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Expressions for the 21 cm wavelength of H and their use to derive two energy-frequency equations to exactly calculate the Planck constant

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Abstract

This work presents a straightforward trigonometric derivation of two fine-structure constant-based analytical expressions to calculate the hydrogen 21 cm radio wavelength, H_{21}. This expression is transformed into two equations with an energy-frequency interpretation for the Planck constant whose exact numerical value can be obtained.

Keywords: Intergalactic H 21 cm radio wave, Rydberg wavelength unit, Planck constant, Sommerfeld or fine-structure constant

Introduction

Since the intriguing 21 cm radio wavelength H_{21} was first theoretically predicted in 1945 [1] and experimentally detected in 1951 [2], no expression for its calculation has been proposed. Recently, a basic trigonometric method to determine the fine-structure constant, α, among other parameters, was proposed, [3]. This expression derivation approach is again implemented here for the H_{21} case.

The one sine cycle traces of selected lengths

Figure 1 compares the radians required for the shown sine functions to complete one cycle trace; functions correspond, from left to right, to the hydrogen atom diameter $2a_0$, the Rydberg wavelength, λ_∞ and a tentative H_{21} numerical value; all of them are normalized or referred to λ_∞. α is the Sommerfeld or fine-structure constant.

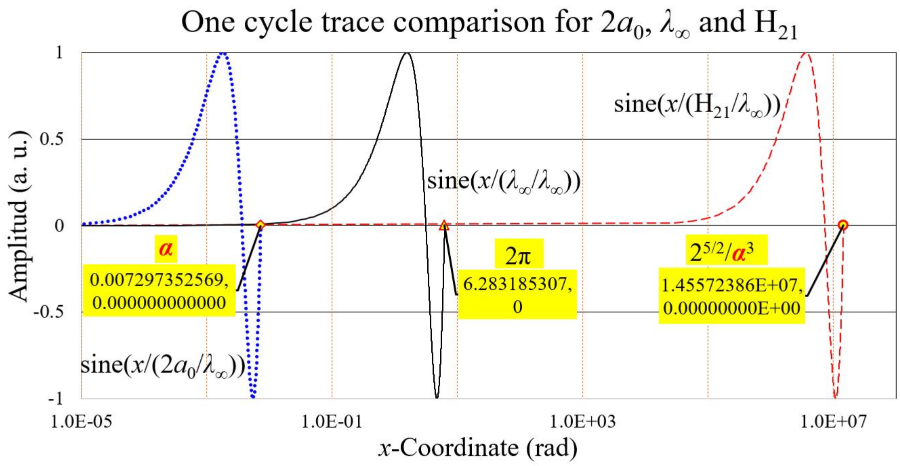

Fig. 1. Method to evaluate the H_{21} numerical value in terms of α, a_0 and λ_∞ by comparing their sine functions one cycle ratios.

From the figure, we define the ratio of the function's parameters of the left and right traces and the ratio of their respective one cycle x-coordinate. Given that both ratios must be equal, we can write

$$\frac{H_{21,a}/\lambda_\infty}{2a_0/\lambda_\infty} = \frac{2^{5/2}/\alpha^3}{\alpha} \quad (1)$$

This provides the following wavelength expression and arithmetic value

$$H_{21,a} = 8\sqrt{2}\frac{a_0}{\alpha^4} = 0.2111\ 2749\ 7\mathbf{262\ 130}\ \text{(m)} \quad (2)$$

whose corresponding frequency is

$$\tau_{21,a} = \frac{c}{H_{21,a}} = 1419.9593\ \mathbf{2262\ 573}\ \text{(Mhz)} \quad (3)$$

where c (m/s) stands for the light speed in vacuum.

Doing again the above process but now using the middle and right traces in Fig. 1 yields

$$\frac{H_{21,b}/\lambda_\infty}{\lambda_\infty/\lambda_\infty} = \frac{2^{5/2}/\alpha^3}{2\pi} \tag{4}$$

and this expression is now obtained

$$H_{21,b} = \frac{2^{3/2}}{\pi}\frac{\lambda_\infty}{\alpha^3} = 0.2111\ \ 2749\ \ 7\mathbf{393\ \ 435}\ \ (\mathrm{m}) \tag{5}$$

whose associated frequency is

$$\tau_{21,b} = \frac{c}{H_{21,b}} = 1419.9593\ \ \mathbf{2174\ \ 263}\ (\mathrm{Mhz}) \tag{6}$$

Finally, let's consider the left and middle functions in Fig. 1 to write

$$\frac{2a_0/\lambda_\infty}{\lambda_\infty/\lambda_\infty} = \frac{\alpha}{2\pi} \tag{7}$$

then

$$\alpha = \frac{4\pi a_0}{\lambda_\infty} \tag{8}$$

so that (2) and (5) can be converted into each other and we can elaborate on (2) as follows

$$H_{21,c} = 8\sqrt{2}\frac{1}{\alpha^3}\frac{a_0}{\alpha} = \frac{2^{3/2}}{\pi}\frac{1}{\alpha^3}\lambda_\infty = \frac{2^{3/2}}{\pi}\frac{1}{\alpha^3}\frac{hc}{eR_y}\ \ (\mathrm{m}) \tag{9}$$

where the following equivalence was used

$$\lambda_\infty = \frac{hc}{eR_y}\ \ (\mathrm{m}) \tag{10}$$

which is the Einstein energy-wavelength expression; here *h* (J·s) is the Planck constant, *Ry* (eV) is the Rydberg energy or the H atom ionization energy and *e* (C) is the electron charge. (9) delivers

$$H_{21} = \frac{2^{3/2}}{\pi\alpha^3}\frac{hc}{eR_y} = 0.2111\ \ 2749\ \ 7\mathbf{262\ \ 318}\ \ (\mathrm{m}) \tag{11}$$

Solving (11) for *h*, we can write the following energy/frequency equation

$$h = \frac{\pi\alpha^3}{2^{3/2}}\frac{eR_y}{\tau_{21}} = (6.6260\ \ 7015)10^{-34}\ \ (\mathrm{J\cdot s}) \tag{12}$$

which gives an exact value for *h*, and its related frequency τ_{21} is

$$\tau_{21} = \frac{c}{H_{21}} = 1419.9593\ \ \mathbf{2262\ \ 447}\ (\mathrm{Mhz}) \tag{13}$$

which matches fairly well the frequency calculated in [4].

As shown in [3]

$$\alpha^2 = \frac{r_e}{a_0} \tag{14}$$

then, another equation equivalent to (12) is

$$h = \frac{\pi\alpha}{2^{3/2}} \frac{r_e}{a_0} \frac{eR_y}{\tau_{21}} = (6.6260\ 7015) 10^{-34}\ (\mathrm{J \cdot s}) \tag{15}$$

Conclusions

Three H_{21} expressions were trigonometrically derived including one which was converted into two Planck constant expressions having an energy/frequency interpretation and provide an exact numerical value for h.

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Conflict of Interest Statement

The authors state that there are no conflicts of interest with respect to the funding, research, authorship and publication of this article.

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