Examining Einstein’s Discrepancy Concerning Clock Synchronization That Involves His Second Relativity Postulate

Steven D. Deines 1

1 Donatech Corporation, Inc., Fairfield, Iowa USA
Email: physicspublishing@gmail.com

Abstract:
In his 1905 special relativity paper, Einstein defined clock synchronization occurs when photon transmissions traverse a distance (or rod) with equal time spans in either direction. With a universal photon speed, he discovered clocks on the ends of a rod moving parallel with uniform velocity cannot be synchronized, but clocks attached to a stationary rod can be, using an inertial resting frame. He claimed the discrepancy was due to simultaneous observations in one frame, but nonsimultaneous in another inertial frame. His claim is invalid, since his derivation used one stationary observer in the same resting frame for both cases without extra observers or frames. His first relativity postulate mandates the system’s state (e.g., clock synchronization) is unaffected by a constant velocity. Equating Einstein’s synchronization for both the stationary and uniformly moving rods, the photon velocity obeys vector velocity addition when the photon source moves relative to the detector, but a roundtrip traverse of photons has an average speed that is identical to the standard magnitude of $c$. Early photon speed tests performed by Fizeau, Foucault, and Michelson were horizontal roundtrip flights that indicated photon speed is universally constant. Six discrepancies and contradictions involving the second relativity postulate are discussed, including special relativity does not
preserve the numerical speed of photons between moving inertial frames. Einstein’s synchronization explains the constructive interference output from the Michelson-Morley interferometer. Past special relativity analyses claimed only length contraction parallel to the interferometer’s constant velocity makes the traverses over the arms equal to output constructive interference. That ignores parallel time dilation projected on each arm that differently alters photon frequency to cause destructive interference. Photon velocities with vector velocity addition seem to resolve many dilemmas from ultraprecise tests including three Pioneer anomalies.

**Keywords:** Special relativity, length contraction, time dilation, photon speed, gravitation, interferometer, Pioneer anomalies

1. Preface

Albert Einstein wrote his early papers in German, especially his famous 1905 paper on special relativity. He hardly knew English until 1913 when he began to take English lessons [1, page 178]. To avoid lengthy translations from Einstein’s original German into English, the author chose the translation of Arthur I. Miller, who was a professor of physics at Harvard and Lowell Universities and who later transitioned as a historian of science due to his keen interest in Einstein’s special relativity paper. In Miller’s book *Albert Einstein’s Special Theory of Relativity*, he translated numerous correspondences involving Einstein and discussed the many issues debated by Einstein’s many supporting and opposing contemporaries. Miller has translated Einstein’s 1905 paper in the appendix of that book, which differs from previous (and, in places, unacceptable) English translations. Typographical errors in the original *Annalen* version are flagged, which went into the Teubner edition, and additional errors that appear in the Dover reprinted volume *The Principle of Relativity*. Footnotes from Einstein and Sommerfeld are annotated. This appendix will be the English translation with demarcations for sections, §, as used in the original German publication and line numbers within each section (a new section resets line numbering to 1 in the translation). References in that book to other related topics outside the appendix are listed with the actual pages.

In this manuscript, velocity of a photon combines the speed of a photon with a specific vector direction. Photon speed refers to the magnitude in m/s of photon velocity.
The author requires that multiple observers concurrently recording an experiment must have the same results, especially when those results are transformed to one common reference frame. Else, experimental physics is a waste of time if multiple observers collecting such concurrent data do not agree on the results. Theoretical physics is then useless since there is no agreed lab result for theory to explain the outcome.

2. Development of Special Relativity

Isaac Newton published the first unified theory of physics that combined the laws of mechanics with Newtonian gravity in his *Principia* [2]. The axioms of mechanics did not fit well with the electric and magnetic research of the 19th century that developed field theory. Steady-state experimental results produced Gauss’s law (a generalized consequence of Coulomb’s law), and Ampère’s law. Interestingly, Weber and Kohlrausch measured the ratio of magnetic and electrostatic units of charge to get the ratio \((\varepsilon \mu)^{0.5}\) that was numerically close to Fizeau’s average speed of light [3]. James Clerk Maxwell found that Faraday’s induction law for dynamical conditions was inconsistent with the Ampère law, which he modified by adding a displacement current to Ampère’s electric current density. This was crucial in the differential as it predicted rapidly fluctuating fields can produce electromagnetic radiation. Maxwell published his derivations that consisted of 20 coupled partial differential equations [4]. Oliver Heaviside employed the curl and divergence operators from vector calculus to reduce Maxwell’s theory into six independent operator equations [5]. Maxwell’s equations show that \(c = (\varepsilon \mu)^{0.5}\). Maxwell proposed that an emitted light ray could be used to standardize both the meter and the second [4, p. 3]. He asserted that electromagnetic waves propagate in a vacuum with one speed of light and predicted light was an electromagnetic wave. This was the second unified theory, which combined classical electromagnetism, electric circuits, and optics.

While many experimental tests were conducted to confirm Maxwell’s theory, Albert Einstein was beginning his studies at the Eidgenössische Technische Hochschule (ETH) in Zurich. Although he failed the entrance exam initially, he passed that exam a year later in 1896. The ETH curriculum was limited, so Einstein often skipped classes to study research papers privately [1, p. 6]. Louis Kollros, a classmate of Einstein at the ETH, recalled “we waited in vain for an exposition of Maxwell’s theory.” Einstein strove to fill this gap by reading independently “the works of Helmholtz, Maxwell, Hertz, Boltzmann and Lorentz.” Kollros
continued, “He neglected his courses knowing that at examination time, he could study the notes taken concisely by his friend Grossmann” [6][1, p.151]. After earning his ETH diploma in 1900, he was denied an ETH assistantship and was unemployed between odd teaching positions. With the help of Marcel Grossman’s father, Einstein secured a position in June 1902 at the Swiss Federal Patent Office in Bern as a technical expert, third class [1, p. 6-7]. Joseph Sauter, a colleague of Einstein at the Patent Office, recalled that “Einstein was admitted to the post without possessing an engineering diploma, but as a physicist au courant with Maxwell’s theory” [7]. Gerald Holton mentions [8] “an almost forgotten teacher of Einstein”; namely Föppl, the author of a widely-read text of 1894 [9] on the Hertz-Heaviside version of Maxwell’s theory. Einstein learned enough of Maxwell’s electromagnetism to competently review applications at the patent office [1, p. 151]. Holton noted that Föppl’s book was “just the kind of book an interested student would want if he were deprived of Maxwell’s theory in course lectures.[8]” The chapter of interest is Chapter V: “Die Elektrodynamik bewegter Leiter” (The Electrodynamics of Moving Conductors), of which Föppl analyzed the arrangement of a magnet and conductor, which Einstein began the relativity paper [1, p. 151]. Very probably, Einstein could have scanned the updated and rewritten version by Abraham of Föppl’s original text on Maxwell’s theory [10].

Before Einstein published his 1905 relativity paper, several books, monographs, and journals about electrodynamics were definitely read, very probably read, and possibly read by him [1, p. 87-92]. While in Bern, he participated in the “Olympia Academy”, which was an informal study group composed of Einstein and his friends. One reviewed book as mentioned by Carl Seeling [11] was Science and Hypothesis by Henri Poincaré, written in German [12]. Poincaré was critical that Maxwell’s equations would ever be derived from mechanics. “Poincaré not only identified the pressing problems of the time as the failure of the either-drift experiments and the lack of explanation for Brownian motion and ‘the cause which produces the electric spark under the action of ultraviolet light’ (i.e., photoelectric effect), he also sketched a brilliant proof that any system satisfying the principle of least action admitted of an unlimited number of mechanical explanations. [1, p.129]” Einstein wrote two papers explaining the Brownian motion and photoelectric effect in 1905 and concluded his 1905 submissions in the Annalen with special relativity and his energy-mass equation. The next section will examine Einstein’s discussion of special relativity in his 1905 paper and the problems that have been uncovered.
3. Einstein’s Thought Experiment: Reflecting Light Between Rod Ends

Einstein’s 1905 relativity paper does not cite any reference to other papers or experiments, but listed four footnotes. Instead, it gives a logical discourse based on personal experiences, which can easily mislead. “Let us consider a coordinate system in which the equations of Newtonian mechanics hold. For precision of demonstration and to distinguish this coordinate system verbally from others which will be introduced later, we call it the ‘resting system’.” [1, §1, lines 1-5]

He considered simultaneous events occur with a time associated at a single location. Einstein’s example was a train arrived at 7 o’clock, which he meant the small hand of his watch pointed at 7 as a train pulled in [1, §1, lines 13-17]. Einstein footnoted there is an inexactitude of two events occurring at (approximately) the same place. He admitted that an observer with a clock will not independently determine time of a remote event communicated to the observer by light [1, §1, lines 24-32].

Einstein considered a clock A at position A and an identical clock B at location B, but there was no way to connect “A time” to “B time” with a common time for locations A and B [1, §1, lines 33-41]. He stated, “The latter time can now be defined by requiring that by definition the ‘time’ necessary for light to travel from A to B be identical to the ‘time’ necessary to travel from B to A. Let a ray of light start at the ‘A time’ \( t_A \) from A toward B, let it at the ‘B time’ \( t_B \) be reflected at B in the direction of A, and arrive again at A at the ‘A time’ \( t_A' \).” [1, §1, lines 41-45] He concluded that the two clocks run in synchronization if [1, §1, Eq. (1.1)]

\[
t_B - t_A = t_A' - t_B
\]

This definition only uses one-way transmissions. Einstein assumed this definition of synchronization was free of any contradictions. He further claimed that (1) if a clock at B synchronizes with the clock at A, the clock at A synchronizes with the clock at B, and (2) if the clock at A synchronizes with the clock at B and also the clock at C, then the clocks at B and C synchronize with each other [1, §1, lines 48-60]. He added the requirement [1, §1, Eq. (1.2)] that for length \( AB \)

\[
\frac{2AB}{t_A - t_A'} = c
\]

where \( c \) is the universal constant for light speed. This is Einstein’s synchronization method based on his definition. Equation 1.1 stipulates that opposite one-way traverses over a fixed distance are equal. The roundtrip time span, \( t_A' - t_A \), is divided by 2, which matches the one-
way transmission span, to find the extra time to advance the broadcast time tag from the master clock to set the remote clock. This synchronizes the remote clock to the master clock time. Einstein stated, “The ‘time’ of an event is the reading simultaneous with the event of a clock at rest and located at the position of the event, this clock being synchronous, and indeed synchronous for all time determinations, with a specified clock at rest.” [1, §1, lines 57-60] Although he did not state it, other physicists considered this to be coordinate time throughout the reference frame. In any case, the time \( t \) is the time recorded by stationary synchronized clocks in the resting frame encompassing the system.

In § 2, Einstein defined his two postulates of special relativity [1, §2, lines 1-12]. The translation lists

(1) “The laws of which the states of physical systems undergo changes are independent of whether these changes of state are referred to one or the other of two coordinate systems moving relatively to each other in uniform translational motion.” and,

(2) “Any ray of light moves in the ‘resting’ coordinate system with the definite velocity \( c \), which is independent of whether the ray was emitted by a resting or by a moving body.”

He added, “Consequently,

\[
velocity = \frac{light\ path}{time\ interval}
\]

where time interval is to be understood in the sense of the definition in §1”. Newtonian forces are the derivatives of momentum, which a constant velocity \( v \) of an unchanging mass results in zero force for any inertial reference frame (those frames that translate linearly by a constant velocity). The Newtonian force equations remain the same for all inertial frames. The first postulate preserves the state of a system, as a zero force will not change the equations of motion of the physical state.

Einstein considered that a rigid rod at rest of length \( L \) is measured by a measuring rod or ruler that is also at rest. The observer would conclude the rigid rod was of length \( L \). He then moved this rigid rod with a velocity \( v \) and orienting it parallel to its velocity. The observer of the moving rod would ascertain at time \( t \) using the stationary, synchronized clocks of the resting system where the ends of the rod would be located and then measure the distance between the two end locations. He predicted this distance would not be \( L \). [1, §2, lines 13-35] He then added two clocks to the two ends, \( A \) and \( B \) of the rod, that were synchronized with the clocks of the resting system. He further added two moving observers, each fixed to each moving clock. He stated, “Let a ray of light depart from \( A \) at the time \( t_A \), let it be reflected at \( B \) at the
time \( t_B \), and reach \( A \) again at the time \( t'_A \). Taking into consideration the principle of the constancy of the velocity of light we find that

\[
t_B - t_A = \frac{r_{AB}}{c - v} \quad \text{and} \quad t'_A - t_B = \frac{r_{AB}}{c + v}
\]

where \( r_{AB} \) denotes the length of the moving rod—measured in the resting system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the resting system would declare the clocks to be synchronous.” [1, §2, lines 47-54] More details of Einstein’s nonsynchronous claim and examples are given in the Appendix.

This is the first discrepancy that Einstein had discovered, which should have yielded equal time spans according to his definition in Eq (1.1) for both photon transmissions between endpoints \( A \) and \( B \). Einstein gave no derivation for the two unnumbered, above equations, but the derivation is shown later in this paper when assuming a universal constant \( c \) with only the stationary observer in the resting frame. His formulas predict a contradiction between time spans that his definition for synchronization by Eq (1.1) required. He did not research into the source of this inconsistency. The first postulate requires states of a system are unaffected by a constant linear velocity between frames. This means that equal time transmission intervals of photons traversing \( A \rightarrow B \) and \( B \rightarrow A \) are maintained between internal frames. The first postulate makes it impossible to determine the absolute constant velocity of an inertial frame, but the clocks attached to ends \( A \) and \( B \) determine the velocity of the uniformly moving rod and its attached inertial frame relative to the resting frame. These two postulates are inconsistent.

Einstein’s claim is contradictory that stationary observers agree the clocks are synchronized versus moving observers arguing the clocks \( A \) and \( B \) are not synchronized. All resting clocks were synchronized initially. The two clocks at the rod’s ends at \( A \) and \( B \) were synchronized to the resting clocks before the rod moved. There is no need to resynchronize the clocks attached to \( A \) and \( B \), which Einstein stated the moving observers performed again [1, §2, lines 45-47]. According to Einstein’s lemma (2), clock \( A \) is synchronized to the resting clocks and clock \( B \) is synchronized to the same resting clocks so that clock \( A \) is synchronized with clock \( B \). When clocks \( A \) and \( B \) move, time dilation is the same for both clocks, so these two clocks still remain synchronized to each other in either the resting or the rod’s moving frame. The time \( t \) is from the resting clock time, which Einstein footnoted that “time” here denotes “time of the resting system”. Also, \( t \) is the time variable in the above equations. At time \( t \), endpoint \( A \) is at the location of one resting clock, which broadcasts its location with its time. Likewise,
endpoint $B$ is at its location of another resting clock, which broadcasts its location with its
time $t$, which is also the same $t$ as the first resting clock where endpoint $A$ happens to be at
this instant. The two messages from clocks $A$ and $B$ at endpoints $A$ and $B$ are the same for
both stationary and moving observers (i.e., a single coordinate point at some instantaneous
time $t$ for $A$ and another coordinate point for $B$), so all observers have the two resting
locations at the resting time $t$, which must be identical between all observers. Einstein’s
dismissal is not valid. Moreover, the two equations for his discrepancy are derived from one
stationary observer fixed in the resting reference frame recording photons traversing over the
moving rod as shown later in the next section.

Einstein considered a resting frame $K$ and a moving frame $k$ with coordinate axes mutually
parallel between frames where $k$ was constrained to move its origin and $\zeta$-axis along the $x$-
axis of $K$ at a constant velocity $v$ [1, §3, lines 4-20]. Consider a rigid rod at rest in the $k$ frame
with a length $L_0$ measured with a “ruler” by an observer in the $k$ frame. Two observers in the
$K$ frame synchronously measure the length $L$ of the moving rod by locating the moving rod’s
ends at different locations in $K$ at the same coordinate time of $t_1 = t_2$. Einstein derived the
transformation equations between time $t$ and coordinate $x$ in the $K$ frame with time $\tau$ and
coordinate $\zeta$ in the $k$ frame as $\tau = \gamma(t-vx/c^2)$ [§3.26] and $\zeta = \gamma(x-vt)$ [§3.27] in his paper. Get
the inverse transformation (interchange $x$ and $\zeta$, interchange $t$ and $\tau$, and replace -$v$ with +$v$)
to get the difference equations

\[ x_2 - x_1 = \gamma[\zeta_2 - \zeta_1] + v(t_2 - t_1), \]  \hspace{1cm} (1)

\[ t_2 - t_1 = \gamma[(\zeta_2 - \zeta_1)] + (v/c^2)(\zeta_2 - \zeta_1). \]  \hspace{1cm} (2)

For synchronization in the $K$ frame, $t_1 = t_2$ in (2), which can be rearranged to get:

\[ \tau_2 - \tau_1 = -\left(\frac{v}{c^2}\right)(\zeta_2 - \zeta_1) \]  \hspace{1cm} (3)

Substitute $(\zeta_2-\zeta_1)/c$ with $(\tau_2-\tau_1)$ into Equation (3) to get

\[ (\tau_2 - \tau_1)(1 + \frac{v}{c^2}) = 0 \hspace{0.5cm} \text{or} \hspace{0.5cm} \tau_1 = \tau_2 \]  \hspace{1cm} (4)

The Lorentz transformation proves synchronized events remain synchronized between inertial
reference frames, which satisfies the first postulate of relativity. Einstein’s claim that
synchronized events in one inertial frame are nonsynchronized in a moving inertial frame is
false.

The author defines simultaneous events occurring when two or more phenomena separate or
intersect at one coordinate point at one instant of coordinate time. A transformation of a three-
dimensional point is still a three-dimensional point. An instant of time from a time scale is
still an instant on a different time scale. Neither time dilation nor length contraction can
change the size of an instant of time or the length of a point between reference frames, so
simultaneity is preserved under Einstein’s first postulate. To be practical, simultaneous events
require an acceptably small neighborhood around a point with the same coordinate time as
given by a perfect clock in that neighborhood. Such clocks scattered in a coordinate frame
must be synchronized to a master time standard or timescale.

Simultaneous events are detected when an observer or detector records the arrival of
phenomena occurring at the same time instant. But, the sources of the phenomena are usually
at different coordinate locations, and the times of emission or occurrence are at different
coordinate times. It is an easy misconception to apply the arrival of simultaneous events
recorded by a detector to mean the sources of the phenomena are also simultaneous, which is
usually untrue.

Simultaneous events can sometimes be scattered from a point at one instant of time. For
example, a shotgun cartridge can discharge the pellets simultaneously when fired, but the
pellets scatter to hit different targets at different later times.

Synchronous phenomena occur at different coordinate positions, but have the same coordinate
time of a reference frame. It is often misstated that separate phenomena occurring at the same
coordinate time are simultaneous, when synchronous is the correct term. For example, two
clocks are synchronous in the same coordinate time after adjustments to a master time
standard. The clocks occupy different locations, but operate synchronously—not simultaneously.

Einstein found an inconsistency in his thought experiment, but he incorrectly claimed it was
due to simultaneous versus nonsimultaneous observations of the same event between different
inertial frames. An error exists somewhere, whether in a derivation, a false assumption, or
misinterpretation of the experiment. Einstein’s discrepancy of photons traversing a moving
versus a stationary rod is revealed in the next section.

4. Photon Paths between Ends of Stationary and Uniformly Moving Rods

The equivalent formula after Einstein’s second postulate is
time interval = \frac{\text{light path}}{\text{velocity}}.

In quantum electrodynamics (QED), photons emitted from endpoint A combine with the reflective atoms at endpoint B to create excited electrons. New photons are emitted from deexcited electrons in B and travel to endpoint A, so that virtually all photons from B to A reinforce various paths with different amplitudes. The square of the amplitudes is the probability where photons traveled, which usually is the path of least action via Snell’s law. In the roundtrip setup, one beam traverses the distance $A \rightarrow B$ and the other beam $B \rightarrow A$ on the macroscopic scale. Any reflection involves two beams. Einstein incorrectly assumed photons were reflected by a perfect mirror at the rod’s end to maintain one continuous beam because quantum mechanics was unknown in 1905.

Einstein’s definition of synchronized clocks according to his (1.1) explains the null results of constructive interference output from the Michelson-Morley (MM) interferometer. Einstein’s definition requires that photons traverse a rod in either direction in equal time spans. The MM interferometer has equal arms of length $L$. An electromagnetic wave is split in the interferometer to travel along two equal arms. Fresnel’s equations explain the electric and magnetic vectors are reflected with the same frequency, but $\pi$ radians out of phase than the initial propagated waves as the index of refraction of the reflective surface is greater than air or a vacuum. At the recombination point of the MM interferometer, the two reflected waves have the same frequency, same mutual phase angles, and same transmission times due to the equal lengths of the arms to maintain synchronization by Einstein’s definition. The combined reflected waves output constructive interference with the MM interferometer, regardless if it is stationary or moving with uniform velocity, as required by the first postulate of relativity. Einstein’s definition is a far simpler explanation than the standard answer that requires only length contraction from special relativity without time dilation that is applied only in the component parallel with the uniform velocity of the MM interferometer.

As Einstein stated, orient a uniformly moving rod with its length parallel to its velocity, $v$, relative to the resting frame. When the rod is resting, the photons traverse a distance of $L$, the length of the rod. Each one-way time span is $\Delta t = L/c$, and each roundtrip distance over the resting rod is $2L$.

However, photons traveling with or opposite to the moving rod experience different distances. For example, runners know that a footrace covers more (or less) ground if the finish line is moved away from (or toward) the runners during the race. This happens also in Zeno’s
paradox of Achilles and the tortoise. Zeno’s paradox relates how the tortoise challenged Achilles to a race with a head start of \( L \), claiming the tortoise would win. Achilles conceded the race without running due to the tortoise’s logic that Achilles was always behind the tortoise for an infinite number of time intervals, despite that Achilles was faster than the tortoise. Zeno did not consider that a finite length can be divided into an infinite number of lengths, but the sum of those spans would still be a finite length. Algebra can directly solve the distances that photons traverse to intercept the opposite end of the moving rod. In general, the photon’s constant speed from endpoint \( A \) to endpoint \( B \) is the constant \( c_{AB} \), and the photon’s speed from endpoint \( B \) to endpoint \( A \) is the constant \( c_{BA} \), which these speeds may be equal or different to magnitude of \( c \). In the resting frame, the rod moves with velocity \( v \) so that the endpoint \( B \) is at \( B' \) where the parallel photons from \( A \) overtake the receding endpoint at \( B' \) in the time span of \( \Delta t_{AB} \) as shown in Figure 1. Also, the newly emitted antiparallel photons from endpoint \( B \) intercept the approaching endpoint \( A \) at the position \( A' \) in the time span of \( \Delta t_{BA} \) as shown in Figure 2.

\[
L_{AB} = L + D
\]

\[\text{Figure 1: Photons Overtaking Receding Endpoint}\]

Solve for \( D \) in \( D/v = (L+D)/c_{AB} \) and replace \( D \) in \( L_{AB} = L+D \).

\[
L_{AB} = L - d
\]

\[\text{Figure 2: Photons Intercepting Approaching Endpoint}\]

Solve for \( d \) in \( d/v = (L-d)/c_{BA} \) and replace \( d \) in \( L_{BA} = L-d \). The resulting distances in the resting frame are:

\[
L_{AB} = \frac{Lc_{AB}}{c_{AB}-v}, \text{ and (5)}
\]

\[
L_{BA} = \frac{Lc_{BA}}{c_{BA}+v} \quad (6)
\]

It is immediately apparent that \( L_{AB} > L \) if \( c_{AB} > v \), and \( L_{BA} < L \) if \( v > 0 \). Moreover, if both \( c_{AB} \) and \( c_{BA} \) are equal to the standard \( c \), the roundtrip distance is greater than \( 2L \). Simply add equations (5) and (6) with the standard velocity \( c \) for the roundtrip distance:
\[
\frac{Lc}{c-v} + \frac{Lc}{c+v} = \frac{2Lc^2}{c^2-v^2} = \frac{2L}{1-\frac{v^2}{c^2}} = 2L\gamma^2 > 2L \quad (7)
\]

With a universal \(c\), the photons have a longer roundtrip distance traversing the uniformly moving rod in the resting frame than if that rod was stationary. Length contraction from special relativity undercompensates, as length contraction is \(L' = L/\gamma\) for the moving rod relative to the stationary observer, leaving an extra \(\gamma\) in the roundtrip distance with the moving rod. This is the second contradiction with special relativity. If one divided (7) by the universal \(c\) and let \(2L/c = \Delta t\) for the roundtrip time span of the stationary rod, then the roundtrip time span for the uniformly moving rod is \(\Delta \tau = \gamma^2 \Delta t\). Time dilation predicts \(\Delta \tau = \gamma \Delta t\), which leaves an unaccounted \(\gamma\) factor. If photons always move with a universal constant \(c\), then Equation (7) shows that velocity of the inertial frame attached to the moving rod can be independently determined within the resting frame. It is also contrary to the first postulate of relativity concerning the state of a system (e.g., the equal parallel and antiparallel time transmissions by traversing over a resting rod) is unaffected when the system (e.g., the rod) undergoes a constant velocity.

If one divides the one-way distances \(L_{AB}\) and \(L_{BA}\) by their respective constant photon speeds, unequal time spans to intercept the opposite ends of the moving rod occur for the stationary observer while an observer fixed with the moving rod concurrently records equal time spans. The results are:

\[
\Delta t_{AB} = \frac{L}{c_{AB}-v} \quad \text{and} \quad (8)
\]
\[
\Delta t_{BA} = \frac{L}{c_{BA}+v} \quad (9)
\]

Equations (8) and (9) are identical in form to Einstein’s unnumbered equations for his unequal time intervals that the photons traverse the moving rod, except he assumed \(c\) in place of \(c_{AB}\) and \(c_{BA}\), which are not necessarily equal.

Einstein used one observer stationary in the same resting frame to get equal time spans for a resting rod and to get unequal time intervals for the uniformly moving rod. His argument to dismiss the discrepancy is unfounded as his derivation did not need a second observer or a second inertial frame. It was proven in Equation (2) through (4) with the Lorentz transformation the synchronization between clocks is preserved between moving inertial frames (i.e., equal time spans of transmission between points \(A\) and \(B\) in both directions for either the resting or moving inertial frames). Thus, his claim that events can be simultaneous
in one inertial frame and be nonsimultaneous in another inertial frame is invalid.

Einstein correctly required equal time spans, $\Delta t_{AB} = \Delta t_{BA}$, in his Equation (1.1) for synchronizing remote clocks. All observers must concurrently record equal time spans in either direction between synchronized clocks for resting or uniformly moving cases as mandated by the first postulate when witnessing the same event. The Equations (8) and (9) with universal $c$ from the second postulate contradict (1.1), which this implies the second postulate is inconsistent with the first postulate.

Use Einstein’s requirement in (1.1) that $\Delta t_{AB} = \Delta t_{BA}$ for photon traverses between ends $A$ and $B$ of the rod. Equate $\Delta t_{AB}$ in (8) to $\Delta t = L/c$ time intervals and, next, $\Delta t_{BA}$ in (9) to the same $\Delta t$ traverse time span.

$$\frac{L_{AB}}{c_{AB}} = \frac{L}{c_{AB} - v} = \frac{L}{c} \implies c_{AB} = c + v, \text{ and } (10)$$

$$\frac{L_{BA}}{c_{BA}} = \frac{L}{c_{BA} + v} = \frac{L}{c} \implies c_{BA} = c - v \quad (11)$$

For a common time between remote points (i.e., valid coordinate time throughout the domain of a reference frame), Einstein required that light (i.e., electromagnetic transmission) must travel from $A$ to $B$ in a time span identical to travel from $B$ to $A$ to synchronize remote clocks. He required by definition that clocks at $A$ and $B$ run in synchronization when $\Delta t_{AB} = \Delta t_{BA}$, which is his Equation (1.1) [1, §1, lines 38-66]. For a roundtrip traverse from $A$ to $B$ and back to $A$, the total time is twice the time span for a one-way traverse. So, an operator at the master clock records the roundtrip traverse time span, divides that by two, then transmits another time tag from the master clock and the one-way time span to the operator of the remote clock to reset the remote clock and match the master clock. This is Einstein’s synchronization procedure. A horologist at $B$ would need multiple messages to adjust the clock at $B$ to replicate the size of its second to achieve a final synchronization with the master clock’s second.

Reiterating with the uniformly moving rod, $c_{AB}$ is the photon’s velocity when emitted from $A$ to intercept the receding endpoint $B$ at $B'$, and $c_{BA}$ is the photon’s velocity when emitted from $B$ to intercept the approaching endpoint $A$ at $A'$. Either velocity could equal the standard $c$ or be different. Einstein concluded clock synchronization requires $\Delta t_{AB} = \Delta t_{BA}$, so that separate clocks fixed relative to each other on an inertial rod can be synchronized to maintain identical coordinate times. To satisfy Einstein’s requirement of equal time spans, Equations (10) and (11) show that parallel and antiparallel photon velocities differ from the standard $c$ for moving
photon sources relative to the photon detectors. The emitted photons from endpoint \( A \) obtain the additional \(+v\) in the parallel velocity to overtake endpoint \( B \). The second beam of ejected photons from endpoint \( B \) after absorption gets the opposite velocity, \(-v\), in the antiparallel direction of the uniformly moving rod relative to the photon detector. These results comply to the addition law of photon velocities, due to the mutual velocity between the source and detector. When the rod is stationary in the resting frame, \( v = 0 \) and the one-way time span is \( L/c \) for either direction by Equations (8) and (9). The addition law of velocities makes \( c_{AB} = c + v \) and \( c_{BA} = c - v \), so that both Equations (8) and (9) predict a one-way time span of \( L/c \) for either direction. As long as the distance between clocks is fixed, synchronization is possible whether the clocks are stationary or uniformly moving relative to an inertial frame.

Roundtrip transmissions involving a single reflection have an average speed equal to the standard \( c \) as \( (|c+v| + |c-v|)/2 = c \). By induction, all roundtrip traverses have an average speed over multiple reflections identical to the standard speed \( c \) [14]. Einstein’s discrepancy is not due to synchronization versus nonsynchronization between observers in different frames. Einstein’s discrepancy was derived with a universal photon speed involving only one fixed observer in the resting frame. A universal speed \( c \) prevents synchronization between moving clocks that maintain a fixed distance between the clocks. Einstein’s discrepancy with his Equation (1.1) is the third contradiction with the second postulate.

Einstein’s second postulate is an excellent approximation due to the high speed of photons, but it makes some doubtful predictions. For example, his last section of his 1905 paper predicted that masses measured parallel versus transversely along the electron’s velocity will be different [1, §10, lines 1-81]. He mathematically derived the dynamics of the slowly accelerated electron, which resulted in a mathematical difference in parallel and perpendicular mass measurements. No one has published any counter derivation showing any mathematical error in §10. Earth experiences a gravitational acceleration that creates its orbit, so any test mass on Earth should vary in mass on a daily basis. No national institute of standards has published any finding that a test mass has a diurnal variation, which is expected as Earth’s rotation would displace the laboratory’s local vertical for a hanging test mass with a significant component parallel or perpendicular to Earth's orbital velocity. With no known experimental verification of a different mass measurement depending on the direction of a moving mass and no error in the derivation, one must question if one of Einstein’s postulates is not precisely true. The lack of verification for a difference in measuring mass parallel versus perpendicular to its orbital velocity is a fourth shortcoming with predictions from
special relativity.

One can test directly if Einstein’s second postulate is totally correct in one direction. According to special relativity, length contraction is \( \Delta L_{\text{resting}} = \gamma \Delta L_{\text{moving}} \) and time dilation is \( \gamma \Delta \tau_{\text{resting}} = \Delta \tau_{\text{moving}} \). Let the integer \( k = 299792458 \), \( \Delta L_{\text{resting}} = 1 \) meter, \( \Delta \tau_{\text{resting}} = 1 \) second. As the speed of light is currently defined,

\[
c = k \text{ meters/s} = k \frac{\gamma \Delta L_{\text{moving}}}{\Delta \tau_{\text{moving}} / \gamma} = k \gamma^2 \frac{\Delta L_{\text{moving}}}{\Delta \tau_{\text{moving}}} = c' \quad (12)
\]

This means that the speed of photons in a constant moving inertial frame has shorter meters and longer second intervals than a resting frame such that one-way photon speeds do not have the same universal constant as \( k \neq k \gamma^2 \). For example, the orbital speed of Earth at apogee and perigee is respectively about 29,300 and 30,300 m/s, which the numerical difference between apogee and perigee of \( k \gamma^2 \) would be about 0.199. It would take at least 11 significant figures in measurement to verify if special relativity would produce this result.

Equation (12) produces a unique paradox that is more confounding than the relativistic twin paradox. Put two spacecraft containing timing laboratories with atomic clocks sufficiently far between galaxies so that gravity can be ignored. When they are docked together, the crews check that all precise timing devices are synchronized and all measuring standards are calibrated (e.g., meter, gram, coulomb, etc.) between laboratories. After separating, the second spacecraft fires its rockets to accelerate until a desired velocity \( v < c \) is achieved, and all acceleration is terminated to allow the second spacecraft to coast at a constant velocity \( v \). Both laboratories perform a one-way traverse of photons to measure photon speed in the units of the spacecraft’s laboratory. The first spacecraft obtains the expected \( k \) m/s where \( k = 299792458 \). The crew of the second spacecraft know they are moving at \( +v \) relative to the first spacecraft, as they initiated the acceleration. Special relativity in (12) predicts the second crew measured photon speed parallel to its velocity \( v \) as \( k \gamma^2 \). The results are exchanged by electromagnetic communication between the crews. Neither spacecraft will execute another acceleration for a future rendezvous. According to the equivalence principle, the value of the photon speed is the same, \( k \), for both spacecraft. Special relativity has been rigorously derived from the second relativity postulate. According to special relativity, the constant for light speed differs as \( k \) or \( k \gamma^2 \). The predicted result from special relativity allows the second crew to determine its velocity relative to the first spacecraft via \( \gamma \), violating the equivalence principle.

Equation (12) is a contradiction, the fifth discrepancy, that special relativity does not maintain
the universal numerical photon speed between moving inertial frames. This indicates the second postulate as currently worded is not correct as length contraction and time dilation are derived from a universal $c$ in a vacuum environment, regardless of the relative velocity between the photon source and photon detector.

5. Analyzing Interferometer Output

The null results of outputting constructive interference in the MM interferometer are directly explained by Einstein’s requirement that photons traverse a rod in either direction in equal time spans. The MM interferometer has equal arms of length $L$. Designate one arm with endpoints $A$ and $B$, and the other arm has endpoints $A$ and $C$. By Einstein’s Equation (1.1), $\Delta t_{AB} = \Delta t_{BA}$ and $\Delta t_{CA} = \Delta t_{AC}$. With equal arms, $L_{AC}/c = L_{AB}/c$ in the resting frame of the MM interferometer, implying $\Delta t_{AB} = \Delta t_{BA} = \Delta t_{CA} = \Delta t_{AC}$. The MM interferometer outputs constructive interference as the split beams have the same frequency, begin with the same phase angle, and combine with the same phase angle due to identical time spans after the initial splitting. With a constant velocity applied to the MM interferometer, constructive interference is still output according to the first postulate of relativity.

The photon speed is so high that the photon velocity is virtually constant over most transverses despite any external acceleration applied to the laboratory. For example, the maximum Coriolis effect is on the equator, which has a tangential speed of 465 m/s. Einstein stated a freely falling frame is sufficiently inertial. Let Earth’s center of mass be the frame’s origin with nonrotating axes for an inertial ECI (Earth-centered inertial) frame. Orient a one-meter leg longitudinally on the equator, and Earth’s rotation would displace the far end 1.55 μm eastward as a photon travels 1 meter. The hypotenuse that the photon travels in the freely falling nonrotational Earth frame is longer than 1 m by 1.20E-12m. One needs a precision of 13 or more significant figures in the laboratory to detect a difference in apparent photon speeds due to this Coriolis effect. Setting a leg of one meter parallel to the equatorial circle would displace the opposite end by 1.55 μm east in a one-way measurement.

A more rigorous demonstration reveals that photons precisely obey vector velocity addition. The Laser Interferometer Gravitational-Wave Observatory (LIGO) is a large-scale physics experiment with two observatories to detect cosmic gravitational waves [14]. Twin observatories are near Hanford, Washington, and Livingston, Louisiana, with 4 km long arms within ultrahigh vacuum chambers allowing laser beams to detect gravity waves. VIRGO,
located near Pisa, Italy, joined LIGO in May 2007 and has 3 km arms. KAGRA, located near Hida, Japan, began joint collaboration on 24 May 2023. Table 1 lists the first three locations and azimuth in degrees counterclockwise of each arm from local east at each facility.

<table>
<thead>
<tr>
<th>LIGO Facility</th>
<th>Location</th>
<th>Local Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hanford</td>
<td>46°27'19&quot;N 119°24'28&quot;W</td>
<td>126° 216°</td>
</tr>
<tr>
<td>Livingston</td>
<td>30°33'46&quot;N 90°46'27&quot;W</td>
<td>198° 288°</td>
</tr>
<tr>
<td>Virgo</td>
<td>43°37'53&quot;N 10°30'16&quot;E</td>
<td>71° 161°</td>
</tr>
</tbody>
</table>

Table 1: LIGO Observatories with Arm Orientations

Each LIGO facility uses a continuous laser beam that is amplified from 40 watts to 750 watts with power reflecting mirrors. The signals are also magnified with signal recycling mirrors. LIGO has enhanced vibration absorption mechanisms to remove ground vibrations, tremors, solid Earth tides, etc., to isolate the signals. To increase the arm lengths from 4 km, Fabry-Perot cavities are placed near the beam splitting mirror and at the hanging reflection mirror at the end of each arm so that 280 reflections inside the cavities increase the effective arm length to almost 1120 km. An ultrahigh vacuum is maintained so that any gaseous molecule is removed promptly to avoid interference with the photon beams or remove dust from landing on the mirror by the laser beam from burning a cavity into the mirror. Also, complete destructive interference is created when recombining the two beams, which can be accomplished by moving one mirror half a wavelength. The original MM interferometer gives bright constructive interference, but this enhancement allows far easier detections with a black background after merging signals. “At its most sensitive state, LIGO will be able to detect a change in distance between its mirrors 1/10,000th the width of a proton.” [14]. (i.e., resolution of 1.7E-19 m.)

The author has found no compensation listed in its website or its publications describing the LIGO construction or processing to correct the different Coriolis effects on the two separate LIGO arms at either location. The LIGO observatories are at significantly different latitudes and each long arm is oriented at different azimuths. Each endpoint is at a different latitude and experiences a different west-to-east increase and an east-to-west decrease in ECI distance to create noncoherent interference.

For approximate calculations, let the Earth’s surface be spherical and rotate within the ECI frame to get the tangential velocity using the geocentric latitude of each observatory. The beams are straight lengths conforming to plane geometry. The Hanford observatory has a
tangential velocity of 320.4178 m/s at the splitting point, the X-axis endpoint is northward with 320.2467 m/s, and the Y-axis endpoint is southward with a 320.5420 m/s tangential speed. The resulting roundtrip distance of one reflection for photons on the X-axis arm is 8000.000013446 m and the roundtrip on the Y-axis arm is 7999.999986604 m in the ECI frame. With a universal photon speed, the LIGO observatory has the claimed precision to detect this difference in coherence in the merged beams. Yet, LIGO has a total of over 22 months of steady destructive interference (excluding the daily glitches that fail to correspond to the rare gravity wave detections between the LIGO observatories).

As done with LIGO, the two axes of the MM interferometer are oriented perpendicular to the local gravity, which affects both beams equally. The apparent frequency shift of the waves accompanying the photons ascending or descending vertically in Earth’s gravity was found in the Pound-Rebka experiment [15]. Any interferometer on Earth undergoes a velocity through the cosmos consisting of Earth’s tangential rotation, Earth’s orbital velocity around the Sun, the rotational and translational velocity of the Milky Way, the rotational and translational velocity with the Local Group, etc. That instantaneous velocity, \( V \), has components that are parallel to the \( x \) body axis, \( y \) body axis, and perpendicular to the \( xy \) body plane (equivalently, parallel with the \( z \) body axis). Figure 3 shows the basic laboratory setup.

![Figure 3. Michelson-Morley apparatus in lab frame.](image)

The beams recombine at \( A \), and the merged output is monitored at \( O \). With equal lengths \( L=AB=AC \), the combined beams produce constructive wave interference, indicating the two beams arrive at \( A \) at the same time. If the lengths \( AB \) or \( AC \) changed gradually to make arrival times different, destructive interference would be detected by the MM interferometer. Einstein’s requirement of equal time spans to traverse a length in either direction explains the MM interferometer’s constructive output, as the merged beams are synchronized upon merger when traversing equal length arms.
The Kennedy-Thorndike experiment was an interferometer with unequal arms, such that its output was a static destructive interference pattern over a long test period of nearly a year [16]. With unequal arms, a photon’s traverse will have unequal time spans of $\Delta t = L/c$ and $\Delta \tau = L'/c$. The output will remain static as the ratio of $\Delta \tau$ and $\Delta t$ will remain constant in the Kennedy-Thorndike test.

For vector velocity addition, Figure 4 illustrates an arm of length $L$ with ends $AB$, which is moved perpendicular with a constant velocity component $V$ relative to an external inertial frame. Observers fixed to this inertial frame will record a photon moving along the hypotenuse $A'B$, while observers stationary relative to the device see a photon moving along the arm $AB$. When the photon from $A$ reaches $B$ in the coordinate time interval, $\Delta \tau$, the arm has moved sideways by the distance $V \Delta \tau$. The time intervals are identical coordinate spans of $\Delta \tau$, so that $C' = (C^2 + V^2)^{0.5}$ in magnitude. The distance the photons traverse in the inertial frame is proportional as $D = C'\Delta \tau = \Delta \tau(C^2 + V^2)^{0.5}$, while $L = \Delta \tau C$ in the interferometer’s frame. After the photons are absorbed at $B$ and reemitted to travel back to $A$ of the arm, there is a mirror image of Figure 4 to repeat the vector diagram.

![Figure 4: Vector Addition of Light](image)

The instantaneous velocity $V$ of the interferometer relative to the inertial frame fixed in the cosmos can be represented as $V = V_x + V_y + V_z$ in the body axes of the interferometer. For the $V_z$ component, the photon velocity $C$ along the $x$-axis or the $y$-axis (e.g., the $AB$ line in Figure 4) would be increased identically to $C_z' = (C^2 + V_z^2)^{0.5}$ in the external inertial frame for both the traverses along the $x$-axis or $y$-axis. In both cases, the distances involved would be the same $D_z = C_z'\Delta \tau$, and the time spans traversed would be $\Delta \tau = [D_z/C_z] = [L/C]$ along the $z$-
axis. Merging the two waves associated with the photons at the recombination point would result in identical later times of $2\Delta \tau$ at arrival with constructive interference.

Let $AB$ represent the $x$-axis moving right with velocity $V_y$ as the photons traverse the $AB$ length of $L$ on the $x$-axis in Figure 4. With the same argument from the previous paragraph, the photon velocity $C$ along the $x$-axis would be increased to a velocity of $C_x' = (C^2 + V_y^2)^{0.5}$ in the external inertial frame. In the external frame, the photon’s distance would be $D_x = C_x' \Delta \tau$ and the time span would be $\Delta \tau = [D_x/C_x'] = [L/C]$ along the $x$-axis. The same applies to the photon traversing the $y$-axis in Figure 4. The photon velocity $C$ along the $y$-axis would be increased to a velocity of $C_y' = (C^2 + V_x^2)^{0.5}$ in the external inertial frame. In the external frame, the photon’s distance would be $D_y = C_y' \Delta \tau$ and the time span would be $\Delta \tau = [D_y/C_y'] = [L/C]$ along the $y$-axis. The roundtrip would be $2\Delta \tau$ for either arm with perpendicular traverses of the photons.

Consider the parallel movement of photons traversing the $x$- or $y$-axis arms. Equations (6) and (7) showed that

$$\frac{L_{AB}}{c_{AB}} = \frac{L}{c_{AB} - v} = \frac{L}{c} = \Delta \tau \Leftrightarrow c_{AB} = c + v \quad \text{and}$$

$$\frac{L_{BA}}{c_{BA}} = \frac{L}{c_{BA} + v} = \frac{L}{c} = \Delta \tau \Leftrightarrow c_{BA} = c - v .$$

When assuming vector velocity addition of a photon source added to the emitted photon speed, then Equations (8) and (9) are derived, where $\Delta \tau = L/C$ is defined in the interferometer’s frame. With $L_{AB} = L_{BA} = L$, this obtains $L_{AB}/c_{AB} = L_{BA}/c_{BA} = L/c = \Delta \tau$ as the same time traverse $\Delta \tau$ and preserved in the external frame. This verifies the first postulate of relativity that the state of a system (e.g., photons traverse equal time intervals over equal lengths in any inertial frame) is unaffected by a constant translational velocity between inertial frames. The startling result is the MM interferometer can neither prove nor disprove the existence of an æther for transmitting electromagnetic waves in space, because its output is always constructive interference. This interferometer result is precisely Einstein’s requirement that photons traverse a rod in equal time spans in either direction, in accordance with the first postulate of relativity.

Even if an æther has a relative wind or a varying density that would affect the velocity of photons (i.e., $c' = c/n$), the modified velocity would only add another velocity component to the total velocity to get the same $\Delta \tau$, which would not affect the output of constructive
interference. As Equations (10) and (11) show for the moving rod, the photon distances over each leg obey vector displacement addition and photon velocity obeys vector velocity addition due to the relative motion between the source and the detector, so that the traverse time is the same for both legs of the MM interferometer for either an observer fixed with the device or observing the moving device with a constant velocity. The MM interferometer is not capable to detect an æther.

The null results for the MM interferometer are usually explained by length contraction applied to the parallel leg, which would be precisely the roundtrip parallel traverse to equal the roundtrip distance on the perpendicular leg. The roundtrip distance of $2\gamma^2L$ is given in Equation (7) for the parallel leg. A mirror image of the triangle in Figure 4 would complete the roundtrip along the perpendicular leg. The perpendicular leg is $L = c\Delta t$ where $\Delta t > \Delta t$. The roundtrip distance along the perpendicular leg is $2D = 2\gamma L$ as shown in Figure 4 for one traverse. The two roundtrip distances would be the same if the parallel roundtrip was shorter from length contraction. However, this explanation ignores time dilation along the parallel leg, which would make the photon frequency differ from the other leg. No one has justified why time dilation is omitted whenever length contraction is used.

Einstein never analyzed the MM interferometer himself, but he deferred to the explanation of Lorentz and FitzGerald. “Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result—a fact very perplexing to physicists. Lorentz and FitzGerald rescued the theory from this difficulty by assuming the motion of the body relative to the æther produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the difference in time mentioned above.” [13] The Lorentz and FitzGerald explanation omits time dilation applied to the interferometer in the direction of motion, which would cause different $\gamma$ factors. Photons in one arm would experience $v \cos \theta$ for a time dilation factor of $\Delta t\gamma_\parallel$ and the other photons experience $v \cos (\theta + \pi/2)$ for a time dilation factor $\Delta t/\gamma_\perp$. Time dilation creates an output of destructive interference by causing different frequencies in the arms.

To show the failure in the standard explanation, consider an observer that operates the MM interferometer in deep space that is sufficiently far from galaxies with the observer located at point $O$ in Figure 3. Let four other observers be in spacecraft. Observers 1 and 2 travel parallel with the $x$-axis of the interferometer with velocities $0.6c$ and $0.8c$, respectively.
Observers 3 and 4 travel parallel with the \( y \)-axis with the same respective velocities. Each observer considers their spacecraft stationary as allowed by the first postulate. The observer at \( O \) shares the continuous output concurrently with the other four observers to affirm the output is constructive interference as indicated by a simple sinusoidal wave output. To get constructive interference, Observer 1 claims the \( x \)-axis was contracted by a factor 0.8 (i.e., \( 1/\gamma \)) while Observer 2 calculated the same axis had to be contracted by 0.6. Observer 3 argues the \( y \)-axis experienced a contraction factor of 0.8, but Observer 4 believes that the \( y \)-axis was shorter by a factor of 0.6. All observers agree the output exhibited constructive interference, but none agree what magnitude or direction of length contraction produced this output. This shows the standard explanation for the null result of the MM interferometer is incorrect.

6. Implications of Related Relativity Effects

The formula \( c = f \lambda \) reveals that special relativity does not preserve the same photon speed between inertial frames. As frequency \( f \) denotes cycles/second and \( \lambda \) is wavelength in meters, \( n \) and \( N \) are the real numbers of units for frequency and wavelengths, \( \text{second} \) and \( \text{meter} \) are the units in the first inertial frame, \( \text{Second} \) and \( \text{Meter} \) are the units in the moving inertial frame with a velocity \( v \) relative to the first inertial frame. Then,

\[
c = n \frac{\text{cycles}}{\text{second}} \cdot N \text{ meters} = n \frac{\text{cycles}}{\text{Second}} \cdot N \gamma \text{ Meters} = \gamma^2 n \frac{\text{cycles}}{\text{Second}} \cdot N \text{ Meters} = \gamma^2 c'
\]  

(13)

Special relativity causes length contraction and time dilation, but it has no effect on \( \text{cycles} \) or numbers (\( n \) and \( N \)). Equation (13), the sixth discrepancy, reveals that the velocity of photons is different between moving inertial frames, and it has the same effect as Equation (12), which is the current definition of universal photon speed where \( k \) represents 299792458 exactly.

These are two contradictions that one-way photon speeds are not numerically identical between moving inertial frames as required by the second postulate of relativity. Equations (5) and (6) are the distances that photons traversed over a moving rod having a velocity \( v \) relative to the “resting” inertial frame, which were derived with the condition that the photon velocities are finite. Equations (4) and (5) are the time intervals that photons take to traverse the moving rod in both directions. In general, \( c_{AB} \neq c \neq c_{BA} \) if the rod moves with constant \( v \) in the inertial frame. Einstein required that the time for photons to traverse the length of a rod must be the same in either direction in his Equation (1.1) for synchronization. The first postulate of relativity requires that Einstein’s (1.1) must hold for a uniformly moving rod.
Einstein was the first to discover the discrepancy in his theory when he assumed \( c_{AB} = c = c_{BA} \), but he made the unfounded claim that unequal time intervals for photons to traverse the moving rod in either direction was due to non simultaneity versus simultaneity between moving observers to cause his discrepancy. The first postulate does preserve simultaneity of coordinate time between synchronized clocks maintaining a fixed distance within different inertial frames, which has been proven in this paper. To satisfy Einstein’s requirement of equal time intervals for the traversing photons, Equations (10) and (11) prove that \( c_{AB} = c + v \) and \( c_{BA} = c - v \), which is the addition law of velocities for the total photon velocity due to the photon source moving relative to the detector. For the moving rod with uniform velocity, photons moving \( A \to B \) have a longer distance to traverse, \( L_{AB} > L \), and a faster one-way photon velocity \( c_{AB} > c \), such that \( L_{AB}/c_{AB} = \Delta t = L/c \). For the photon beam moving \( B \to A \), photons have a shorter distance to traverse \( L_{BA} < L \) and a slower one-way photon velocity \( c_{BA} < c \), such that \( L_{BA}/c_{BA} = \Delta t = L/c \). The satisfies Einstein’s synchronization requirement of equal time spans to traverse the moving rod in either direction. This contradicts Einstein’s second postulate that “the ray of light moves with a definite velocity \( c \)...independent of whether the ray was emitted by a resting or by a moving body.” [1, §2, lines 9-11]. Einstein’s second postulate is restrictive and is not general.

Although this text is redundant, it is extremely important to understand how early measurements of photon speed indicated it was a universal constant. Prior experiments performed by Fizeau, Foucault, and Michelson horizontally measured average photon speeds that involved a single reflection over a length \( L \) by recording the total time interval, \( \Delta t \), to get \( c = 2L/\Delta t \). A roundtrip traverse of photons back to the point of origin does result in an average speed equal to the magnitude of \( c \). Roundtrip transmissions involving a single reflection have an average speed equal to the standard speed \( c \) as \( ([c+v] + [c-v])/2 = c \). This produces an apparent constant speed that can easily mislead physicists to assume rays of light had only one universal speed, while photon flight velocities can vary, but were not measured. This concept is likely why Maxwell treated \( c \) as a constant in his partial differential equations for electromagnetic theory.

Other photon speed tests relied on other methods. Using the electromagnetic values for permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \), Maxwell’s theory predicts \( c^2 = 1/(\varepsilon_0 \mu_0) \). The electromagnetic source is stationary within the laboratory, so the calculated photon speed will be the standard \( c \) as the detectors will be stationary relative to the source. Cavity resonance tests measure the value \( c \) as \( c = f\lambda \). Again, the cavity measurements are taken with the detector,
cavity and photon source stationary in the laboratory, which obtains the standard $c$. In Mössbauer spectroscopy, nuclear resonance (emission and absorption of the same gamma ray by identical nuclei) is observed when using a solid. A nucleus in a gas has a recoil when emitting a gamma ray, but conservation of momentum produces a gamma ray that does not match absorption of a gaseous target nucleus. A significant fraction of emission and absorption events will be recoil-free in a solid. Mössbauer spectroscopy succeeds because photons as gamma rays emitted by one nucleus in a solid can be absorbed by another solid containing nuclei of the same isotope, and this absorption can be measured. The photon source’s and target’s nuclei are stationary in the solid, which the photon speed is the expected $c$. All these tests reinforce that photons have an apparent universal speed of $c$, but none of these tests measured total photon velocities, which are one-way transmissions before absorption while the source and detector were moving relative to each other.

If photon velocities are affected by the velocity of the photon source relative to the detector according to vector velocity addition, then a faster photon velocity will elongate the wavelength without changing the frequency. This means $v' = f \lambda'$, where $v'$ is any photon speed, $f$ is the associated frequency and $\lambda'$ is the corresponding wavelength. Photons would now behave the same in a vacuum as in a transparent medium. In a medium, the photon speed is $c/n = v'$ where $n$ is the index of refraction and $n \geq 1$ with 1 for a vacuum. Only the speed and wavelength change in a medium. As photons penetrate different layers of transparent media, the speed and wavelength change with each material. No waves stack up at the interfaces between materials. This applies as well to photons entering a medium from a vacuum or exiting a medium into a vacuum.

If photon speeds conform to $v' = f \lambda'$ in a vacuum, then photons gain speed the farther they move from a gravitated body. In the Pound-Rebka test [15], photons that traveled up 22.5 m to the absorber had a longer wavelength and a shorter wavelength when traveling down. As a photon moves farther away from a mass, less gravitational deceleration on the photon causes the photon to move faster. Beams bent by the Sun demonstrate this. Despite the tiny size of a photon, the side of the photon opposite to the Sun has a slightly greater velocity than the side facing the Sun, which makes the photon’s path bend around the Sun. This is similar to a tank with its left tread moving slightly faster than the right tread, causing the tank to turn right from a straight path. This effect impacts all cosmological models and observations, as “blueshift” observations are due to stellar objects moving from Earth—not towards it—as emitted photons are red shifted when leaving a gravitational source. An earlier paper gives
more details [13].

If the speed of photons increases with a further distance or elevation from a center of gravity, then recent ultraprecise tests at the National Institute of Standards and Technology (NIST) [17,18] may misidentify the results as time dilation when the actual effect is a slight increase in the photon speed as the elevation of the photon relative to Earth’s center is increased. The Al⁺ optical clock at NIST is tuned initially to a frequency-quadrupled Yb-doped fiber laser that is locked to the $^1S_0 \leftrightarrow ^3P_0$ transition ($\lambda_L \approx 267$ nm for the laser). The $^{27}$Al⁺ ion is interrogated alternately on these transitions with $m_f = \pm 5/2$ to generate a clock frequency that is insensitive with external magnetic fields in the first-order effects. Many detailed compensations are given [19], but the laser must output the stable wavelength, $\lambda_L$, for this optical clock. If photon speed increases as the test platform is elevated (i.e., $c' = f \lambda'$), then the laser’s wavelength, $\lambda_L$, is longer, which implies the laser’s frequency, $f_L$, is smaller (i.e., $f_L \approx \gamma f_0$) when assuming $c$ is a universal constant velocity. The computed lower clock frequency can be misinterpreted as due to time dilation by making the time unit longer in the denominator. NIST has the ultraprecise clocks and equipment to do a precise photon speed test. If photons complete a roundtrip traverse by a reflection over a horizontal track in a shorter time when the track is elevated within the laboratory than at a lower elevation relative to Earth’s center, this would verify that the average photon speed increases the farther distance photons are away from a center of gravity. The clock should not be elevated during the track test, but fiber optic cables can be calibrated for transmission times and can be connected between the optical clock and the track. The cables can be calibrated by connecting the cables into a loop and observe the time to transmit photons through the loop starting from the optical clock and circle back to the clock. The cable calibration should be done once when the loop is horizontal at the lower elevation of the track test and a second calibration by transmitting through the loop of joined cables starting at the optical clock and the other side of the loop placed at the higher elevation of the second track test. If the average speed of photons increases with increased elevation from Earth’s center, then $c$ is neither universal nor constant, even when measured horizontally. The present definition of velocity $c$ should stipulate an elevation, such as mean sea level, to be more accurate.

General relativity is not the only theory that predicts photon deflection accurately. The photon deflection of 1.75″ by the Sun when the photon’s path grazes the Sun can be derived from a Newtonian solution of the two-body problem when considering the Sun and photon move in hyperbolic orbits due to the gravitational attraction between them. Starting with a photon
infinitely far from the Sun, their barycenter is infinitely far between them. Gravity that the Sun pulls on the photon is also responsible for the photon’s pull on the Sun. This is a two-body problem. Both bodies execute hyperbolic orbits about the barycenter. When the photon approaches the solar system, the barycenter moves inside the Sun. The mathematics predict that the photon and solar deflections measured against the orbital asymptotes will obtain a total deflection of 1.75” [21]. Other classical derivations assumed a stationary Sun with an orbiting photon, which only obtains half of the observed deflection.

If a photon can have a higher velocity other than the standard \( c \), then it is possible that high velocity particles could be misinterpreted as different objects. The muon and tauon are two objects in the standard model of particle physics that are identical in all characteristics to the electron, except their masses are different according to the special relativity formula \( E = mc^2 \). If an electron is ejected with ultrahigh energy after an atomic collision, that electron would be misidentified as a different particle with a different mass. This would also allow a different explanation of high energy \( \gamma \) rays colliding with the atmosphere. High energy cosmic radiation impacts atmospheric molecules creating pions decaying into muons. These muons interact with other atmospheric particles before disappearing with a typical average lifetime of 2.194 \( \mu s \). With a velocity slightly below \( c \), 50 \( \mu s \) would be needed to reach sea level, where muons are detected [22]. If electron velocity can exceed \( c \), the cosmic rays may eject electrons with higher velocities, such as 25\( c \), to allow those electrons to reach sea level without time dilation to dissipate their high energies.

The entanglement experiments where two or more particles interact over very long distances in a significantly short time span \( \Delta t \) can be explained if one-way photon velocities exceed the standard \( c \) (i.e., \( d = c' \Delta t > c \Delta t \)).

If photons can have different velocities than the standard \( c \), then some so-called constants in physics are not actually constants. If photon speed is not a constant according to the Einstein’s synchronization requirement that photon speeds must traverse over a rod in identical time spans in either direction for all observers, then permittivity of free space, \( \varepsilon_0 \), must be a variable, because the product \( 4 \pi \varepsilon_0 c^2 \) is defined to be exactly 1E7 [22]. The fine structure constant, \( \alpha \), equals \( e^2/(2 \varepsilon_0 \hbar c) \) where \( e \) is the elementary charge and \( h \) is Planck’s constant. Equivalently, \( \alpha = 2 \pi c e^2/(h \, 1 \text{E7}) \), implying \( \alpha \) varies proportionally as \( c \) varies, or \( h \) varies inversely with \( c \) or both. Assigning photon velocity with a universal constant speed of \( c \) is usually adequate for 5 or 6 significant figures as most photon emitters move with a velocity <
1000 m/s.

Maxwell’s electromagnetic equations will need to be generalized because $c$ and $\varepsilon_0$ are treated as constants in the current theory. Curls and divergences contain combinations of derivatives that can make additional terms from the products that contain $c$ or $\varepsilon_0$. It is beyond the scope of this paper to delve into the generalized derivation of Maxwell’s equations.

The meter must not be defined by the photon velocity transmitted in one direction, as the velocity of the emitter relative to the observer must be compensated. An alternative definition of the meter is suggested with details for consideration [13].

A broadcasting satellite can boost photon velocities to exceed $c$ that would provide an alternative explanation of the flyby anomalies. Several deep space satellites have exhibited a tiny increase in velocity when approaching the perigee of a flyby of Earth, which is in the range of 3 to 11 mm/s or about $2\times10^{-6} = dV/V$. A proposed empirical equation for the anomalous flyby velocity change is [23]

$$\frac{dV}{V} = \frac{2\omega_E R_E (\cos \theta_i - \cos \theta_0)}{c}$$  \hspace{1cm} (14)

where $\omega_E$ is the Earth’s angular velocity, $R_E$ is the Earth’s equatorial radius, and $\theta_i$ and $\theta_0$ are the inbound and outbound equatorial angles of the spacecraft. The equator’s tangential velocity from Earth’s rotation, $\omega_E R_E$, is 465 m/s. Equation (14) is $3.1\times10^{-6}$ or slightly less, depending on the cosine projection. Equation (14) predicts that the transmitted photon arrived at the ground station slightly sooner than anticipated, indicating a slightly faster photon velocity (~($1+3\times10^{-6}$) $c$). This is a miniscule effect, so long as other effects such as atmospheric drag do not overpower it.

Some prior tests claim that the same standard velocity of $c$ was measured for a moving particle’s photon emission. One example is $\gamma$ rays from the decay of $\pi^0$ mesons with more than 6 GeV were measured absolutely by timing over a known distance [24]. The test was designed to measure $c+kv$ with $k = (-3\pm13)\times10^{-5}$ for moving mesons ($\gamma$>45). Two unidentified detectors were spaced 31.450 m apart to measure the time interval the $\gamma$ rays traveled, resulting in the standard $c$. According to quantum electrodynamics (QED), the first detector intercepted the initial photons by absorption, creating excited orbital electrons in the atoms of the detector that triggers timing pulses, which deexcite and emit new photons with the standard velocity $c$. The second photon detector measured the time after photons were emitted by the first detector (i.e., absorbed $\gamma$ rays and reemitted new photons of velocity $c$), resulting in the standard
photon velocity. Both photon detectors were stationary in the laboratory, which no mutual velocity exists between these detectors. This and similar tests must be scrutinized to ensure the photon velocities were measured correctly without prior interception as the initial incident γ rays were not measured.

General relativity, which is derived from the second postulate, is not exact for predicting orbital perigee precision, but it is an excellent approximation. General relativity predicts Mercury’s excess perihelion shift is 42.98″/cy, but Meisner, Thorne and Wheeler [25, p. 1112] determined the extra relativistic perihelion shift was 41.4 ± 0.9″/cy that excludes general relativity without any explanation. A post-Newtonian approximation with coordinate time predicted 40.73″/cy, which is inside the error boundary [26].

Shapiro et al [27] chose Icarus for its large orbital eccentricity and periodic close approaches to Earth for observations to compare against general relativity’s orbital perihelion prediction that contains an orbital eccentricity term. That team used a dimensionless parameter λ to determine how well perihelion predictions compared with observations (unity for general relativity and zero for Newtonian theory). Their first solution for the orbital elements of Icarus resulted in $\lambda = 0.75 \pm 0.08$, which excluded general relativity, but the post-Newtonian approximation of 0.83 fits [26, 27]. Shapiro knew that light speed slows down moving in the presence of the Sun. He predicted this effect seven years earlier than the Icarus study [28], and the Shapiro time delay effect was verified in experiment by radar reflections off of Venus [29]. As both Shapiro and Ash were team members of the Venus radar test and the Icarus study, the solar gravitational effect on light was incorporated in the modeling of the Icarus observations. Shapiro et al [27] wrote, “we must conclude that either general relativity is incorrect or some other aspect of either our theoretical model or the observations differed from our presumptions.” These ardent relativists attempted many more solutions with different models and even changed the angular spread between background stars to make λ unitary within error bounds. They concluded that the FK4 star catalog had a distortion that caused the anomaly, but subsequent star catalogs such as FK5 show no such regional distortion when compared to the FK4. All coordinate transformations from FK4 axes are direct rotations to later star catalogs’ axes with no regional warping.

If the photon velocities in a vacuum have different speeds than c, then many four-dimensional quantities are no longer invariant. When the photon speed is not the constant c, these squared vectors are not invariant quantities, such as the squared four-displacement ($F = c^2t^2 - x^2 - y^2 - z^2$), the squared four-momentum ($p^2 = m^2c^2$), $c^2B^2 - E^2$, the squared four-current density ($J^2 - c^2\rho^2$),
the squared vector potential \((A^2-V^2/c^2)\), and the squared Laplacian \((\nabla^2-1/c^2 \partial^2/\partial t^2)\) [22]. Any derivation based on these quantities is only approximate and limited to the case when photons complete a horizontal roundtrip traverse to guarantee invariance with an average speed of \(c\). Many derivations using these terms as invariant in quantum mechanics must be scrutinized for valid predictions when using one-way photon velocities. Minkowski used a few of these invariances to define and describe his four-dimensional space-time model (later used in general relativity), which are based on Poincaré’s group properties of the Lorentz transformation. Minkowski used Lorentz’s \(F = \pm 1\) to seek other invariant quantities under the Lorentz group, such as \((x, y, z, ict)\) [1, p. 238-243]. With a varying photon velocity due to the velocity of a photon source relative to the observer or due to a changing distance of the photon relative to a center of gravity, the light-time axis is no longer rigid in the space-time reference frame that Minkowski defined. Time is a scalar and not a static vector quantity that is used as an axis to define space-time reference frames.

There are probably more ramifications if the photon velocity does not maintain a constant speed equal to the universal constant \(c\) in a vacuum. In any case, it will take time to consider these results thoroughly. It would not be surprising if other accepted concepts of physics may need revisions besides revising the second postulate of relativity.

7. New Explanation for the Pioneer Anomalies

The Pioneer 10 and Pioneer 11 spacecraft achieved the most precise deep space navigation to date in their trajectories while passing the outer planets and exiting the solar system. A small linear Doppler frequency drift was detected that did not match the expected Pioneer downlink transmissions compared to the received downlink. This was the first anomaly, which revealed each Pioneer received the uplink transmissions from Earth sooner than expected (e.g., the Pioneers appeared closer to the Sun than expected if light velocity was constant). It was assumed a tiny unmodeled acceleration that varied between 2E-10 m/s² and 8E-10 m/s² toward the Sun caused this. Additional anomalies revealed the downlink signals exhibited annual and diurnal variations in the Doppler residuals.

These anomalies were detected due to the high precision that was designed in the Pioneer spacecraft. “Due to the long distances from the Sun, the spin-stabilized attitude control, the long continuous Doppler data history, and the fact that the spacecraft communication systems utilize coherent radio-tracking, the Pioneers allow for a very
sensitive and precise positioning on the sky. For some cases, the Pioneer 10 coherent Doppler data provides accuracy which is even better than that achieved with VLBI observing natural sources (e.g., some pulsar emissions are more precise than many atomic clocks). In summary, the Pioneers are simply much more sensitive detectors of a number of solar system modeling errors than other spacecraft.” [30]

Pioneer 10 and Pioneer 11 were launched respectively on March 3, 1972 and April 5, 1973, and their Jupiter encounters for their orbital boost occurred respectively on December 3, 1973, and December 3, 1974. The Saturn flyby of Pioneer 11 occurred on September 1, 1979. Their last telemetry received was on April 27, 2002, and November 24, 1995, respectively. Their trajectories are shown in the following figure.

![Figure 5: Pioneer 10, Pioneer 11, and Voyager trajectories.](image)

The spacecrafts’ velocities relative to the Deep Space Network (DSN) stations on Earth allowed the computation of the line-of-sight based on prior trajectory information and repeated observations from the spacecrafts’ radio transmissions. A pair of onboard redundant Sun sensors and a star sensor provided a roll reference in each spacecraft’s orientation, which each had closed loop navigation capability by locking onto the Earth-based uplink signal using a conical scan. The radio signals transmitted by both DSN and Pioneer vehicles were circularly polarized. The spin axis of the Pioneers coincides with the antenna axis. Every revolution of the spacecraft adds a cycle to both the uplink reception and downlink transmission. Pioneer 10 had a 4.8 rpm spin, which would add 0.08 Hz to the frequency in each direction.

Each Pioneer had two receivers and two transmitters onboard with an effective power of 70 dBm. The radio systems operated in the S-band of 2.111 GHz for uplink communication and
2.292 GHz downlink. With no precise onboard frequency oscillator, the downlink was synchronized phase-coherently to the uplink by an exact ratio of 240/221 for turnaround frequency during Doppler tracking. This allowed precise Doppler frequency measurements with millihertz accuracy. A DSN station would transmit a continuous signal, which the spacecraft returned a downlink signal that is phase-locked to the received DSN signal. This and other related features of the Pioneers are described in the Jet Propulsion Laboratory (JPL) report [30].

JPL’s Orbit Determination Program (ODP) and the Aerospace Corporation’s Compact High Accuracy Satellite Motion Program (CHASMP) obtained the same results after analyzing the Doppler data. “Although, by necessity, both ODP and CHASMP use the same physical principles, planetary ephemeris, and timing and polar motion inputs, the algorithms are otherwise quite different. If there were an error in either program, they would not agree.” [30] The initial results are shown in the following figure.

![Figure 6: Early Unmodeled Pioneer Accelerations](image)

Two additional anomalies embedded in the Doppler data are annual and diurnal variations that JPL has not unexplained, which are shown together in one segment in the next figure.

![Figure 7: CHASMP Residuals (23 Nov 1996 – 23 Dec 1996)](image)
A clear modeling error is seen by the solid diurnal curve with a segment of the annual curve shown in the background. “The annual and diurnal terms are very likely different manifestations of the same modeling problem. The magnitude of the Pioneer 10 post-fit weighted RMS residuals of $\approx0.1$ mm/s, implies that the spacecraft angular position on the sky is known to $\leq 1.0$ milliarcseconds (mas). (Pioneer 11, with $\approx0.18$ mm/s, yields the result $\approx1.75$ mas.) At their great distances, the trajectories of the Pioneers are not gravitationally affected by the Earth. (The round-trip light time is now about 24 hours for Pioneer 10.) This suggests that the sources of the annual and diurnal terms are both Earth related.” [30]

Some studies attributed the presumed acceleration was due to thermal radiation causing a recoiling force that was not included in the initial thermal models of the Pioneers. All thermal recoil models predict a decrease over time as the Pioneer radioactive power sources diminished from the initial 155 watts at launch, which did not agree with well with Figure 6. The last definitive study, published in 2012, was by Turyshev et al [31] that constructed a thermal model that was numerically solved for the many thermal conduction and radiation terms of the Pioneers and compared the result to the flight telemetry. There are far more unknowns for the thermal flows and effective radiation emission terms from various thermal heat sources without actual Pioneer data for each thermal flow. Arbitrary choices for the numerous unknowns are made to produce a reasonable collective result. That same team also developed a parameterized model for thermal recoil force and estimated the coefficients of that model to match against the Pioneer navigation data. Inserting arbitrary numbers into a model to match an anomaly only shows that it could be a possible cause for the anomaly, but it is not proof. None of the thermal recoil force models explain why the supposed acceleration began at Jupiter ($\sim5$ AU) after a year in the mission and increased to its maximum between Saturn and Uranus ($\sim15$ AU). No thermal recoil model explains why the Pioneer anomaly was not visible earlier in each mission, such as $\sim1.5$ AU) when Pioneer spacecraft power was higher than near Jupiter. No thermal recoil model can explain the annual and diurnal variations in the Doppler residual data.

Equations (10) and (11) reveal that photon velocity is affected by the velocity of the light source relative to the light detector via vector velocity addition. This immediately explains the annual and diurnal variations found in the Pioneers’ Doppler residuals. The DSN imparts an additional vector velocity on the uplink due to Earth’s rotational velocity and orbital velocity. Any motion of the DSN during reception of the downlink is opposite to the DSN transmission velocity of the uplink. Einstein’s requirement of equal time intervals to traverse between
points A to B or B to A is valid in inertial frames (e.g., stationary or moving with constant velocity). The downlink transmission time interval is nearly the same as the uplink transmission time interval due to the Earth’s movement over the hours that the roundtrip transmission required. If the DSN was stationary relative to the solar barycenter, then the average photon velocity would be the expected standard c. The vector velocity difference between Earth’s orbital velocity and rotational velocity at transmission and reception time would result in the average speed of the photon being different than the standard speed c in a roundtrip. The difference in the total DSN velocity between transmission and reception will result in Doppler residuals exhibiting both an annual and diurnal variation.

The region of outer space covering the first detected anomaly has no practical difference between Newtonian gravity and general relativity. Newton defined momentum (i.e. “quantity of motion”) of a particle as its velocity multiplied by a proportionality constant, which he assigned as mass. In special relativity, momentum and energy are still well-defined properties, except the mass is variable, which will be treated as \( m_{\text{rel}} \) in this paper for a photon. The four-vector derivation produces \( E = m_{\text{rel}}c^2 \) and \( E^2 = p^2c^2 + m^2_{\text{rel}}c^4 \) [22, Chapter 5, Relativity 1]. For a photon at rest, no energy is being transported, so the rest mass of a photon is assigned zero as \( 0 = E = m_{\text{rest}}c^2 \). If the rest mass of a photon is zero, then \( E \) is \( pc \) as derived in classical electromagnetic theory. In special relativity, the momentum of a moving photon is \( p = m_{\text{rel}}\gamma \),

Assume Earth’s gravitational force \( F \) on a moving photon is Newton’s law of gravity.

\[
F_{\text{Earth}} = G \frac{m_{\text{Earth}} m_{\text{rel}}}{r^2} = m_{\text{rel}} g_r
\]

where \( G \) is the gravitational constant, \( m_{\text{Earth}} \) is the mass of the Earth, \( m_{\text{rel}} \) is the relativistic mass associated with the moving photon, and \( r \) is the ray length from Earth’s center of mass to the photon’s relativistic mass. Let \( R \) be the radius of Earth, so that \( Gm_{\text{Earth}}/R^2 = g_R \) at Earth’s radius where \( g_r = g_{\text{Earth}}(R^2/r^2) \). The gravitational work as potential energy done on the moving photon of mass \( m_{\text{rel}} \) (or just \( m \) for brevity) is:

\[
E_{\text{potential}} = \int_R^r F_{\text{Earth}} \, dr = m g_R R^2 \int_R^r \frac{dr}{r^2} = m g_R R^2 \left( \frac{1}{R} - \frac{1}{r} \right) = m g_R \left( 1 - \frac{R}{r} \right)
\]

\[
E_{\text{kinetic}} + E_{\text{potential}} = \frac{1}{2} mc^2 + m g_R \left( 1 - \frac{R}{r} \right) = \frac{1}{2} mc^2
\]

The relativistic mass of the moving photon will cancel out. The result gives the one-way photon velocity originating at Earth’s surface intercepting the Pioneer spacecraft at a distance \( r \) from Earth. Although counterintuitive, \( c^* > c \) as the photon velocity is slower when the
gravitational field becomes denser. This is borne out by the photon velocities in transparent medium, which experiments confirm \( c' = c/n \) for the refraction index \( n > 1 \) of the medium. When photons enter a denser transparent medium of \( n_2 > n_1 \) from a less dense medium, the photon speed is slower as \( c/n_2 < c/n_1 \). A denser medium means more mass and a denser gravitational field internally within that substance. Photon trajectories also bend around the Sun as explained earlier due to the photon speed being slightly faster the farther away the photon is from a center of gravity. In general relativity, the gravitational field slows down the velocity of the photon as the photon gets closer to the Sun. “Note that this velocity is independent of the frequency; hence there is no dispersion, and the group velocity (signal velocity) coincides with the phase velocity.” [32, p.197] This Newtonian derivation agrees with the general outcome of the Pound-Rebka results if \( c' = f \lambda' \).

The following formula gives the photon velocity in terms of speed starting from Earth and anywhere between Earth and the Pioneer spacecraft.

\[
c' = c \sqrt{1 + \frac{2GmR}{c^2} \left(1 - \frac{R}{r}\right)} \tag{15}
\]

The term with the parenthesis is tiny with Earth’s mass. The larger contribution is from the Sun, which the Earth mass would be replaced in (15) with the solar mass. Equation (15) would need to be integrated from \( c \) on Earth at 1 AU to the uplink signal bypassing the Sun at closest approach (e.g., \( R \)) if the Earth was on the opposite side from the Pioneer, then calculate the changing photon speed to the Pioneer. The uplink arrival at each Pioneer would be a faster \( c' \) than the standard \( c \), which would explain why the Pioneer intercepted the uplink sooner than computed based on the standard \( c \) speed. Also, the onset of the Pioneer anomaly occurred in the proximity of Jupiter and later Saturn for Pioneer 11, so a third gravitational field of a planet needs to compensated with (15) to get the combined photon velocity for both the uplink and downlink transmissions when a Pioneer was near a planet.

To summarize, the photon velocity increases as the gravitational field becomes less dense, which occurs as a photon moves away from the center of gravity of a mass. Equation (15) shows one explanation this can occur. Also, the velocity of the photon source relative to the detector (or observer) will affect the recorded reception time due to the photon velocity differs from the standard \( c \), which Equations (10) and (11) confirm. The source of the annual and diurnal variations in the DSN Doppler residuals is found by the differences of the orbital velocity and rotational velocity at uplink transmission and downlink reception times. JPL should compare this theory to the DSN Doppler residuals.
8. Summary and Conclusion

A horologist at an ultraprecise timekeeping facility has two main duties: (1) synchronize all atomic clocks in the facility to maintain one master timescale, and (2) determine the offset in s and rate of change in s/s of the facility’s timescale to International Atomic Time (TAI) and to Earth’s rotation time (formerly Greenwich Mean Time and now effectively Coordinated Universal Time (UTC)). This allows outside users to check and calibrate their time using the facility. A dedicated horologist would maintain a constant temperature in the facility and isolate all vibrations from affecting the clocks. Before synchronizing the clocks, all electrical wiring and circuitry would be calibrated in roundtrip connections to a stable clock to determine the electrical delays. Then, synchronization of a second clock to a primary clock begins by measuring the electrical time difference between the two clocks and adjusting the second clock to the primary clock. Electrical pulses from the primary clock allows the horologists to synchronize pulses with the secondary clocks by accounting for the electrical delays. Routine checking between clocks over long periods allows for finer adjustments to maintain synchronization between all of the facility’s clocks. This is the practical application of Einstein’s synchronization method to synchronize atomic clocks within the facility.

In Einstein’s 1905 paper that defined special relativity, Einstein devoted the first section to time and how to synchronize remote clocks so that one can set a common coordinate time for timing separated events. Einstein conceived a thought experiment of photons traversing a rod of length $L$ in both directions. He required that photons traversing from end $A$ to end $B$ must take the same time when traversing from end $B$ to end $A$ (his Equation 1.1), which allows the process to synchronize remote clocks. When the rod is stationary in the resting frame of reference, each transmission span is simply $\Delta t = L/c$. The first relativity postulate requires equal time spans when the rod undergoes a constant velocity. However, when Einstein derived the transmission intervals for the rod moving at a constant uniform velocity $v$ relative to the same observer fixed in the resting inertial frame, he uncovered a discrepancy of unequal time spans while assuming a photon velocity has a universal constant speed. Einstein’s discrepancy means separated clocks uniformly moving cannot be synchronized by electromagnetic transmissions assuming a universal photon speed. However, the International Bureau of Weights and Measures (BIPM: Bureau International des Poids et Mesures) located in Sèvres, France, receives time difference data from participating atomic time laboratories throughout the world. BIPM routinely informs each participating atomic time laboratory its offset and rate from International Atomic Time (TAI). Timing institutions routinely exchange time
difference data with time from a satellite in common view, such as a Global Positioning System (GPS) satellite. The timing laboratories compute the exchanged data to discover the time difference of the facilities by \([\text{Lab}(1) – \text{GPS}] – [\text{Lab}(2) – \text{GPS}] = [\text{Lab}(1) – \text{Lab}(2)]\). Earth’s surface is not sufficiently inertial with ultraprecise synchronization. Foucault’s pendulum demonstrates Earth’s rotation with varying tangential speeds between the poles and the equator. Even the tides demonstrate the Earth’s surface is accelerated due to the solar and lunar gravitational pulls and centrifugal accelerations exerted on the orbiting Earth. A timing facility at 45° latitude would experience about 328 m/s tangential velocity from Earth’s rotation, which would make a pulse from a primary atomic clock arriving 1.82E-14 s later to a secondary atomic clock located 5 m east of the primary, according to Einstein’s discrepancy (c in Equation 4 – \(L/c\)). If one considered Earth’s orbital velocity, which varies between 29,300 and 30,300 m/s, a velocity projection onto the 5 m distance could be around 10,000 m/s that would make a pulse from the primary atomic clock to be off by 5.56E-13 s. Timing labs often maintain atomic clock synchronization around 1E-15 s/s. Even NIST’s optical clocks are capable of a precision of 1E-17 s/s. The timekeeping community would argue they can synchronize clocks and maintain ultraprecise time adequately between timing institutions—contrary to Einstein’s formulas that moving clocks cannot be synchronized with a universal constant \(c\).

This paper showed Einstein’s derivation had no algebraic error. The cause of the discrepancy is not dismissed by Einstein’s unsupported claim that a timed event is synchronized in one inertial frame while not synchronized in a different inertial frame, because the discrepancy was derived using only one resting frame with the same fixed observer recording the two cases of a stationary rod and uniformly moving rod. Technology relies on synchronization for all electrical systems in communication, experimentation, finance, transportation, navigation, etc. No consistent anomaly or reoccurring discrepancy has been found with synchronization on the complex moving Earth.

Therefore, some assumption is not valid to derive Einstein’s discrepancy, which assumed the relativity postulates: (1) the equivalence principle, and (2) the universal speed of light in a vacuum for all inertial reference frames, regardless of the motion of the light source or detector. The one-way photon transmission for synchronizing two clocks fixed relative to each other is not accomplished by photons having a universal constant \(c\) via Einstein’s (1.1).

Six discrepancies and contradictions are discussed in this paper involving the second relativity postulate. The first discrepancy was discovered by Einstein himself, when he showed that the
parallel time interval to transverse over a moving rod differed from the antiparallel time interval when assuming photon speed is the universal constant \( c \), which contradicted his requirement of equal time spans for synchronization (his Equation 1.1) and failed the first postulate. His derivation was confirmed in this paper by Equations (4) and (5), which allowed the parallel \( c_{AB} \) and antiparallel \( c_{BA} \) to be constant velocities of the photons that could be different or equal to \( c \). It is proven that the Lorentz transformation does preserve clock synchronization between inertial frames (i.e., if \( t_1 = t_2 \) for synchronized clocks in the resting frame, then \( t_1 = t_2 \) in the moving inertial frame.)

If Einstein had multiplied his equations by \( c \) and added them together, the roundtrip distances would be \( 2L \) for the stationary rod and \( 2\gamma^2L \) for the moving rod in his resting frame (add (5) and (6) using \( c \) to get Equation (7)). Length contraction is \( L' = L_0/\gamma \) for the moving rod relative to the stationary observer, leaving an extra \( \gamma \) in the roundtrip distance within the moving rod. This discrepancy is a contradiction of the first postulate. If one divided (7) by the universal \( c \) and let \( 2L/c = \Delta t \) for the roundtrip time span of the stationary rod, then the roundtrip time span for the uniformly moving rod is \( \Delta \tau = \gamma^2 \Delta t \). If the clocks on the rod ends were synchronized when stationary and then accelerated to a uniform speed \( v \), time dilation would affect both clocks equally and still keep their synchronization. Time dilation undercompensates for the difference between \( \Delta \tau \) and \( \Delta t \). However, comparing the measured time spans of \( \Delta \tau \) and \( \Delta t \), one can compute the relative speed of the moving frame attached to the moving rod and the resting frame attached to the stationary rod. This again contradicts the equivalence postulate. “Measurements made entirely within a given system must be incapable of distinguishing that system from all others moving uniformly with respect to it. This postulate of equivalence requires that physical laws must be phrased in an identical manner for all uniformly moving systems.” [33] This is the second contradiction.

To satisfy Einstein’s synchronization requirement that photons always traverse a rod both ways in equal time intervals, equate \( L_{AB}/c_{AB} \) to \( L/c \) and \( L_{BA}/c_{BA} \) to \( L/c \). Equations (6) and (7) show the photon velocities obey vector velocity addition for a moving photon source relative to the detector (i.e., \( c' = c \pm v \)). Following Einstein’s requirement for equal photon transmission time intervals in either direction between points \( A \) and \( B \) to synchronize clocks, the second relativity postulate is expanded concerning moving photon sources relative to the photon detectors. Then, photon traverses will have the same time spans, which are required for ultraprecise clock synchronization. Einstein proved the speed of the universal velocity \( c \)
prevents synchronization between moving clocks that maintain a fixed distance between them. This analysis is the third discrepancy due to the original wording of the second postulate and the present worldwide synchronization between atomic clocks that disputes Einstein’s original discrepancy.

The fourth discrepancy is special relativity predicts that masses measured parallel to the moving mass’s velocity will be \( \gamma \) times more massive than measured perpendicular to that velocity. No one has contradicted Einstein’s derivation in §10 of his 1905 paper for different longitudinal and transverse masses with a slowly accelerated electron, and no national standards institute has ever reported a diurnal variation in mass measurements. For example, if measuring a 1000 kg mass parallel to Earth’s orbital velocity and measuring it again 6 hours later to change the local vertical by Earth’s rotation relative to outer space, a variation could be a 5 \( \mu \)g change with electrical measurements and without using balance beams.

The most important theoretical result is special relativity does not maintain the same numerical constant speed for the same photon velocity between moving inertial frames. This is the fifth contradiction. Length contraction predicts shorter meters, and time dilation causes longer time seconds. Equation (12) shows that the current definition of a photon speed in one inertial frame compared to another inertial frame moving with a uniform velocity is \( c_1 = k \) m/s and \( c_2 = \gamma^2 k \) m’/s’. If length spans and time intervals were either both dilated or both contracted together, then cancellation of the changes in meter and second would produce the identical numerical photon speed in both inertial frames. However, they are not, and this is the cause for this most significant contradiction failing the second postulate.

Finally, the sixth contradiction is given by Equation (13) as the product of frequency and wavelength associated with a photon is not preserved between inertial frames. This is borne out by the equation \( c = f\lambda \) as \( c_1 = f\lambda = \gamma^2 f\lambda’ = \gamma^2 c_2 \) between moving inertial frames.

Some may claim that one-way photon velocities were found to be the standard \( c \). For example, two photon detectors were fixed 31.450 m apart to measure the speed of \( \gamma \) rays [24]. According to the tenets of QED, the high energy photons were intercepted by atoms in the first detector, which generated excited orbiting electrons by absorption to send timing pulses. Later, those electrons deexcited to emit new, secondary photons that were detected by the second detector. As both detectors were mutually stationary within the laboratory, the relative velocity \( v \) between the first detector (i.e., the actual emitter of the secondary photons) and the second detector is zero. So, the expected result was \( c \pm v = c \), which that test confirmed.
Unfortunately, such tests fail to measure the actual speed of the incident γ rays or high energy photons due to interception prior to measuring.

Many ramifications due to this generalization will affect different fields in physics. The MM interferometer results for static constructive output contradict the standard explanation that only length contraction applied on the parallel component of the interferometer arms will cause the appropriate distance changes to output constructive interference, but ignores time dilation applied to the parallel components that would differently change each arm’s frequency to output destructive interference. The LIGO observatories have output the equivalent of constructive interference, which the one-way photon velocities explain those results. A universal constant c would result in dynamic interference due to Earth’s changing orbital velocity relative to outer space and due to Earth’s tangential velocity from its rotation, which Einstein discovered in his unequal time interval discrepancy between beams traversing a moving rod in either direction using a universal c. In the freely falling, Earth centered ECI inertial frame, which is nonrotational, the northern and southern endpoints of each arm in LIGO have different tangential velocities from Earth’s rotation, causing the endpoints to approach or recede differently from the ECI origination point of each photon emission at each mirror, which results in different roundtrip arm distances in the ECI frame and predicts destructive interference with a universal c. However, LIGO has collected over 22 months of equivalent constructive interference, excluding the daily glitches and rare gravitational wave detections.

The Pioneer anomaly is the most controversial discovery in classical physics with its three contradictory results. JPL and Aerospace Corporation discovered that both Pioneer spacecraft detected the uplink signal from the DSN ground stations sooner than expected. Downlink signals were compared continuously to uplink transmissions in real time. In addition, the Doppler residuals revealed an annual and diurnal variation in the precise circularized signals, which also remain unexplained. If photon velocity obeys vector velocity addition, then the annual and diurnal variations are due to the differences between uplink transmission and downlink reception with Earth’s orbital velocity and tangential rotational velocity. The roundtrip transmission time was many hours, which the DSN network was not at the origination point of uplink transmission when it received the downlink. Only when the roundtrip of several traverses ends at the origination point will the average photon speed equal the standard speed c. Contained in this paper is a derivation that shows the photon speed increases when traversing away from a center of gravity. This would explain why the Pioneer
spacecraft each detected the uplink sooner than expected, and the discrepancy between predicted uplink reception and actual downlink transmission increased the further away each Pioneer exited the solar system.

The only time that the average photon speed in any inertial frame is consistently the standard speed of \( c \) is when photons traverse a roundtrip excursion back to the point of origin if the effects of gravity can be ignored or gravity is perpendicular to all photon traverses. (Photon velocity for a roundtrip is zero as the vector addition of displacements is zero.) Early light speed tests by Fizeau, Foucault, and Michelson were horizontal roundtrip flights that indicated photon speed is universal and fixed. This could have influenced Maxwell and Einstein to believe the magnitude of photon velocity has a constant universal speed, despite the relative movement between a photon source and detector.

In conclusion, the objective of this paper was to resolve Einstein’s original discrepancy that two clocks with a fixed distance between them could only be synchronized if stationary in a resting inertial frame, but unable to be synchronized when moving within the resting frame. The simplest case was attaching two perfect clocks to a rod at endpoints \( A \) and \( B \). When the rod was stationary, the time span to traverse the rod between the points \( A \) and \( B \) is the same in either direction, which was Einstein’s requirement for synchronization. Equations (8) and (9) were rigorously derived with one stationary observer fixed in the resting frame, showing for a rod moving with uniform velocity, \( v \), that \( \Delta t_{AB} = L/(c_{AB} - v) \) and \( \Delta t_{BA} = L/(c_{BA} + v) \). This confirmed Einstein’s algebraic results if one replaced the general one-way speeds of photon velocities with the universal speed of velocity \( c \). The first relativity postulate mandates that the physical laws governing the states of a system (e.g., equal transmission intervals over a rod in both directions) are the same under uniform translational velocities.

Synchronization between the clocks at endpoints \( A \) and \( B \) is kept whether the rod is stationary or moving uniformly as required by the first relativity postulate and as demonstrated by actual synchronization between atomic clocks in either direction. Einstein’s thought experiment had the clocks synchronized when the rod was stationary. When the rod was accelerated to the desired uniform velocity and orientation, both clocks had the same effects of acceleration, time dilation, changes in gravity (if any), etc., so the clocks still maintained their mutual synchronization. The clocks broadcasted the time when a photon was emitted from end \( A \) with \( t_A(1) \), the time when the photon was absorbed according to QED to excite electrons at end \( B \) with \( t_B(1) \) on the first leg, the time the deexcited electrons at \( B \) emitted new photons with \( t_B(2) \), and the time when end \( A \) absorbed photons with \( t_A(2) \) on the second leg. An observer
stationary in the resting frame receives the same four messages of times as an observer fixed to the uniformly moving rod in its moving reference frame. In the moving frame, the second observer is stationary and will calculate equal traversing time spans of photons in both directions between the rod’s ends, which satisfy Einstein’s definition of synchronization. The same four messages of time tags are transmitted to the first observer stationary in the resting frame. The calculated times to traverse the rods in both directions must be identical between the two observers. Equations (8) and (9) give the time spans of the moving rod in each direction for the stationary observer in the resting frame with the general photon velocities of $c_{AB}$ and $c_{BA}$.

When equalizing $\Delta t_{AB} = \Delta t_{BA}$ with $\Delta t = L/c$ from the stationary case, $c_{AB} = c + v$ and $c_{BA} = c - v$ due to this relative velocity, $v$, between the photon source and detector. So, equating the one-way time intervals for both the stationary and moving cases resulted in the discovery that one-way photon velocities obey vector velocity addition when the photon source has a relative velocity with the photon detector. This concept of the one-way photon velocity resolves several issues or explains the contradictory results of very precisely measured events, especially the null results of the MM interferometer, long-term LIGO data, and the three Pioneer anomalies.

The second postulate of relativity should be expanded by allowing one-way photon velocities to change due to the velocity of the photon source relative to the detector by vector velocity addition in the absence of gravity. As derived in this paper (Equation (15)), photon velocity increases as its distance from a center of gravity increases. If true, photon velocities are neither constant nor universal in the presence of gravity. It will take time for many of these issues to be reviewed thoroughly. “For example, we speak of the physical constants, to denote those numbers that never change. These include the most basic parameters of the laws of physics, such as the speed of light or the charge of the electron. But are these constants actually constant?” [34] “In the unlikely event that there is new physics, one does not want to miss it because one had the wrong mind set.” [32] It would not be surprising if other aspects of physics will need revision as well.

**Appendix: Simultaneity/Nonsimultaneity**

In December 1916, Einstein published a book introducing relativity theory to interested readers having a general scientific background [35]. Among several topics, Einstein
considered a mental experiment to illustrate that events can be simultaneous in one inertial frame and nonsimultaneous in another inertial frame. He described dual lightning strikes hitting synchronously the ground of a long embankment at points A and B that were parallel to a straight railroad track. A person was standing at the midpoint M between A and B, which that person reported the lightning strikes hit the ground simultaneously. Concurrently, a long train was traveling at a constant velocity v on the railroad. Einstein’s description stated, “But the events A and B also correspond to positions A and B on the train.” A passenger on the train was at midpoint $M'$, between A and B at the instant the dual lightning strikes occurred (i.e., the passenger was in the plane of the perpendicular bisector where the person on the ground was also located between A and B in the ground frame at the same instant the strikes hit the ground). In the finite amount of time for light to traverse the equal distances d of $A \rightarrow M$ and $B \rightarrow M$ with the constant velocity c, the train moved the passenger to a new distance approximately $vd/c$ toward B (or precisely $vd/(c+v)$ by Equation (6)). Einstein pointed out that the passenger is closer to B and that the passenger reports light from B was seen earlier than from A. Two nonsimultaneous lightning strikes were reported by the passenger, but the person on the embankment saw simultaneous strikes. (The author’s definition of simultaneous is satisfied as photons from the two bolts arrived at the person’s position on the ground at the same time instant.)

It would seem Einstein demonstrated, “Events which are simultaneous with reference to the embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). Every reference-body (co-ordinate system) has its own particular time; unless we are told the reference-body to which the statement of time refers, there is no meaning in a statement of the time of an event.” [35, p. 30-31]. Consider the following mental experiment involving a flat Earth with still air. A hiker stands on a bridge spanning over a long passenger train that is moving west to east at a uniform velocity v. Two radio towers with identical station clocks are located 5 km north at A and 5 km south at B from the bridge. The clocks in each radio station are synchronized. Due to poor insulation and grounding, dual synchronous lightning strikes hit the two stations and short out the clocks, which stopped at the same instant of coordinate time. The hiker and all passengers on the train are located in the plane of the perpendicular bisector of the length $AB$. All points on this plane are equidistant from A and B, so that the arrival of light and slower sound from each strike arrives to each observer simultaneously. The hiker saw and heard the earliest simultaneous events of lightning and thunder. Some passengers closer to the line $AB$ may report earlier simultaneous lightning
strikes and thunder than other passengers further away. All observers, whether fixed in the ground frame or in the train frame, reported simultaneous events of lightning and thunder from the dual strikes. Einstein’s claim of simultaneous observations of a pair of events in one inertial frame are nonsimultaneous sightings in another moving inertial frame is not universal between resting or moving inertial frames.

Einstein described the locations of the two witnesses in terms of the embankment frame in his scenario [35, p. 26-30]. One gets the same result with thunder claps if the air is still in the embankment frame. The speed of light is mimicked by the much slower speed of sound in this case. However, Einstein’s mental test has a paradox in the train frame. Einstein wrote that the dual lightning strikes at $A$ and $B$ are also located on the train’s frame at the instant of the strikes with the sitting passenger located at $M'$ midway between $A$ and $B$. By the second postulate, relativity theory predicts light from the two strikes in the train frame will merge simultaneously at $M'$. Einstein wrote the passenger reported nonsimultaneous lightning strikes, which is a contradictory paradox in the train frame.

To resolve this paradox, embellish Einstein’s dual lightning scenario while retaining the same concepts and the same geometry. The passenger had opened a window in the train car to see lightning directly and hear thunder during the storm. The railroad company had installed ground monitors at regular intervals beside the rails to tell the time and where the train was during its destination. The monitors had clocks that were synchronized to a common timescale. The railroad company also installed similar monitors on each railcar with synchronized clocks on a different timescale to check their status. Two monitors were at ground points $A$ and $B$ where the dual lightning strikes occurred, and those clocks were shorted out, indicating identical time outage from the strikes (i.e., synchronized strikes at the same identical coordinate time). The two strikes grazed the two railcars $A$ and $B$ and hit the ground, causing the same damage to those two railcar monitors. As Einstein stipulated, the person at $M$ saw a single light flash and, later, heard a single clap of thunder, while the passenger at $M'$ witnessed two lightning flashes and, later, two claps of thunder. The train engineer stopped the train at the next station to inspect the damage, and the passenger walked to each damaged site to record what happened. The two railcars, $A$ and $B$, had charred marks along the sides where each bolt grazed the railcar, and those railcar clocks were stopped at the same time. The passenger measured the distances between the marks and the passenger’s seat, which were the same (i.e., identical lengths for $AM'$ and $BM'$). The passenger noticed there was no wind after the train stopped. Physics calculations predict the passenger at $M'$ should
only see one lightning flash and hear one thunder clap being simultaneous, as the synchronous lightning strikes were equal distances from the passenger in the train frame for both light speed and the slower sound speed.

Then, the passenger recalled that there was an apparent wind with lots of turbulence from the open window when the two thunder claps were heard. If the train was stationary relative to the ground, then a wind from B moved the sound faster to $M'$ and retarded the effective sound speed from A to $M'$. If the train has an opposite ground velocity $v$ (noted by the train engineer) while the air was still, this would create the identical effect in the arrival times between the thunder claps from $A$ or $B$ relative to $M'$. This would explain the two thunder claps heard by the passenger, according to Equations (8) and (9). In still air with constant temperature and pressure, the horizontal speed of sound is constant in all radial directions. The velocities of sound between the sources and detector do explain the result of two thunder claps heard by the passenger on the moving train. The passenger realized that the light from the two lightning strikes did not penetrate through all the cars between car $A$ and car $M'$ or between car $B$ and car $M'$. The passenger saw light diffused from the atoms in the atmosphere that went through the open window. If velocities of light are affected by the relative velocity between the light source and the light detector as indicated by Equations (10) and (11), which holds for the velocity of sound, then the paradox is resolved so that the passenger does report two lightning flashes and two thunder claps in both the embankment frame and in the train frame. This requires a generalization for the second postulate of relativity that the relative velocity of light between the source and detector does add a vector velocity to the standard velocity of $c$. In the author’s opinion, leaving this paradox unresolved is worse than modifying the second postulate of relativity (i.e., it is impossible for the passenger to observe one and two lightning strikes for one event based on different inertial reference frames).

References


[7] Sauter, J., *Erinnerungen an Albert Einstein*. This pamphlet (unpaginated) was published in 1965 by the Patent Office in Bern, and contains documents pertaining to Einstein’s years at that office as well as a note by Sauter.


[14] LIGO caltech.edu website (operated by Caltech and MIT), Facts subpage lists the extreme engineering to construct LIGO and its operation.


