SCIREA Journal of Physics
ISSN: 2706-8862
http://www.scirea.org/journal/Physics
April 2, 2024
Volume 9, Issue 2, April 2024
https://doi.org/10.54647/physics140620

How Rayleigh and Jeans Arrived at Their Radiation Formula

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Abstract

In 1900, Rayleigh put forward a complete expression of blackbody radiation distribution function, but the specific coefficient value was not given. It was not until 1905 that Rayleigh reconsidered the problem of blackbody radiation in the long wave region that he calculated the specific coefficient value, but the coefficient value he gave was 8 times larger than that in the formula we are familiar with today. In the same year, this coefficient was revised by Jeans, so it was called Rayleigh-Jenkins formula.

Keywords: blackbody radiation, Rayleigh-Jeans formula

1. background of blackbody radiation

As the British physicist Lord Kelvin said, two small, disturbing clouds appeared in the sky of the theory of the thermal and light energy in the 19th century. In fact, if we carefully review the new discoveries made at the end of the 19th century, we will find that "the beautiful and clear sky has long been overcast with clouds, with a strong 'wind before the coming of rain' potential. Here we only review the background of blackbody radiation.

In the mid-19th century, physicists had already realized the importance of studying the relationship between light emission and light absorption. In 1859, German physicist Kirchhoff obtained the following law: "At the same temperature, the ratio of emissivity to absorptivity for the same wavelength radiation is the same for all objects." In 1860, Kirchhoff defined an ideal object with an absorption coefficient 1 as an "absolute blackbody", which can absorb all radiation falling on it at any temperature. For an absolute blackbody, its surface brightness is equal to its emission ability. Kirchhoff called for the most important task to find the function of emission ability. In 1879, Stefan derived from his measurements that the total thermal radiation energy of a blackbody per unit area in unit time is proportional to the fourth power of absolute temperature. This result was demonstrated by Stefan's student Boltzmann in 1884 using electromagnetism and thermodynamics, so this is also called "Stefan-Boltzmann law". In 1881, American physicist Langley obtained a rough distribution curve of thermal radiation using carbon-coated platinum wire. This curve is asymmetric, and the maximum radiation intensity moves to the short-wave direction as the temperature increases. In 1893, German physicist Wilhelm Wien (Figure 1)participated in designing an open cavity radiation experiment, and soon obtained a displacement law of thermal radiation consistent with Langley's. In 1896, German physicist Paschen used various methods to measure thermal radiation on many solids such as carbon, copper oxide, and platinum, obtaining relatively accurate blackbody radiation spectra. This spectrum is almost similar to a bell-shaped curve, just like the dorsal fin of a shark. it was time to establish its mathematical expression after the actual blackbody radiation spectrum was obtained,. In 1896, using thermodynamic principles, Wien proved that the blackbody radiation spectrum must obey the following formula:

$$\rho(\lambda, T) = c\lambda^{-5}\psi(\lambda T) \quad (1)$$

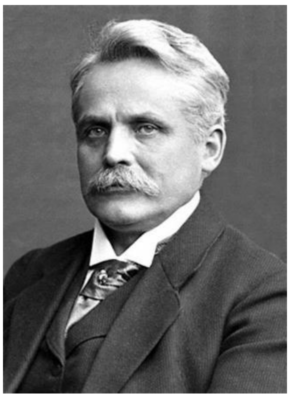

Fig.1 Wilhelm Wien(1864-1928)

The expression of ψ cannot be finally determined yet, but from formula (1), the Stefan-Boltzmann law and Wien's displacement law can be derived. Soon after that, Wien made an assumption that blackbody radiation was actually emitted by the molecules obeying the Maxwell speed distribution, and the wavelength of the radiation emitted by a single molecule is only a function of the molecular speed. This assumption leads to a formula for the blackbody radiation spectrum:

$$u(\lambda,T)=\frac{c_1}{\lambda^5}e^{\frac{-c_2}{\lambda T}} \tag{2}$$

2. The initial proposal of Rayleigh formula in 1900

In 1900, Wien's blackbody radiation formula was most discussed. But as early as early November 1899, Otto Lummer and Ernst Pringsheim (Figure 2) had reported that there was a systematic inconsistency between Wien's formula and experiment results. They found that Wien's formula perfectly matched the experiment in the short-wavelength part, but the radiation intensity obtained from Wien's formula was always lower than the experimental measurement in the long-wavelength range. However, within a few weeks, another experimental physicist Friedrich Paschen (Figure 3) offered a different opinion, claiming that Wien's formula was a rigorous and effective natural law, and presented a new set of experimental data, which was in excellent agreement with Wien's formula.

At a biweekly meeting in Berlin on February 2, 1900, Lummer and Pringsheim announced their latest measurement data. They firmly believed that there was indeed a systematic inconsistency between the data they measured and the prediction made by the Wien's formula in the infrared region, and this inconsistency could not be caused by experimental errors. Lord Rayleigh (Figure 4) was attracted by the discussion about the Wien's formula. Before that, Lord Rayleigh had written an article to defend the equipartition theorem (also named as the Maxwell-Boltzmann theorem at the time), which was opposed by Lord Kelvin.

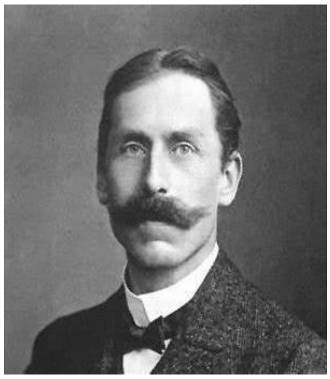

Fig.2 Lummer （left 1860-1925） Pringsheim（right 1859-1917）

Fig.3 Friedrich Paschen （1865-1947)

Raleigh believed that the proof in his defense article was sound, but he also fully understood that nature did not always agree with the conclusions from the equipartition theorem. For example, diatomic gas molecules behaved as if they had only five degrees of freedom, seemingly providing no energy for rotation around the atomic bonds. Raleigh's explanation was that the binding of the two atoms in a diatomic molecule might not be infinitely strong. Raleigh believed that the equipartition theorem was still applicable to any dynamic system if properly applied. Therefore, as a special case of the equipartition theorem, he carefully examined Wien's blackbody radiation formula and made a brief comment, "From a theoretical point of view, this result is almost a guess to me."[1] He specifically pointed out that at high temperatures, the energy of waves with long wavelengths was too small according to the Wien's formula, which was inconsistent with the equipartition theorem of Maxwell-Boltzmann(Figure 5) theorem .

Fig.4 Lord Rayleigh（1842-1919）

Fig.5 Maxwell（left 1831-1879） Boltzmann（right 1844-1906）

Rayleigh believed that every vibration mode should be equally weighted, according to the equipartition theorem. Although the equipartition theorem did not always hold in all cases, it seemed to work for the more important vibration modes. In other words, he believed that Wien's formula provided a basically correct description for short wave lengths region, but not for long wave lengths region. He expected the equipartition theorem to hold for the long wavelength part. To derive the distribution function of blackbody radiation from the equipartition theorem, Rayleigh compared the system of cavity radiation with the acoustic vibration of a cubic air. For a one-dimensional string of length *L*, according to the features of standing waves $L = m\frac{\lambda}{2}(m = 1,2,3...)$, and it can only vibrate with a vibration mode of $\lambda = \frac{2L}{m}$. Similarly, for the vibration of a cube with a side length of *L*, the same conditions apply, but the possible vibration modes need to be determined by three numbers n, p, q:

$$\begin{cases} L = n\frac{\lambda}{2} \\ L = p\frac{\lambda}{2} \\ L = q\frac{\lambda}{2} \end{cases} \Rightarrow \begin{cases} k_1 = \frac{n\pi}{L} \\ k_2 = \frac{p\pi}{L} \\ k_3 = \frac{q\pi}{L} \end{cases} \tag{3}$$

where n, p, q are positive integers. and we can get

$$k^2 = k_1^2 + k_2^2 + k_3^2 = (n^2 + p^2 + q^2)\left(\frac{\pi}{L}\right)^2 \tag{4}$$

Since $k = \frac{2\pi}{\lambda}$, the wavelength can be determined to be $\lambda = \frac{2L}{\sqrt{n^2 + p^2 + q^2}}$.

In fact, any specific vibration is characterized by a wave number *m*, and the vibration mode number of wave number between *m* and *m*+Δ*m* is determined by $m^2\Delta m$. By interpreting the wave number *m* as the blackbody radiation frequency ν, and assigning a kinetic energy proportional *T* to each frequency mode (the equipartition theorem), Rayleigh concluded that the energy of each mode with a frequency between ν and $\nu+\Delta\nu$ is necessarily proportional to $T\nu^2\Delta\nu$.

Considering $\nu = \frac{c}{\lambda}$, $\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$, the distribution function $u(\lambda, T)$ is determined by the expression $\frac{T}{\lambda^4}$, except for a constant factor c_1'. Eventually, Rayleigh proposed the "complete expression" for the distribution formula of blackbody radiation.

$$u(\lambda,T)=c_1'\frac{T}{\lambda^4}e^{\frac{-c_2}{\lambda T}} \quad (5)$$

The reason this formula is called a complete expression is that Rayleigh simply inserted an exponential factor to make the radiation distribution applicable even in the short-wavelength or high-frequency region, after he noticed that the exponent in Wien's formula fitted the data well. From Rayleigh's formula (5), for long wavelength region or large values of λT, the exponential part $e^{\frac{-c_2}{\lambda T}} \to 1$, so $u(\lambda,T)\propto kT$, which agrees with the equipartition theorem. At the same time, even in short wavelength region, if Wien's displacement law is taken into account, equation (5) can be transformed into Wien's formula, and the constant factor c_1' in (5) and the constant c_1 in Wien's formula (2) are connected. However, the constant c_1 in Wien's formula does not have a specific value, so the constant factor c_1' in (5) is not specified.

3. Reconsideration of Rayleigh Formula in 1905

Lord Rayleigh's formula of total radiation, equation (5), immediately attracted the attention of experimental physicists. For example, Otto Lummer and Eugen Jahnke, Heinrich Rubens and Ferdinand Kuhlbaum (Figure 6), all cited it in 1900. But unfortunately, Rayleigh's formula of total radiation (5) did not leave a lasting impression, because it was quickly replaced by the blackbody radiation formula proposed by Max Planck at the end of 1900.

Fig. 6 Jahnke（left 1861-1921）Rubens（middle1865-1922 ） Kuhlbaum（right 1857-1927）

Based on his proposed energy quantum hypothesis, Max Planck (Figure 7) derived a new formula for blackbody radiation $u(\lambda,T)=\frac{c_1}{\lambda^5}\frac{1}{e^{\frac{c_2}{\lambda T}}-1}$, and Rubens found that it perfectly matched experiment results. Even so, Rayleigh's theory on blackbody radiation were re-raised by Rayleigh himself and James Jeans (Figure 8) about five years later, and left a significant mark in the history of physics.

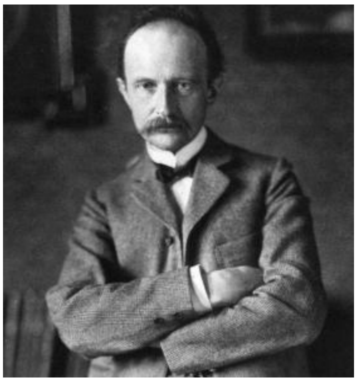

Fig. 7 Max Planck（1858-1949）

Fig. 8 James Jeans（1877-1946）

In Nature released on May 18, 1905, Lord Rayleigh published a letter dealing with the "dynamic theory of gases and radiation"[2]. In this letter, Rayleigh re-derived his previous formula, i.e. equation (5). He also calculated the value of the coefficient c_1'. His idea five years ago is that if n,p,q represented the integers in three different directions in a cube, the condition for standing waves in the cube could be written as

$$n^2+p^2+q^2=\frac{4L^2}{\lambda^2} \qquad (6)$$

Using spherical coordinate system, Rayleigh calculated the number of combinations of (n,p,q) within R and $R+dR$. The volume between R and $R+dR$ can be written as

$$dV=4\pi R^2 dR \qquad (7)$$

If $R=\sqrt{n^2+p^2+q^2}$, $R=\frac{2L}{\lambda}$, so $|dR|=\frac{2L}{\lambda^2}d\lambda$. Equation (7) can be written as

$$dV=4\pi\cdot\frac{4L^2}{\lambda^2}\cdot\frac{2L}{\lambda^2}d\lambda=\frac{32\pi L^3}{\lambda^4}d\lambda \qquad (8)$$

Note that in the npq space, the number density of points or the number of (n,p,q) combinations is one per unit volume (i.e. $dN=dV\cdot 1=dV$), but considering the two polarization directions (B, E) of electromagnetic waves, the number of combinations is twice that in the above equation, i.e. $dN = \frac{64\pi L^3}{\lambda^4} d\lambda$, or the number of vibrational modes (the number of degrees of freedom of the harmonic oscillator) per unit volume and wavelength is

$$\frac{dN}{L^3 d\lambda} = \frac{64\pi}{\lambda^4} \qquad (9)$$

According to the equipartition theorem of energy, the average kinetic energy of each harmonic oscillator is $\frac{1}{2}kT$, where *k* is the Boltzmann constant, and the average potential energy and average kinetic energy of the harmonic oscillator are equal, so the average energy of each degree of freedom is kT. Eventually, the radiated energy equals to the product of number of vibrational modes and the average energy per mode:

$$u(\lambda, T) = \frac{64\pi}{\lambda^4} \cdot kT \qquad (10)$$

Comparing with the formula proposed in 1900, Lord Rayleigh obtained the value of coefficient c', and $c_1' = 64\pi K$.

Soon after that, Rayleigh made two comments on his new radiation formula (10): first, he raised a question about why the coefficient in equation (10) was inconsistent with that in Planck's formula in the limit of long wavelengths; secondly, he keenly realized a great difficulty brought by the new formula (10), that is, no equilibrium could exist between blackbody radiation and molecules, because all energy would eventually be absorbed by short-wavelength or high-frequency radiation, which was later known as the ultraviolet catastrophe. The two comments made by Rayleigh were soon answered by James Jeans, lecturer in applied mathematics in Cambridge University.

4. Modification to Rayleigh formula by James Jeans

Just in 1905, Jeans also discussed the balance between blackbody radiation and molecules, and prepared to submit two papers. But before the submission, he saw the first letter in Nature from Rayleigh, and the second letter soon after. He decided to answer the two questions from Rayleigh. In a letter dated May 20, 1905, but published in the June 1 issue of Nature[3], the title was still "The Kinetic Theory of Gases and Radiation", and in the postscript of his paper in Philosophical Magazine dated June 7, 1905, he pointed out an error in Rayleigh's new formula (10), and obtained the following expression for the radiation distribution function:

$$u(\lambda,T)=\frac{8\pi}{\lambda^4}\cdot kT \tag{11}$$

This formula is later known as the Rayleigh-Jeans formula. Jeans commented, "This is one-eighth of the amount found by Lord Rayleigh, but agrees exactly with that given by Planck for large value of λ."[4]

So, Jeans stated that Lord Rayleigh's first difficulty originated from a calculation error: Rayleigh forgot the non-negativity of n,p,q when deriving his formula. Because n,p,q must take positive values, only the volume of the sphere in the first quadrant should be considered, which is exactly one eighth of the original. Shortly afterwards, Lord Rayleigh affirmed the correctness of Jeans' formula (11) in his letter entitled "The constant of radiation as calculated from molecular data" dated July 7, 1905 and published in the July 13, 1905 issue of Nature. He admitted his own mistake[5]. At the same time, in the letter to Nature, Jeans also discussed the second difficulty, i.e., the existence of balance between blackbody radiation and molecules. He mentioned that although equation (11) represents a stable energy distribution of radiation, this distribution cannot be observed experimentally because the kinetic energy transfer to short-wavelength radiation modes in a cavity at a given temperature *T* is very slow. Jeans argued that "if the average collision time between two molecules in a gas system is a large multiple *N* of the vibration period, it can be shown that the average energy transferred to vibration per collision contains a small factor of order e^{-N} . As a result, in any finite time *t*, only long-wavelength vibrational modes will have full energy allowed by the equipartition theorem (i.e. kT), but short-wavelength vibrational modes do not. The contribution of the latter to radiation energy depends on a time share $f(\lambda,T,t)$, which gradually increases during the process of time until it reaches the value $\frac{8\pi}{\lambda^4}\cdot kT$. In addition, this share will decrease rapidly to 0 as the wavelength decreases."[4]

5. the influence of Rayleigh-Jeans formula

Although Jeans correctly modified Rayleigh's formula and seemingly solved the difficulty of divergence, his work did not receive immediate recognition among scientists. There may be several reasons for this: First, Planck's blackbody radiation formula has already described the experimental results perfectly. Secondly, there has never been any indication that the energy distribution in the cavity changes over time. Thirdly, if Jeans' explanation is correct, the well-established thermodynamic proofs of several blackbody radiation laws such as Stefan-Boltzmann's law and Wien's displacement law will collapse, and it will be difficult to establish these laws under non-equilibrium conditions. Lastly, it seems that Jeans used an elastic ether vibration to describe blackbody radiation, just like Rayleigh, but according to Einstein's theory of relativity, the elastic ether had become a questionable concept at that time. Although Planck's blackbody radiation formula has surpassed the Rayleigh-Jeans formula, in the early 20th century, the hypothesis of energy quanta was a shock to classical physics. Even Planck himself was hesitant, not to mention most physicists, so the energy quantum hypothesis did not receive much attention in the first few years, and people reacted indifferently to it. Only young Einstein enthusiastically accepted it. In contrast, the Rayleigh-Jeans formula is based on classical electromagnetic theory, so people were more willing to accept it psychologically.

The Rayleigh-Jeans formula is still of great significance for studying electromagnetic radiation. It can be used not only to simulate various types of radiation, but also to help us understand how electromagnetic radiation affects the environment. From a historical perspective, the Rayleigh-Jeans formula could be viewed as a scaffold for the understanding of quantum theory. Because there are irreconcilable contradictions between experimental results and short-wavelength or high-frequency range, the occurrence of ultraviolet catastrophe is inevitable. Only by accepting quantum theory can we find a new way out, thus promoting our understanding of the natural world. Eventually, the contribution of the Rayleigh-Jeans formula in history cannot be erased.

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