula, and additionally carried out simulations with the codes GEF, TALYS, qualitatively agree with each other and reflect the structure, characteristic of the available experimental data of actinoid fission.

Keywords: *photofission, total average prompt neutron yield, number of neutrons from fission fragments, separation energy, Wahl method, Talys, GEF*

1. Introduction

The number of prompt neutrons is an important nuclear-physical parameter that characterizes the fission process of actinides and is necessary for practical applications [1]. At some time, values of neutron multiplicity were studied only for spontaneous and neutron fission of a limited number of actinide nuclei. The experimental data and evaluation of prompt neutron yield in the case of actinides photofission are practically absent at the present time [2-4]. Data about the yields of prompt neutrons is necessary to solve the current problems of atomic engineering, such as the methods of nuclear fuel burning and long-lived actinide decontamination, detecting and studying the characteristics of special nuclear materials, and others [5-7]. Especially this is true for photonuclear data [8,9]. To solve these problems, it was necessary to create methods for estimating (calculating) the dependence of the total average number of prompt neutrons $\bar{\nu}(A_F)$ and their dependence $\nu(A)$ on the mass of photofission fragments of a wide range of actinides at fixed excitation energies. To obtain the mass and charge distribution of actinide nuclei fission products and to convert the secondary fragments (products) yields into the primary ones the same data is also used [10-12].

Generally, the phenomenological Wahl method [13], which is widely used till present for the estimation of the number of neutrons $\nu(A)$ during neutron- and gamma-induced fission, emitted by fixed fission fragments with mass A , is applied. It should be pointed out that the Wahl method does not reflect the complex structure of “sawtooth-like” $\nu(A)$ dependence due to the nuclear shells effect [1].

At the present time, there is only data on the total yields of prompt neutrons from photofission for the limited number of actinides, but data about their dependence on fragment mass is practically absent.

The reason for this is the experimental difficulties of time-span or direct neutron measurements in photofission experiments [8]. However, prompt neutron emission curves can be obtained by combining the measurement of mass distribution of pre- and post-photofission products [14].

The important question is whether it is possible to describe (parameterize) some of the main patterns using the calculation results, that depend on such parameters of actinides photofission as charge, mass, photon energy, and shell characteristics [9,15]. According to the latest research [10,12], it is possible to parameterize total prompt neutron yields $\bar{\nu}(A_F)$, which depend on the fragment masses of actinide photofission and other parameters that would reproduce the complex structure of saw-tooth behavior and allow prediction of the $\nu(A)$ dependence for the photofission of a wide range of actinides for the first-chance photofission energy range (without pre-fission neutrons) [16,17].

Thus, the goal of the current work is to solve the problem of parametric description of the dependence of the neutron yield on the first chance actinides photofission fragments mass ($\nu(A)$).

2. Parameterization of $\nu(A)$ and result of calculation

A recent analysis of experimental data prompt neutron yields from fragments of fission of ^{233}U , ^{235}U , ^{239}Pu induced by thermal neutrons and spontaneous fission of ^{252}Cf showed that for a detailed account of “saw-tooth” particularity of dependence of fission neutron yield from a mass, an efficient tool is the value of model function $R(A)$, introduced by Wahl [13], which is defined as

$$R(A) = \frac{\nu_{L,H}(A)}{\bar{\nu}(A_F)} \quad (1)$$

where $\nu_{L,H}(A)$ - prompt neutron yield of light and heavy fragment mass respectively, $\bar{\nu}(A_F)$ - total prompt neutron yield, A - fragment mass and consists of several segments to reflect the observed features, depending on the complexity of the experimental behavior of $R(A)$. Therefore, the whole range of fragments mass was divided for 2 x 4 or more segments [18]. To parameterize prompt neutron emission and identify general prediction patterns for photofission we will use the results of $\nu(A)$ calculation for the photofission ^{235}U and ^{238}U at bremsstrahlung boundary energies of 12 ÷ 30 MeV ($E^* = 9.7\text{-}14.1$ MeV) [14]. In this case, the whole range of fragments mass is sufficient to divide for 2 x 2 segments. As a result the "experimental" values of $R(A)$ (with errors), determined using formula (1) from experimental values of $\nu_{L,H}(A)$ and $\bar{\nu}(A_F)$, experimental or calculated.

Model function $R(A)$ is chosen as a linear function for each segment:

$$R_i^L(A) = a_i^L + b_i^L(A - A_L) \quad (2)$$

$$R_i^H(A) = 1 - R_i^L(A - A_H) \quad (3)$$

for light and heavy fragments, respectively, $i = 1, 2$ (number of the segment), a_i^L , b_i^L , A_L - parameters, $A_H = A_F - A_L$, A_F - the mass of the compound nucleus.

To parameterize prompt neutron emission and identify general prediction patterns we will use the results of $\nu(A)$ calculation for the photofission ^{235}U and ^{238}U at bremsstrahlung boundary energies of $12 \div 30$ MeV ($E^* = 9.7\text{-}14.1$ MeV) [14]. Here we simulate the behavior of neutrons from photofission of ^{235}U and ^{238}U actinides depending on the energy and nucleon composition in the giant dipole resonance energy range. As a result, we get the following picture (Fig. 1).

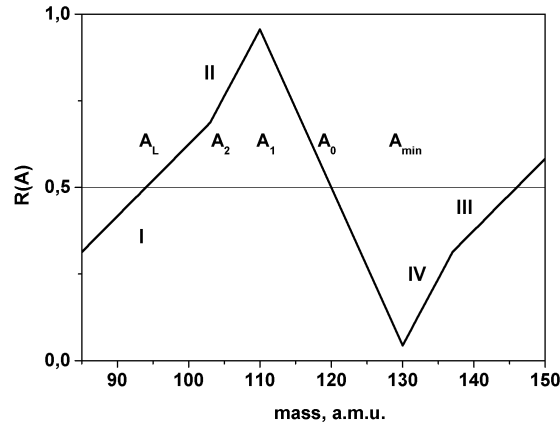


Figure 1. An example of $R(A)$ function.

Let us consider some features of $R(A)$ function. At the point of symmetric fission $A_0 = A_F/2$; $R(A_0) = 0.5$ and $R(A_L) = 0.5$, where A_L is determined from fitting. Kink points A_{min} , A_1 and A_2 are chosen from physical considerations and experiments: $A_{min} = 130$ corresponds to the mass of nearly magic nucleus fragment associated with spherical shells $Z = 50$ and $N = 82$, where fission neutron yield is minimal. Then maximum neutron fission yield for light fragments will match point A_1 , which is symmetrical to A_{min} relative to A_0 ,

$$A_1 = 2A_0 - A_{min} \quad (4)$$

The kink point A_2 corresponds to the average fragment mass of light fragments,

$$A_2 = \langle A_L \rangle = A_F - \langle A_H \rangle \quad (5)$$

and related to the intermediate deformation of the fissioning nuclide.

The parameters a_i , b_i and A_L are determined by calculating the function $R(A)$ for 4 segments I - IV (see Fig.1). The number of free parameters can be reduced using the conditions.

$$a_1 = R(A_L) = 0.5 \quad (6)$$

$$a_2 = a_1 + (b_1 - b_2)(A_2 - A_L) \quad (7)$$

The dependence of b_i slopes on excitation energy is noticed in the Fig. 1 for $R(A)$, so we have chosen $b_i = x_i + yE_\gamma$. The value of A_L varies significantly with changes of actinide mass, at least for neutron-induced actinide fission. Therefore, we chose a similar parameterization [19]:

$$A_L = 90 + B \cdot A_0 \quad (8)$$

To take even-even and even-odd effect into account we introduce the factor

$$P(N_F) = 2 - c[(-1)^{N_F} - (-1)^{Z_F}], \quad N_F = A_F - Z_F \quad (9)$$

As a result, b_i slopes will look as

$$b_i = (x_i - yE_\gamma) \cdot P(N_F), \quad i = 1, 2 \quad (10)$$

We calculate the function $R(A)$ for the photofission of actinides ^{235}U and ^{238}U according to (1) - (10) by fitting 356 "experimental" values of $R(A)$.

Using the least squares method [20], the five parameters x_1 , x_2 , c , B and y were defined to satisfactorily describe the characteristic "saw-tooth" behavior of prompt neutrons from the photofission of actinides with $A_F = 235$ a.m.u. and $A_F = 238$ a.m.u.

Table 1. Calculated parameters of $R(A)$ function.

Parameter	Value	Error
x_1	$1.66 \cdot 10^{-2}$	$0.35 \cdot 10^{-2}$
x_2	$1.08 \cdot 10^{-2}$	$0.29 \cdot 10^{-2}$
y	$-0.384 \cdot 10^{-3}$	$0.212 \cdot 10^{-3}$
c	-0.282	0.159
B	$1.26 \cdot 10^{-2}$	$0.52 \cdot 10^{-2}$

The result of $R(A)$ calculation at the maximum bremsstrahlung energy 12 MeV are shown in Fig. 2.

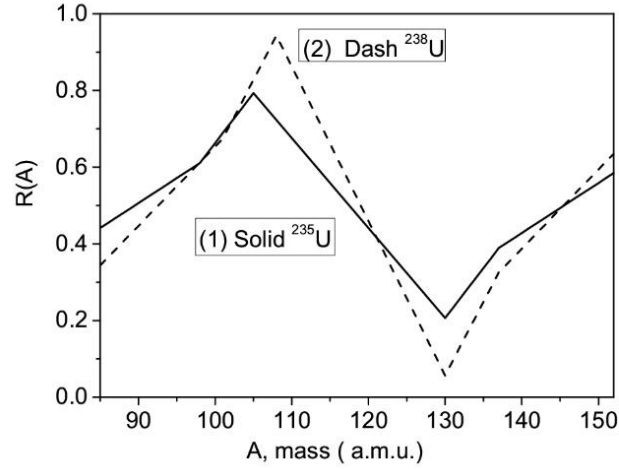


Figure 2. The result of $R(A)$ functions calculation for ^{235}U (1 Solid), ^{238}U (2 Dash) photofission at the maximum bremsstrahlung energy 12 MeV.

The curves for the dependence of prompt neutrons yield $\nu_{L,H}(A)$ from fragment mass can be calculated with help (1), if the value of the prompt neutrons averaged number for photofission of the actinides is known. Otherwise, instead of the experimental values of $\bar{\nu}(A_F)$ in (1) one may use the results of empirical calculations of $\bar{\nu}(A_F)$, presented in [21] and described below.

The initial formula has been chosen:

$$\bar{\nu}(A_F, Z_F, E_\gamma) = \bar{\nu}_0(A_F, Z_F) + a(A_F, Z_F)(E_\gamma - E_S) \quad (11)$$

where the slope $\bar{\nu}_0(A_F, Z_F)$ and the intercept $a(A_F, Z_F)$ are:

$$\bar{\nu}_0(A_F, Z_F) = C_1 + C_2(Z_F - Z_0) + C_3(A_F - A_0) + C_4P(A_F, Z_F) \quad (12)$$

$$a(A_F, Z_F) = C_5 + C_6(Z_F - Z_0) + C_7(A_F - A_0) + C_8P(A_F, Z_F) \quad (13)$$

where $P(A_F, Z_F)$ – parity factor, E_S – nucleon separation [22].

Coefficients C_i was calculated by the least-square method [20]. The final formula for calculating the total average number of prompt neutrons for actinides photofission read as:

$$\bar{\nu}_0(A_F, Z_F) = (1,97 \pm 0,05) + (0,165 \pm 0,028)(Z_F - 90) + (0,0341 \pm 0,0093)(A_F - 232) + (0,0853 \pm 0,0094) * P(A_F, Z_F) \quad (14)$$

$$a(A_F, Z_F) = (0,0963 \pm 0,75 * 10^{-2}) + (0,0371 \pm 0,43 * 10^{-2})(Z_F - 90) + (0,566 \pm 0,138) * 10^{-2}(A_F - 232) \quad (15)$$

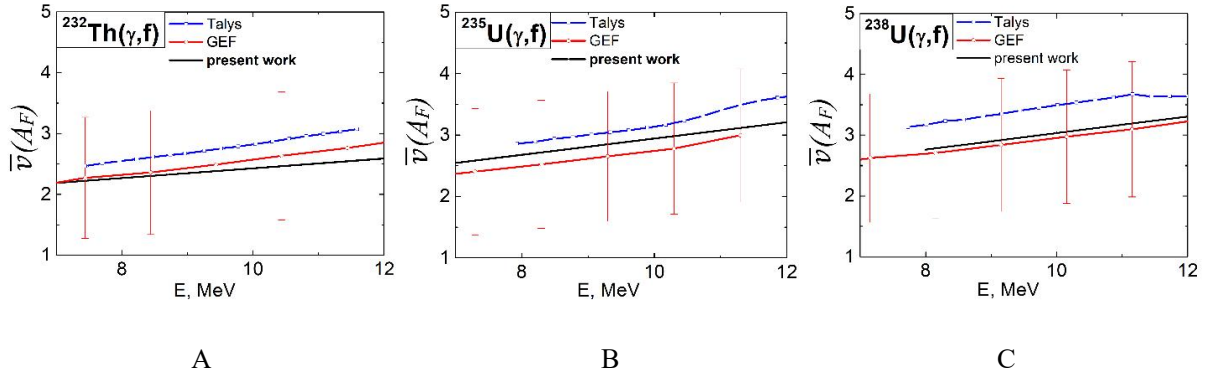
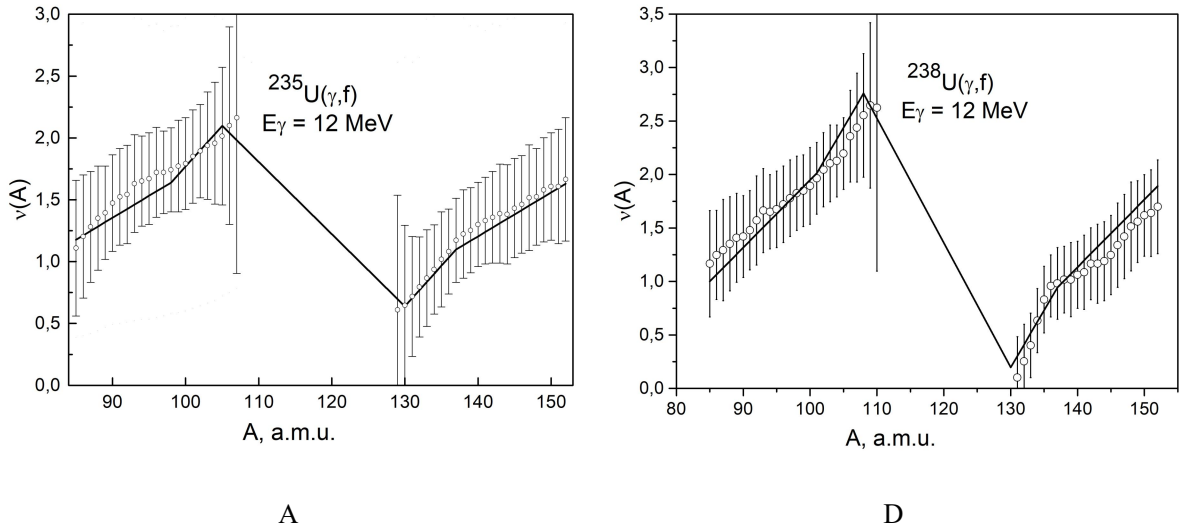


Figure 3. Energy dependence of the total average number of prompt neutrons in photofission of ^{232}Th (A), ^{235}U (B) and ^{238}U (C) actinides

The results of calculations of the energy dependence of the total average number of prompt neutrons ($\bar{\nu}(A_F)$) in the photofission of actinides ^{232}Th , ^{235}U and ^{238}U are shown in Fig. 3. For comparison simulation results using the GEF [23,24] and TALYS [25,26] codes are also given, which are consistent with our data. Calculations with the TALYS code were performed without changing the parameters (by default) for all simulation cases.

The results of the $\nu_{L,H}(A)$ calculation using (1)-(15) and Table 1 are shown in Fig. 2 and Fig. 4. The Obtained results that are shown in figures are consistent with experimental data. Thus, using the modified Terrell method [27] possible values of prompt neutron yield from light and heavy fragments with known total yields can be estimated just through the mass distributions of fission fragments.



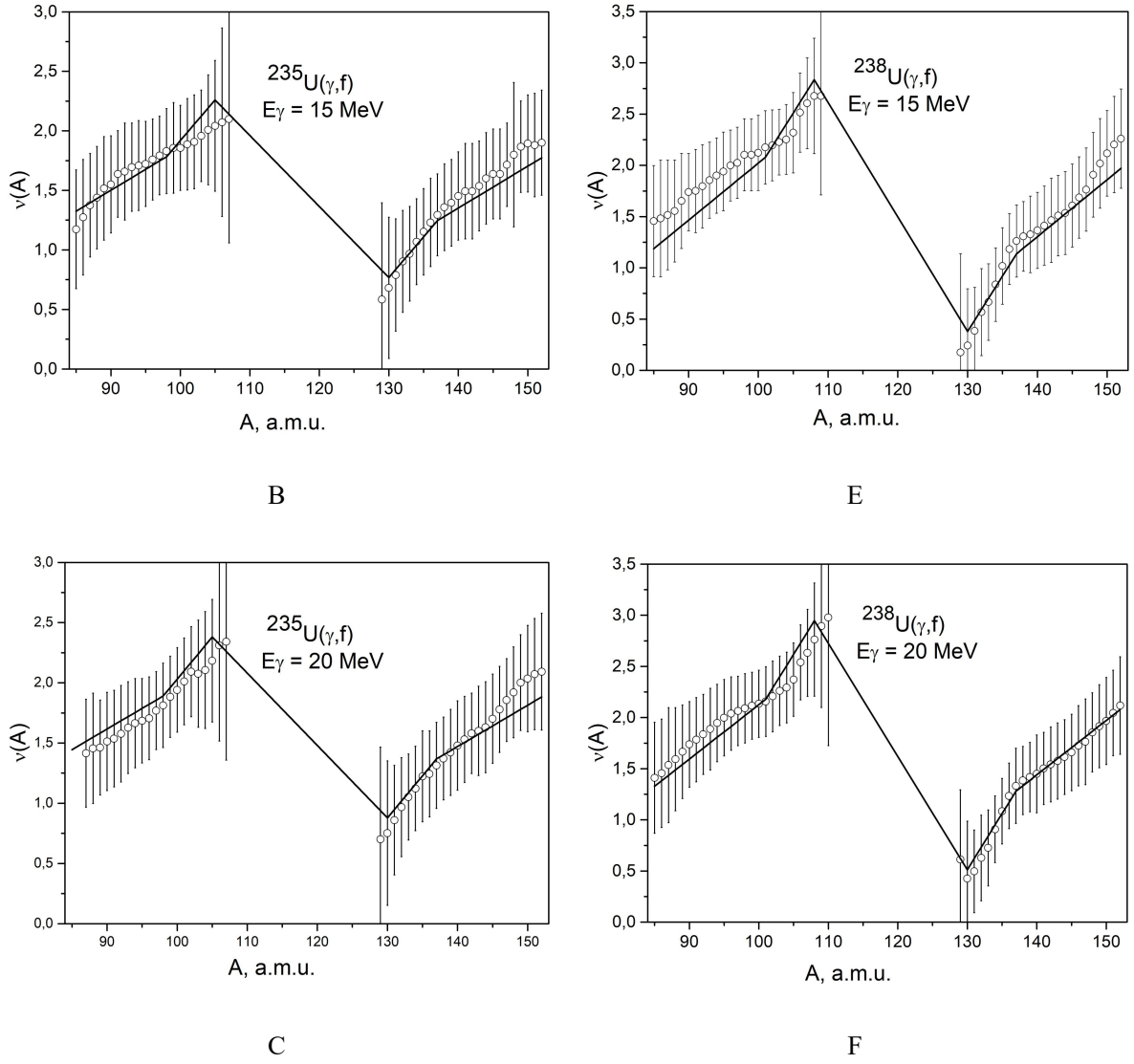


Figure 4. Results of 2x2-segments calculation of $v_{L,H}(A)$ (solid lines) of ^{235}U (A, B, C) and ^{238}U (D, E, F) photofission at maximum bremsstrahlung energy of 12, 15, 20 MeV.

The obtained results of the estimation of the dependence of the prompt neutron yields from light and heavy fragments for the first chance of actinide (^{232}Th [28], ^{235}U [14], ^{238}U [14] at excitation energies 8.8, 9.7 and 9.7 MeV respectively) photofission are compared with the results of calculations (modeling) by the “GEF” [24] and “TALYS” [26] codes (Fig. 5).

In addition, calculations were made of the dependence of prompt neutron yields from individual fragments using the simple Wahl method [29], which is still widely used today [10] (Fig. 5). The values of $v(A)$ for light v_L and heavy v_H masses of fission fragments were calculated by the Wahl method [29] based on the following equations (16)-(17):

$$v_L = 0.531 \langle v \rangle + 0.062(A_L + 143 - A_C), \quad (16)$$

$$v_H = 0.531 \langle v \rangle + 0.062(A_H + 143) \quad (17)$$

where A_C is mass of the compound nucleus and $\langle \nu \rangle$ is the average number of prompt neutrons emitted during the fission.

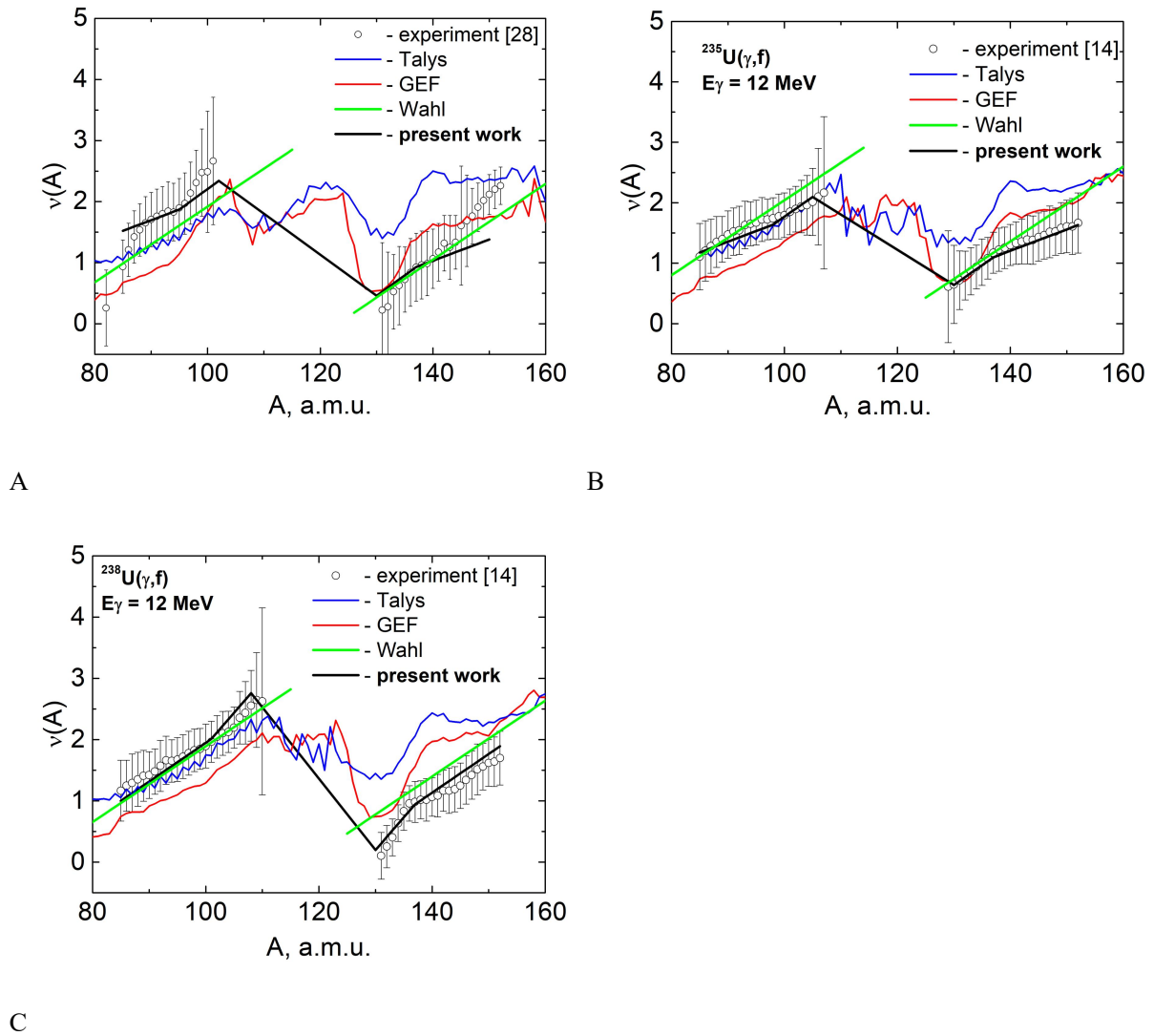


Figure 5. Dependence $\nu_{L,H}(A)$ at fixed excitation energies for fissile nuclei ^{232}Th (A), ^{235}U (B), and ^{238}U (C)

With $R(A)$ function parameterization, which fairly well reproduces the characteristic features of its behavior and parameterization of an average number of prompt neutrons, we can calculate the expected values of prompt neutrons yield for other actinides. Thus, to determine the photofission yield for neutron yield on fragment mass $\nu(A)$ of arbitrary actinides we need to know the value of $\nu(A)$, which is determined by the general empirical formulas (11)-(15). The resulting formulas of prompt neutron yield on fragment mass $\nu(A)$ of photofission of actinide nuclei can be used as initial (seed) for solving the integral equations [19] in these processes. The calculations of GEF [24], TALYS [26] and the simple Wahl method [29] are in only qualitative agreement with data.

3. Approbation and neutron data comparison

To verify the adequacy of the parametric description of the dependence of the neutron yield on the first chance photofission fragments mass ($\nu(A)$) for the wide range of actinides, the available experimental data of neutron fission [3,4] were used. According to [30,31], in order to use the data of the neutron fission reaction (n,f) to describe the photofission reaction (γ ,f), it is necessary to use the data (n,f) for the isotope ($A-1$)X, which has one neutron less:

$${}^AX(\gamma,f){}^AX^* \leftrightarrow ({}^{A-1})X(n,f){}^AX^* \quad (18)$$

The energy of the neutron E_n causing the fission reaction will be equivalent to the average excitation energy E^* of the fissile nucleus formed by photons minus the neutron separation energy S_n [22] for this fissile nucleus AX :

$$E_n \leftrightarrow E^* - E_S \quad (19)$$

The data of $\nu(A)$ were used to analyze the following reactions: ${}^{233}\text{U}(n,f)$ [32], ${}^{235}\text{U}(n,f)$ [33,34], ${}^{237}\text{Np}(n,f)$ [35], ${}^{239}\text{Pu}(n,f)$ [36], as a result of which fissionable nuclei ${}^{234}\text{U}^*$, ${}^{236}\text{U}^*$, ${}^{238}\text{Np}^*$, ${}^{240}\text{Pu}^*$ are formed.

As input data for the calculations of $\nu(A)$ (or $\nu_{L,H}(A)$), the values of $\bar{\nu}(A_F)$, obtained according to formulas (14) – (15) and which are presented in Table 2, were used. The same table presents the values of the estimated $\bar{\nu}(A_F)$ data from the ENDF library [37] for fissile nuclei ${}^{234}\text{U}^*$, ${}^{236}\text{U}^*$, ${}^{238}\text{Np}^*$, ${}^{240}\text{Pu}^*$, formed as a result of photofission, as well as the results of calculations with the GEF [24] and TALYS [26] codes. Simulations with the TALYS code were performed for two separate fission channels (by photons and neutrons) of the formation of the specified fissile nuclei. As can be seen from the table, the results of our calculations agree with the values from the ENDF library of estimated nuclear data [37] and correlate with the results of simulations by the GEF [24] and TALYS [26] codes.

Table 2. Average values of prompt neutron yields ($\bar{\nu}(A_F)$) for fissile nuclei ${}^{234}\text{U}^*$, ${}^{236}\text{U}^*$, ${}^{238}\text{Np}^*$, ${}^{240}\text{Pu}^*$

Reaction	En, MeV	E*, MeV	Present work	ENDF [36]	GEF [24]	TALYS + γ [26]	TALYS +n [26]
${}^{233}\text{U}(n,f){}^{234}\text{U}^*$	n _{th}	6.845	2.370	2.480	2.429	2.808	2.782
${}^{235}\text{U}(n,f){}^{236}\text{U}^*$	n _{th}	6.546	2.460	2.411	2.370	2.864	2.864
${}^{235}\text{U}(n,f){}^{236}\text{U}^*$	0.5	7.046	2.531	2.481	2.509	2.943	2.932
${}^{235}\text{U}(n,f){}^{236}\text{U}^*$	5.5	12.046	3.263	3.031	3.021	3.612	3.471
${}^{237}\text{Np}(n,f){}^{238}\text{Np}^*$	0.5	6.288	2.460	2.625	2.458	3.025	2.923
${}^{237}\text{Np}(n,f){}^{238}\text{Np}^*$	5.5	10.988	3.270	3.224	3.086	3.721	3.692
${}^{239}\text{Pu}(n,f){}^{240}\text{Pu}^*$	n _{th}	6.534	2.750	2.864	2.760	3.354	-

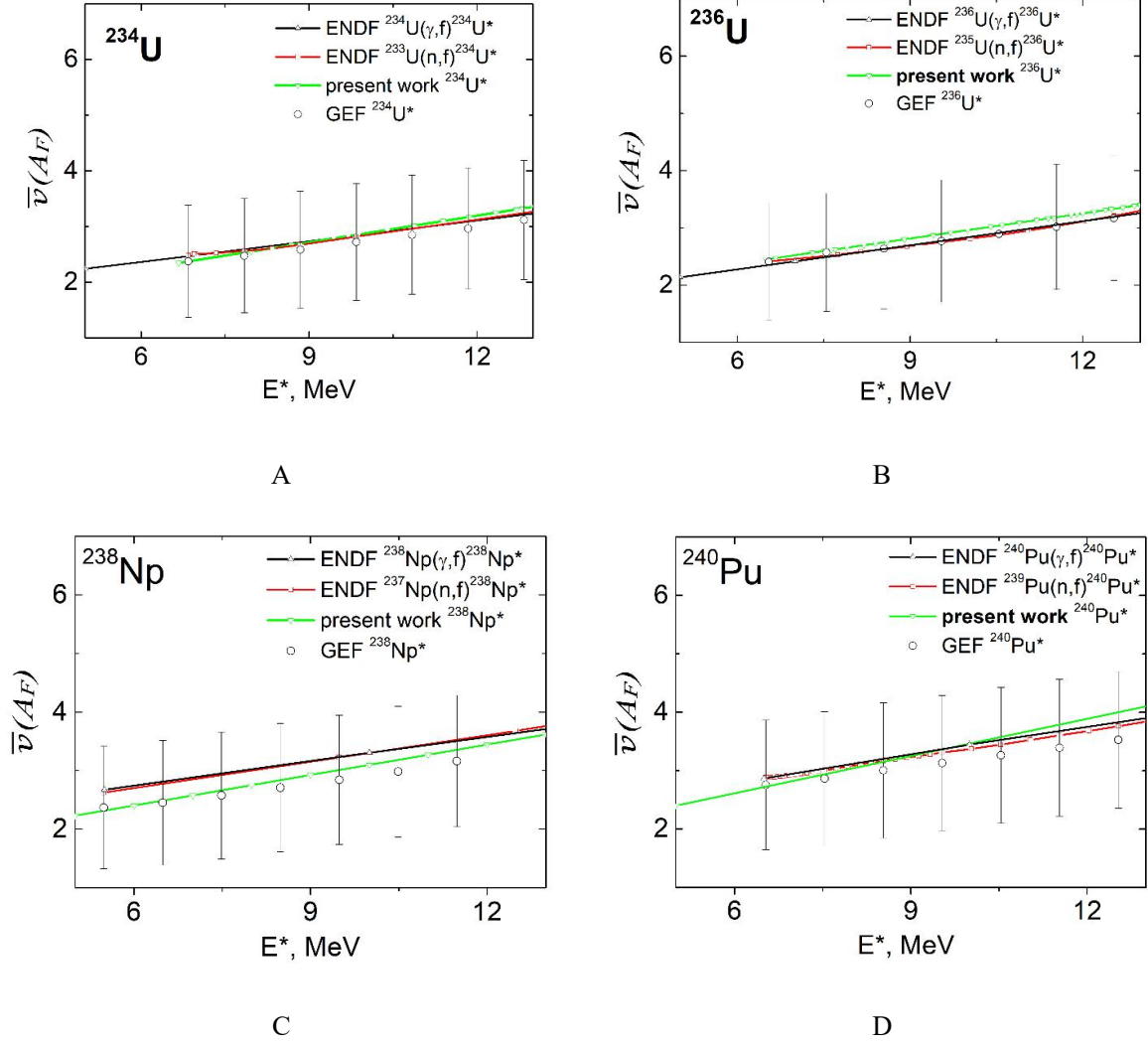


Figure 6. Energy dependence of the total average number of prompt neutrons in photofission of ^{234}U (A), ^{236}U (B), ^{238}Np (C) and ^{240}U (D) actinides

The value of the average excitation energies of fissile nuclei formed by the (n,f)-reaction channels were calculated from the average neutrons energy $\langle E \rangle$ according to the formula (18) [38] :

$$\langle E^* \rangle = (\Delta^A X + \Delta n - \Delta^{A+1} X) + \langle En \rangle \quad (20)$$

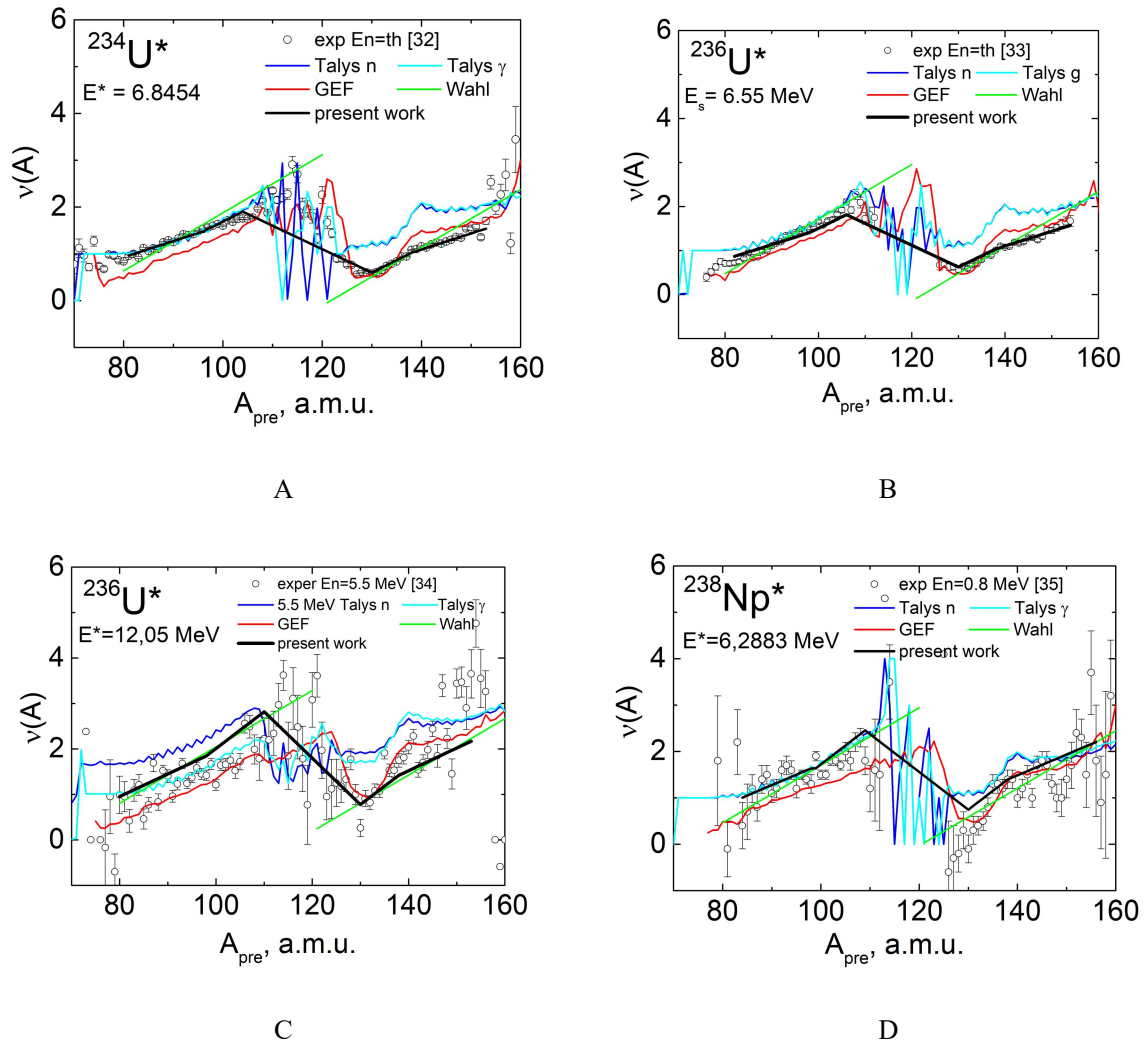
where X – primary actinide nuclei, Δ – excess (or defect) of masses, the values of which were taken from [39].

Fig. 6 shows the dependence of $\bar{\nu}(A_F)$ on the excitation energy (first chance) of fissile nuclei $^{234}\text{U}^*$, $^{236}\text{U}^*$, $^{238}\text{Np}^*$, $^{240}\text{Pu}^*$. The values of $\bar{\nu}(A_F)$ were calculated by formulas (14)-(15) and modeled by the GEF code [24]. Additionally, data from the ENDF library [36] (for photon and neutron fissile nuclei formation channels) are presented for comparison with our calculations. The presented data are consistent with each other.

The results of the calculations obtained by the parametric formula of the dependence of the neutron yield on the fragment mass $\nu(A)$ for fissile nuclei $^{234}\text{U}^*$, $^{236}\text{U}^*$, $^{238}\text{Np}^*$, $^{240}\text{Pu}^*$ are presented in fig. 7. The results are in consistency with experimental data [30-34] within the error limits.

The same figure shows the results of calculations by GEF [24] and TALYS [26] codes and the simple Wahl method [29]. Simulations with the TALYS code were performed for two separate fission channels (by photons and neutrons) of the formation of the fissile nuclei specified above.

It should be noted that the values of the dependence of the prompt neutron yield on individual fragments of photofission for the specified actinides, obtained as a result of calculations by the parametric formula, simulations by codes GEF [24], TALYS [26], and the simple Wahl method [29], qualitatively agree with each other and reflect the structure characteristic of the available experimental data of actinoid fission [1,22,24].



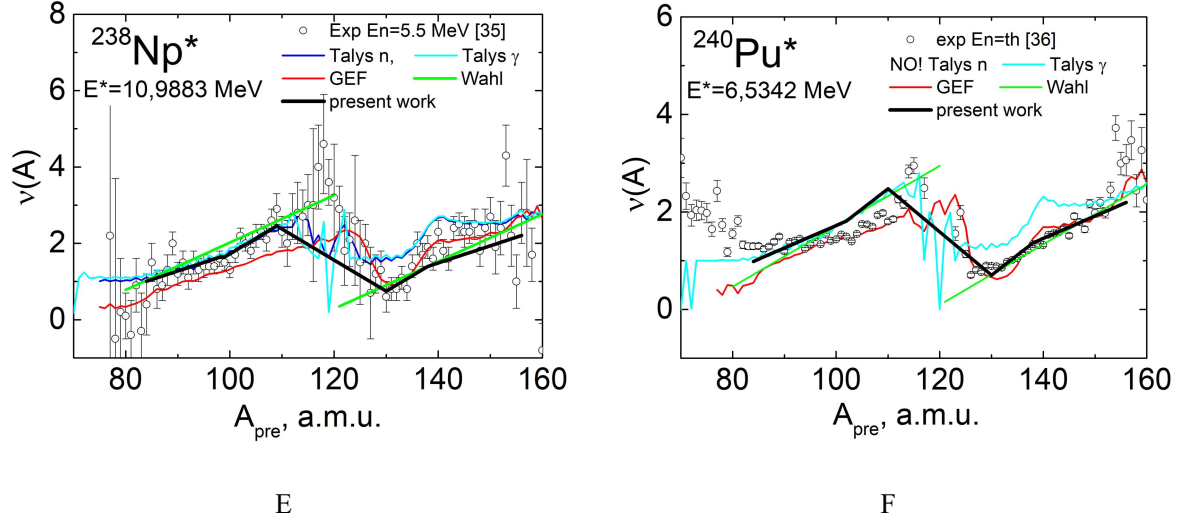


Figure 7. Dependence $\nu_{L,H}(A)$ at fixed excitation energies for fissile nuclei $^{234}\text{U}^*$ (A), $^{236}\text{U}^*$ (B,C), $^{238}\text{Np}^*$ (D,E), and ^{240}Pu (F)

4. Computer programs for the stimulation of the neutron multiplicity of the actinide's photofission

Based on semi-empirical models for describing the neutron multiplicity during photofission of actinides for the range of first-chance energies, a complex of computer programs “NPMA Prompt neutron yield version 1.0.2301” [40] and “NPMU Prompt neutron parametrization version 1.0.2302” [41] was created to carry out calculations of $\langle AF \rangle$ and $\nu(A)$ values.

Computer programs can be installed and run for a wide range of platforms (MSI – installation file for Windows platforms, DMB – installer file for MacOS platforms, RMP – installer file for Linux platforms, JAR – Java archive format for running under the JVM virtual machine), which support the JVM. They can be used by a wide range of end users, as they do not require large computer computing resources.

Computer programs can be used in addition to scientific research to support the educational process in educational institutions when teaching a nuclear physics course.

Programs can be obtained on request from the Institute of Electron Physics.

5. Conclusions

Parametrization was made for the dependence of the average number of prompt neutrons $\nu(A)$ emitted from fission fragments from their mass number A of first-chance photofission of

actinide nuclei ^{232}Th , ^{235}U , and ^{238}U . The phenomenological Wahl method was used as the basis for the proposed parametrization of the dependence from fragment masses. The suggested approach allowed us to describe the observed typical changes of the "saw-tooth" structure of prompt neutron yield from the light and heavy fission products using few energy and nucleon composition-dependent free parameters and to predict $\nu(A)$ for other actinide isotopes in an energy range of first chance.

In present work, the expected values of prompt neutrons yield for arbitrary neighboring actinides, such as fissioning nuclei ^{234}U , ^{236}U , ^{238}Np and ^{240}Pu , were calculated to verify the proposed parametrization. As an input parameter for calculations the total average number of prompt neutrons $\bar{\nu}(A_F)$, which reasonably reproduces the characteristic features of its behavior and parameterization of an average number of prompt neutrons, was taken. The obtained parametrization results of estimated dependence of the first chance prompt neutron fission yields $\nu(A)$ from light and heavy fragment masses A are compared with the results of calculations (modeling) by the program codes GEF, TALYS and simple Wahl method. Both models give the results only qualitatively consistent. The energy dependence of neutron multiplicity is the same in both cases.

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