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STUDY OF STERILE NEUTRINO CONTRIBUTION TO NEUTRINOLESS DOUBLE BETA DECAY

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Abstract

The sterile neutrino contribution to neutrinoless double beta (*0νββ*) decay has been first studied within a specific model with the two sterile neutrinos mixed only with the *ν^e* flavor state. In the present paper, we assume that in addition to the three conventional light neutrinos there exists only one Majorana neutrino mass-eigenstate *νh*, dominated by the sterile neutrino species, with an arbitrary mass *mh*, which may mix with all the active neutrino weak eigenstates *νe,μ,τ.* We study possible contribution of this *ν^h* neutrino state to *0νββ*-decay via a nonzero admixture of *ν^e* weak eigenstate.

Keywords: *Sterile neutrino, double beta decay, neutrino mass, PHFB model, nuclear matrix elements*

1. Introduction

The sterile right-handed neutrinos not participating in the electroweak interactions, are natural candidates for the extension of the standard model (SM) [1]. It has long been recognized that the

sterile neutrinos may have plenty of phenomenological implications. One of them is the seesaw generation of tiny Majorana neutrino masses for the three observable very light neutrinos via a very large Majorana mass term of the right-handed neutrinos. In this case together with the very light neutrinos there also appear very heavy Majorana neutrino mass states with the typical masses of 10¹² GeV. However, in a more general case of the sterile-active neutrino mixing there may also appear additional neutrino states v_h with arbitrary masses m_h . The neutrino mass eigenstates v_h , dominated by the sterile v_R neutrino weak eigenstates, contain some admixture of the active $v_{e,\mu,\tau}$ neutrino species that allows the v_h to contribute to various processes, in particular, to those which are forbidden in the SM by the Lepton Number or Lepton Flavor Violation (LNV or LFV). They may also modify the interpretation of cosmological and astrophysical observations. Therefore, the masses m_h of v_h neutrino states and their mixing with the active neutrinos are subject to various experimental as well as cosmological and astrophysical constraints. Various implications of the sterile neutrinos have been extensively studied in the literature [2-7].

The sterile neutrino contribution to *0νββ*-decay has been first studied in the pioneer work [8] within a specific model with the two sterile neutrinos mixed only with the v_e flavor state. In the present case, we assume that in addition to the three conventional light neutrinos there exists only one Majorana neutrino mass eigen state *νh*, dominated by the sterile neutrino species, with an arbitrary mass *mh*, which may mix with all the active neutrino weak eigenstates, *νe,μ,τ*. We study possible contribution of this *ν^h* neutrino state to *0νββ*-decay via a nonzero admixture of *ν^e* weak eigenstate. The paper is organized as follows. In Section 2, theoretical description of the mechanism and model used to calculate matrix elements is presented. Section 3 deals with results and discussions. Some Concluding remarks are presented in Section 4.

2. Mechanism involving sterile neutrinos

The contribution of the sterile v_h neutrino to the half-life $T_{1/2}^{0\nu}$ for the $0^+ \to 0^+$ transition of $0\nu\beta\beta$ -decay has been derived by considering the exchange of a Majorana neutrino between two nucleons and is given by [9]

$$
\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = G_{01} \left| U_{eh}^2 \frac{m_h}{m_e} M^{(\nu)}(m_h) \right|^2 \tag{1}
$$

Here m_e is the electron mass and G_{01} is the phase space factor given by

$$
G_{01} = \left[\frac{2(G_F g_A)^4 m_e^9}{64\pi^5 (m_e R)^2 \ln(2)}\right] \times \int_{1}^{T+1} F_0(Z_f, \varepsilon_1) F_0(Z_f, \varepsilon_2) p_1 p_2 \varepsilon_1 \varepsilon_2 d\varepsilon_1 \tag{2}
$$

 U_{eh} is the v_h − v_e mixing matrix element and the NTME $M^{(0\nu)}(m_h)$ is written as

$$
M^{(0\nu)}(m_h) = \langle 0_F^+ \left\| \left[-\frac{H_F(m_h, r)}{g_A^2} + \sigma_n \sigma_m H_{GT}(m_h, r) + S_{nm} H_T(m_h, r) \right] \tau_n^+ \tau_m^+ \right\| 0_I^+ \rangle \tag{3}
$$

with $S_{nm} = 3(\sigma_n \cdot \mathbf{r}_{nm})(\sigma_m \cdot \mathbf{r}_{nm}) - \sigma_n \cdot \sigma_m$

The neutrino potentials H_{α} are of the form

$$
H_{\alpha}(m_h, r) = \frac{2R}{\pi} \int_{0}^{\infty} \frac{f_{\alpha}(qr)h_{\alpha}(q^2)q^2dq}{\sqrt{q^2 + m_h^2} \left(\sqrt{q^2 + m_h^2} + \overline{A}\right)}
$$
(4)

where $f_{\alpha}(qr) = j_0(qr)$ and $f_{\alpha}(qr) = j_2(qr)$ for α = Fermi (F)/Gamow Teller (GT) and tensor (T) potentials respectively. The effects due to the finite size (FNS) are incorporated through the dipole form factors [10]. The functions $h_F(q)$, $h_{GT}(q)$ and $h_T(q)$ are given by

2

$$
h_F(q) = g_F^2(q^2)
$$
 (5)
\n
$$
h_{GT}(q) = \frac{g_A^2(q^2)}{g_A^2} \left[1 - \frac{2}{3} \frac{g_F(q^2)q^2}{g_A(q^2)2M_P} + \frac{1}{3} \frac{g_F^2(q^2)q^4}{g_A^2(q^2)4M_P^2} \right] + \frac{2}{3} \frac{g_M^2(q^2)q^2}{g_A^24M_P^2}
$$
\n
$$
\approx \left(\frac{\Lambda_A^2}{q^2 + \Lambda_A^2}\right)^4 \left[1 - \frac{2}{3} \frac{q^2}{(q^2 + m_\pi^2)} + \frac{1}{3} \frac{q^4}{(q^2 + m_\pi^2)^2} \right]
$$
\n
$$
+ \left(\frac{g_V}{g_A}\right)^2 \frac{K^2q^2}{6M_P^2} \left(\frac{\Lambda_V^2}{q^2 + \Lambda_V^2}\right)^4
$$
 (6)
\n
$$
h_T(q) = \frac{g_A^2(q^2)}{g_A^2} \left[\frac{2}{3} \frac{g_P(q^2)q^2}{g_A(q^2)2M_P} - \frac{1}{3} \frac{g_P^2(q^2)q^4}{g_A^2(q^2)4M_P^2} \right] + \frac{1}{3} \frac{g_M^2(q^2)q^2}{g_A^24M_P^2},
$$
\n
$$
\approx \left(\frac{\Lambda_A^2}{q^2 + \Lambda_A^2}\right)^4 \left[\frac{2}{3} \frac{q^2}{(q^2 + m_\pi^2)} - \frac{1}{3} \frac{q^4}{(q^2 + m_\pi^2)^2} \right]
$$
\n
$$
+ \left(\frac{g_V}{g_A}\right)^2 \frac{K^2q^2}{12M_P^2} \left(\frac{\Lambda_V^2}{q^2 + \Lambda_V^2}\right)^4
$$
 (7)

where

$$
g_V(q^2) = g_V \left(\frac{\Lambda_V^2}{q^2 + \Lambda_V^2}\right)^2 \tag{8}
$$

$$
g_A(q^2) = g_A \left(\frac{\Lambda_A^2}{q^2 + \Lambda_A^2}\right)^2 \tag{9}
$$

$$
g_P(q^2) = \frac{2M_P g_A(q^2)}{(q^2 + m_\pi^2)} \left(\frac{\Lambda_A^2 - m_\pi^2}{\Lambda_A^2}\right)
$$
 (10)

$$
g_M(q^2) = K g_V(q^2)
$$
 (11)

with $g_V = 1.0$, $g_A = 1.254$, $K = \mu_p - \mu_n = 3.70$, $\Lambda_V = 0.850$ GeV, and $\Lambda_A = 1.086$ GeV.

The short range correlation (SRC) has been included using the Jastrow correlation with Miller Spencer type of parametrization given by [11],

$$
f(r) = 1 - ce^{-ar^2}(1 - br^2)
$$
 (12)

with $a = 1.1$ fm⁻², $b = 0.68$ fm⁻² and $c = 1.0$. The above matrix elements are calculated with the help of Projected Hartree Fock Bogoliubov (PHFB) model.

2.1 PHFB model

The expression to calculate the NTMEs M_α of $(\beta^-\beta^-)_{0\nu}$ decay for the $0^+ \to 0^+$ transition in the PHFB model is obtained as follows. In the PHFB model, a state with good angular momentum J is obtained from the axially symmetric HFB intrinsic state $|\phi_0\rangle$ with K = 0 through the following relation using the standard projection technique [12]:

$$
|\psi'_{00}\rangle = \left[\frac{(2J+1)}{8\pi^2}\right] \int D'_{00}(\Omega) R(\Omega) |\phi_0\rangle d\Omega \qquad (13)
$$

Where $R(\Omega)$ and $D'_{MK}(\Omega)$ are the rotation operator and the rotation matrix, respectively. The axially symmetric HFB intrinsic state $|\phi_0\rangle$ can be written as

$$
|\phi_0\rangle = \prod_{im} \left(u_{im} + v_{im} b_{im}^\dagger b_{i\widetilde{m}}^\dagger \right) |0\rangle \tag{14}
$$

Where the creation operators b_{im}^{\dagger} and b_{im}^{\dagger} are defined as

$$
b_{im}^{\dagger} = \sum_{\alpha} C_{i\alpha,m} a_{\alpha m}^{\dagger} \tag{15}
$$

And

$$
b_{i\tilde{m}}^{\dagger} = \sum_{\alpha} \left(-1 \right)^{l+j-m} C_{i\alpha,m} a_{\alpha,-m}^{\dagger} \tag{16}
$$

The amplitude (u_{im} , v_{im}) and the expansion coefficient $C_{ij,m}$ are obtained from the HFB calculations. Employing the HFB wave functions, one obtains the following expression for NTMEs M_{α} of $(\beta^{-}\beta^{-})_{0\nu}$ decay [13,14]

$$
M\alpha = \left[n^{j_{i}=0}n^{j_{f}=0}\right]^{-1/2}
$$
\n
$$
\times \int_{0}^{\pi} n_{(Z,N),(Z+2,N-2)}(\theta) \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta|O_{\alpha}|\gamma\delta\rangle \times \sum_{\epsilon n} \frac{\left(f_{Z+2,N-2}^{(\pi)*}\right)_{\epsilon\beta}}{\left[1 + F_{Z,N}^{(\pi)}(\theta)f_{Z+2,N-2}^{(\pi)*}\right]_{\epsilon\alpha}^{-1}}
$$
\n
$$
\times \frac{\left(f_{Z,N}^{(\nu)*}\right)_{\eta\delta}}{\left[1 + F_{Z,N}^{(\nu)}(\theta)f_{Z+2,N-2}^{(\nu)*}\right]_{\gamma\eta}^{-1}} \sin \theta \, d\theta \qquad (17)
$$

where

$$
n^{j} = \int_{0}^{\pi} \left[\det(1 + F^{(\pi)} f^{\pi^{+}}) \right]^{1/2} \times \left[\det(1 + F^{(\nu)} f^{\nu^{+}}) \right]^{1/2} d_{00}^{j}(\theta) \sin(\theta) d\theta \tag{18}
$$

$$
n_{(Z,N),(Z+2,N-2)}(\theta) = \left[\det \left(1 + F_{Z,N}^{(\nu)} f_{Z+2,N-2}^{(\nu)^{\dagger}} \right) \right]^{1/2} \times \left[\det \left(1 + F_{Z,N}^{(\pi)} f_{Z+2,N-2}^{(\pi)^{\dagger}} \right) \right]^{1/2} \tag{19}
$$

The π(ν) represents the proton (neutron) of nuclei involved in the *(ββ)0ν* decay process. The matrix $F_{ZN}(\theta)$ and f_{ZN} are given by

$$
F_{Z,N}(\theta) = \sum_{m'_\alpha,m'_\beta} d_{m_\alpha,m'_\alpha}^{j_\alpha}(\theta) d_{m_\beta,m'_\beta}^{j_\beta}(\theta) f_{j_\alpha m'_\alpha j_\beta m'_\beta} (20)
$$

$$
f_{Z,N} = \sum_i C_{ij_\alpha,m_\alpha} C_{ij_\beta,m_\beta} (v_{im_\alpha}/u_{im_\alpha}) \delta_{m_\alpha,-m_\beta} (21)
$$

The calculations of required NTMEs *M^α* are performed in the following manner. In the first step, matrices $F_{Z,N}$ (θ) and $f_{Z,N}$ are set up for the nuclei involved in the $(\beta\beta)_{0\nu}$ decay making use of 20 Gaussian quadrature points in the range $(0,\pi)$. Finally, the required NTMEs can be calculated in a straightforward manner using Eqn. (21).

3. Results and discussions

We have calculated the NTMEs defined in Eqn. (3) and results are shown in table 1. We have also calculated the expected half-life $T_{1/2}^{0\nu}$ for a single neutrino of mass m_h with coupling U_h , shown in table 2. These half-lives are calculated for axial vector coupling constant $g_A=1.254$ and plotted in fig. 1-3. In the same figures. current limits from the experiment AMoRE-I for 100 Mo [15], for 128 Te [16] and from CUORE for ¹³⁰Te [17] are also shown. The variation of these half-lives with the coupling U_{eh}^2 is plotted in fig. 1-3 for sterile neutrino mass ranging from 100 eV to 10 GeV. It is observed that

half-life increases for lower values of mass m_N in all three cases. Also the half-lives increase when U_{eh}^2 changes from 10^{-2} to 10^{-8} . .

Table 1: Calculated NTMEs $M^{(\nu)}(m_h)$ **for** $\theta\nu\beta\beta$ **-decay of** ^{100}Mo **and** $^{128,130}\text{Te}$ **isotope including short range correlation (SRC)**

	Nuclei Mass (m_h) (GeV)					
	0.001	0.01	0.1	1.0	10	100
100 _{Mo}	6.262	5.91	3.033	0.105	0.998	0.998
128 Te	3.619	3.412	1.64	0.064	0.613	0.613
130Te	4.054	3.815	1.808	0.069	0.659	0.659

Table 2: Half life $T^{0\nu}_{1/2}$ for a single neutrino of mass ${\rm m}_{\rm h}$ with coupling U^2_{eh} from $10^{\text{-}2}$ to $10^{\text{-}8}$ for **Mo, 128,130Te isotopes**

Figure 1: **Half-life** for a single neutrino of mass m_N with coupling $U_{eh}^2 = 10^{-2}$ to **-8 . Green line represents the AMoRE-I experiment limit for ¹⁰⁰Mo**

Figure. 2: Half-life for a single neutrino of mass m_N with coupling $U_{eh}^2 = 10^{-2}$ to 10^{-8} . Green **line represents the Gran Sasso experiment limit for 128Te**

Figure. 3: Half-life for a single neutrino of mass m_N with coupling U_{eh}^2 = 10⁻² to 10⁻⁸. Green **line represents the CUORE experiment limit for 130Te**

The analysis of light sterile neutrino of eV order mass can be done by writing [18]

$$
\langle m_{light} \rangle = \sum_{k=1}^{3} U_{ek}^{2} m_{k} + \sum_{i} U_{ei}^{2} m_{i}
$$
 (22)

where k stands for three known neutrino species and *I* for unknown neutrinos of eV order mass and comparing the experimental limits with the calculated half-lives using

$$
\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = G_{01} \left[\frac{\langle m_{light} \rangle}{m_e}\right]^2 |M^{\nu}|^2 \tag{23}
$$

we get the results shown in table 3. We have presented the extracted limits on effective neutrino mass $\langle m_{light} \rangle$ by using the phase space factors G₀₁ from [23] and available experimental half-lives for considered isotopes. It is observed that best upper limit of 0.22 eV is obtained in case of ¹³⁰Te isotope. Also we have shown the predicted half-lives for considered transitions taking $\langle m_{liaht} \rangle = 50$ meV and shown the results in column 7 of the table 3. The half-lives are of the order of 10^{26} years as observed from the above table.

Table 3: Experimental half-lives along with extracted limits on effective neutrino mass and predicted half-lives for $\langle m_{light} \rangle = 50$ meV

Decay	M^{ν}			$T_{1/2}^{0\nu}$ Ref. G ₀₁ (yr $\langle m_{light} \rangle$ $T_{1/2}^{0\nu}$ (yrs)	
		(yrs)	$\mathbf{1}$	(eV) (predicted)	
		(Exp.)			
$^{96}Zr \rightarrow ^{96}Mo$ 2.624 9.2x10 ²¹ [19] 2.06x10 ⁻ 14.15			14	$7.36x10^{26}$	
100 M0 \rightarrow 100 Ru 5.871 1.1x10 ²⁴ [20] 1.59x10 0.66 1.90x10 ²⁶			14		
$^{110}Pd \rightarrow ^{110}Cd$ 6.977 6.0x10 ¹⁷ [21] 4.83x10 1.36x10 ³ 4.44x10 ²⁵			15		
128 Te \rightarrow 128 Xe 2.906 1.1x10 ²³ [16] 1.66x10 13.01 7.44x10 ²⁷			15		
130 Te \rightarrow 130 Xe 4.029 2.3x10 ²⁵ [17] 1.43x10 0.22				$4.49x10^{26}$	

¹⁵⁰Nd→¹⁵⁰Sm 2.931 2.0x10²² [22] 6.32x10⁻ 1.55 1.92x10²⁶ 1.55 14 1.55 1.92x10 26

14

4. Conclusions

We have calculated the NTMEs of $0\nu\beta\beta$ decay process associated with light sterile neutrino in PHFB model and using Eqn. (5), we have obtained the half-life $T_{1/2}^{0\nu}$ for a single neutrino of mass m_h with v_h ⁻ v_e mixing matrix element coupling U_{eh}^2 from 10⁻² to 10⁻⁸ for ¹⁰⁰Mo, ^{128,130}Te isotopes. We have also shown the effect of U_{eh}^2 on half-lives of these isotopes in fig. 1-3. It is observed that half-life increases for lower values of mass m_N in all three cases. Also the half-lives increase when U_{eh}^2 changes from 10^{-2} to 10^{-8} . The predicted half-lives are of the order of 10^{26} years as shown in table 3 for effective light neutrino mass of 50 meV.

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