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On Variable Quark Masses Derived from Meson Spectra

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ABSTRACT

The masses of the 5 known quarks together with a zero point quark-antiquark interaction energy d_{m0} in the scalar strong interaction hadron theory SSI are determined from a meson mass formula using the masses of 6 mesons. Five different combinations of 6 ground state mesons each led to 5 sets of quark masses and d_{m0}. Using these results, the masses of the remaining 16 ground state mesons were calculated using the same formula. For the "nominal" combination, the root mean square of all deviations from data is < 0.5 %, with a maximum deviation of 1.6 %. The invisible quark masses are not fixed natural constants like the electron mass. Each quark has some natural mean mass, around 0.64 GeV for the *u* and *d* quarks and 0.72 GeV for the *s* quark. These values can however vary ±10% dependent at least upon the flavor of its companion antiquark in the meson and its spin.

The set $\pi^{\pm}$ and π^0 is the only isotriplet in the ground state meson spectra. They share the same quark content and strong interaction not found in the remaining isodoublet mesons. The allows for a perturbational calculation of the mass difference of $\pi^{\pm}$ and π^0 due to the $\pi^{\pm}$ charge. Via a mathematically consistent interpretation of the resulting formula for this difference, the calculated value agrees with the measured 4.6 MeV within 0.02 %.

Keywords: scalar strong interaction hadron theory SSI, pseudoscalar mesons, vector mesons, quark masses, meson masses, meson mass formula, zero point potential, "marble" model for charged mesons, charge contribution to meson mass. pion isotriplet, isodoublet mesons.

1. INTRODUCTION

As quarks are invisible, their masses have to be inferred from hadron data [1] with theoretical models. The quark masses given in [1] consist of two groups. The low masses of the *u*, *d* and *s* quarks come from the chiral perturbation theory CHPT complementing low-energy dynamics of QCD. The high masses of the *c* and *b* quarks are derived from a separate constituent model. The accuracy of these quark masses is 3-4 digits (see Case A in Table 1 below). But ChPT has met such difficulties in Sweden´s Lund University that it had to be abandoned many years ago.

Another set of quark masses comes from the scalar strong interaction hadron theory SSI [2]. In this set, the above dichotomy was removed; all 5 quarks are on equal footing.

The quark masses in SSI appear in the meson mass formula [2 (5,1,1)]

$$E_{Jn} = \pm\sqrt{\left(m_p + m_r\right)^2 - 4d_{m0} + 8d_h\left(s + n' + \frac{3}{2}\right)}, \qquad s = J \tag{1}$$

where E_{Jn} is the strong interaction meson mass, m_p the quark masses with *p=1, 2, 3, 4,* and *5* referring to the *u*, *d*, *s*, *c*, and *b* quarks, respectively. d_h is the confinement strength and d_{m0} a "zero point potential" between the quark and antiquark (see (8)). J=0 for pseudoscalar 0^- and J=1 for vector 1^-mesons. n' is a radial quantum number. Mass contributions from the charges of the mesons do not enter this formula,

Using 6 pseudoscalar 0^- masses, given in Case B in Table 1 below, as input to (1), 5 quark masses and d_{m0} [2 Tables 5.2] were obtained from (1). These led to predictions of ground state meson masses [2 Tables 5.3-5] and radially excited mesons [2 Tables 5.6-7]. The orbitally excited $l > 0$ singlets were treated in [2 §5.6.1, Table 5.8]. These results agree with data to varying degrees of accuracy. Assignments of the orbitally excited $l > 0$ triplets are shown in [2 Table 5.9]. For this Case B, the ground state meson spectra also have been explicitly and largely successfully accounted for recently [3 Table A1]. The accuracy of these quark masses in this Case B is 4 digits.

In SSI, these quark masses have been used to obtain nucleon wave functions which in their turn are the basis for the nucleon size [2 (12.6.22), 3 (A17)]. This size in the hidden space between the quark and diquark in nucleon underlies the treatment of dark matter and energy [2 Ch 15, 16] and nuclear force [4]. Therefore, it is desirable to improve the above 4 digit accuracy.

Now the ground state meson masses [1] used as input in (1) have accuracies up to 6 digits. Therefore, the quark masses derived from them from (1) also will have the same 6 digit accuracy. In this connection, however, the meson charge mass contribution absent in (1) will have effect on the this accuracy. In [2 Table 5.2] or Case B, it was assumed that such contribution is 2.1 MeV. This value came from the classical charge mass e^2/r_m =2.18 MeV where r_m=0.659 fm is the $\pi^{\pm}$ charge radius.

The quark masses and d_{m0} in this Case B were found via trial and error and is approximative. Due to the success in this case, they will be updated here using a systematical and exact determination of their values. Further, the constant 2.1 MeV charge contribution will be replaced by the more specific ones in (2b) below.

Equation (1) with $n'= 0$ for ground state mesons is now modified to [3 (A8)]

$$(E - E_{10})^2 = \left(m_p + m_r\right)^2 - 4d_{m0} + 8d_h\left(J + \tfrac{3}{2}\right) \qquad (2a)$$

$E_{10} = Q^2/R_m$, $R_m = r_m\sqrt{\pi}/2$

$\pi^{\pm}$: r_m=0.659 fm, E_{10}=2.467 MeV,

$K^{\pm}$: r_m= 0.56 fm, E_{10}=2.904 MeV and is assumed to hold for all other charged mesons (2b)

where E stands for E_{Jn} in (1), Q is the meson charge. R_m is the radius of a "marble" model of the meson charge introduced in [3 Sec. 5]. The assumption in (2b) is made because charge radii is known only for $\pi^{\pm}$ and $K^{\pm}$. The so-modified Case B is designated as Case C in Table 1.

2. NEUTRAL INPUT MESONS AND QUARK MASSES

The 6 0^- mesons, 3 neutral and 3 charged, for Cases B and C were chosen because they have the lowest masses containing all 5 flavors and have the same strong interaction form in (1). π^0 does not have this, as is seen in (6) below. It is also hindered by its role in the special case of Sec. 5.

In general, there are 22 charged and neutral ground state mesons to choose 6 input mesons from. A full treatment of this complex situation is beyond the scope of this paper. Here, an attempt will be made to eliminate the uncertainties arising from the unknown charge radii of charged mesons in (2b). In the following Cases D, E, F, and G in Table 1, only neutral meson masses are employed as inputs to fix the 5 quark masses and d_{m0} via (2a). These in their turn serve as inputs in (2a) to predict the masses of the remaining neutral mesons. These masses are then compared to the data [1]. In this way, no charge is involved and a pure test of the strong interaction formula (2a) with E_{10} =0 can be done.

Since there are only 5 neutral 0^- mesons, excluding the η's in [2 table 5.5] because (2a) for them differs too much from data, the 6th neutral meson must be a vector 1^- meson. Most of the vector meson masses are less precise due their larger error margins and widths. But there are 3 candidates, namely $\phi(1020)$, J/ψ and $Y(1S)$. These have been employed to obtain the 5 quark masses and d_{m0} in Cases D, E and F in Table 1 via (2). In Case G there, both $\phi(1020)$ and J/ψ enter in favor of π^0. The results are shown in Table 1.

Table 1. Input mesons for and quark masses and d_{m0} from (2) for Cases B to G
Δ stands for deviations from the mean values. The underlined Case C is the nominal set

Case	Input mesons
A	Particle Data Group [1]
B	0^- mesons $\pi^\pm$, $K^\pm$, K^0, D^0, $D_s^\pm$, and B^0 with E_{10} = 2.1 MeV in approximative [2 Table 5.2]
C	Same mesons as in B but using (2) and obtained systematically with 6 digit accuracy
_D	Neutral mesons π^0, K^0, D^0, B^0, B_s^0, and $\phi(1020)$
E	Neutral mesons π^0, K^0, D^0, B^0, B_s^0, and J/ψ
F	Neutral mesons π^0, K^0, D^0, B^0, B_s^0, and $Y(1S)$
G	Neutral mesons K^0, D^0, B^0, B_s^0, $\phi(1020)$, and J/ψ

	m_1 (GeV)	m_2	m_3	m_4	m_5	d_{m0} (GeV^2)	M_d (4)
A [1 PDG]	0.00216	0.047	0.0935	1.273	4.813		
B [2 Table 5.2]	0.6592	0.66135	0.7431	1.6215	4.7786	0.64113	–0.82067
C [~B]	0.658060	0.66047	0.742149	1.62171	4.77967	0.639932	–0.82146 Nom
D [$\phi(1020)$]	0.594504	0.59989	0.686968	1.61593	4.81156	0.5621	–0.82254
E [J/ψ]	0.595836	0.60106	0.68812	1.61595	4.81095	0.56359	–0.8218
F [$Y(1S)$]	0.695652	0.69160	0.776013	1.62459	4.76563	0.686478	–0.81867
G [ϕ, J/ψ]	0.607698	0.61752	0.70271	1.62224	4.80182	0.583976	–0.83474

Cases C-G

	m_1 (GeV)	m_2	m_3	m_4	m_5	d_{m0} (GeV^2)	M_d (4)
Mean value	0.63515	0.63411	0.71919	1.6201	4.79393	0.607175	–0.82384
Δ_{max}	+5.68%	+9.1%	+7.9%	+0.28%	+0.37%	12.8%	+1.3%
Δ_{min}	–10.3%	+5.4%	–4.5%	–0.26%	–0.6%	–7.4%	–0.6%
Δ_{av}	±8%			±0.27%		±10%	±1%

The quark mass and d_{m0} values in the B and C cases are close to each other, as expected. Those in the D and E Cases also yield nearly the same quarks masses and d_{m0} values. Thus, there are roughly 4 sets, B-C, D-E, F, and G of quark and d_{m0} values. Table 1 also shows that the *c* and *b* quark masses, m_4 and m_5, are nearly the same for all B-G Cases.

But these 4 sets differ in the *u, d, s* mass and d_{m0} values. Relative to the nominal Case C,

the u quark mass differs by about −11% for D-E, +6% for F. −8% for G
the s quark mass differs by about −8% for D-E, +5% for F, −6% for G
d_{m0} value differs by about −14% for D-E, +7% for F, −10% for G (3)

3. PREDICTED MESON MASSES VS DATA

The ground state meson spectra can now be worked out using (2) for Cases C-G in Table 1. The results are compared to data in Tables 2-4 below. Deviations from data are given in Error lines

Table 2. (2) predictions for neutral 0^- mesons in MeV

	π^0	K^0	D^0	B^0	B_s^0	RMS Error
Data [1]	134.9768	497.611	1864.84	5279.72	5366.93	
Case B	139.057	497.956	1864.69	5279.07	5368.93	
Error %	+3.02	+0.069	–0.008	–0.0124	+0.056	1.35%
Case C	137.124	497.611	1864.84	5279.72	5363.84	
Error %	+1.58	0 input	0 input	0 input	– 0.058	0.7%
Case D	134.977	497.611	1864.84	5279.72	5368.93	
Error %	0 input	0 input	0 input	0 input	+0.037 input	0.017%
Case E	134.972	497.609	1864.84	5279.72	5368.93	
Error **%**	0 input	0 input	0 input	0 input	0 input	0 %
Case F	135.034	497.956	1864.84	527972	536693	
Error %	+0.42 input	+0.07 input	0 input	0 input	0 input	0.19%
Case G	79.4955	497.611	1864.84	5279.72	5366.93	
Error %	–41.1	0 input	0 input	0 input	0 input	18.3%

Table 3. (2) predictions for charged 0^- mesons in MeV

	$\pi^\pm$	$K^\pm$	$D^\pm$	$D_s^\pm$	$B^\pm$	$B_c^\pm$	RMS Error
Data [1]	139.57039	493.677	1869.66	1968.35	5279.41	6274.47	
Case B	141.1407	493.959	1869.42	1968.52	5281.17	6266.03	
Error %	+1.125	+0.057	–0.013	–0.01	+0.03	–0.135	.46%
Case C	139.5706	493.677	1870.07	1969.65	5280.14	6368.523	
Error %	0 input	0 input	0.1	0.1 input	+0.14	–0.095	0.07%
Case D	137.336	486.403	1874.13	1976.47	5277.1	6319.88	
Error %	–1.62	–1.5	+0.24	+0.41	–0.044	+0.72	0.97%

Case E	137.339	486.83	1873.93	1976.32	5277.27	6318.81	
Error %	−1.62	−1.4	+0.22	+0.4	–0.04	+0.706	0.94%
Case F	137.444	512.355	1862.84	1966.97	5286.7	6242.34	
Error %	−1.55	+3.78	–0.366	–0.07	+0.138	–0.515	1.69%
Case G	81.3545	473.85	1867.74	1980.15	5272.55	6309.46	
Error %	−41.7	−4	−0.1	+0,6	–0.13	+0.5	17.1%

Table 4. (2) predictions for vector 1^- mesons in MeV. The ρ case is excluded in the RMS error values due to its large width

	ρ	$K^{*\pm}$	K^{*0}	$\phi(1020)$	J/ψ	$D^{*\pm}$	D^{*0}	$D_s^{*\pm}$	B^*	B_s^{*0}	$Y(1S)$	RMS Error %
Data [1]	769±6	893±2	895.55	1019.461	3096.9	2010.26	2006.85	2112.2	5324.75	5415.4	9460.4	
Case B	763.24	897.6	898.87	1021.9	3058.2	2013.79	2009.25	2106.1	5332.89	5415.2	9496.1	
Error %	–0.75	+0.5	+0.1	+0.24	–1.27	+0.18	+0.12	–0.29	+0.15	–0.003	+0.38	0.47
Case C	±763.26	897.81	898.675	1021.48	3059.42	2015.02	2009.39	2107.21	5332.7	5415.79	9.4985	
Error %	±0.8	+0.53	+0.35	+0.2	–1.22	+0.24	+0.13	–0.24	+0.15	+0.007	+0.4	0.47
Case D	762.86	893.84	898.67	1019.461	3.0978	2018.22	2009.39	2113.6	5332.49	5420.8	9578.9	
Error %	–0.8	+0.09	+0.35	0 input	+0.03	+0.4	+0.13	+0.07	+0.145	+0.1	+1.25	0.44
Case E	762.86	894.07	898.67	1019.645	3096.9	2018.04	2009.39	2113.44	5331.29	5420.8	9577.4	
Error %	– 0.8	+0.12	+0.35	0.018	0 input	+0.39	+0.13	+0.06	+0.12	+0.1	+1.24	0.49
Case F	763.31	908.19	898.866	1030.96	3035.26	2007.74	2009.39	2104.7	5339.43	5418.85	9460.4	
Error %	–0.75	+1.7	+0.37	+1.11	–2.03	–0.13	+0.13	–0.36	+0.28	+0.064	0 input	0.93
Case G	752.54	887.09	898.68	1019.461	3.0969	2023.17	2009.39	2117.02	5332.49	5418.9	9.5548	
Error %	–2.1	–0.66	+0.35	0 input	0 input	+0.64	+0.13	+0.23	+0.15	+0.06	+1	0.45

The error lines in these 3 tables are collected and presented in Table 6.

Table 5. RMS error % from Tables 2-4

Case	Neutral 0^-	Charged 0^-	Vector 1^-	Total	Maximum
B	1.35	0.46	0.47	0.774	3.02 for π0
C	0.705	0.069	0.474	0.476	1.58 for π^0 Nominal
D	0.0167	0.968	0.436	0.599	1.62 for $\pi^\pm$
E	0	0.94	0.488	0.605	1.62 for $\pi^\pm$
F	0.19	1.69	0.93	1.11	3.78 for $K^\pm$
G	18.3	17.1	0.454	12.8	41.7 for $\pi^\pm$

4. EVALUATION OF THE RESULTS AND VARIABLE QUARK MASSES

Table 5 shows that Case C has the lowest total error and the lowest maximum error. This case will be taken to be the "nominal" case. Cases D-G all have only neutral meson masses for input in order to avoid uncertain mass contributions from meson charges due to lack of charge radii data. Because all neutral 0^- mesons have been used up as inputs, only neutral 1^- meson masses are available for comparison with predictions from (2). Deviations from data in % collected from error lines in Table 4 are shown in Table 6.

Table 6. Error in % for neutral 1^- meson masses predicted by (2) collected from Table 4

Case	K^{*0}	$\phi(1020)$	J/ψ	D^{*0}	B^*	B_s^{*0}	$Y(1S)$
D [$\phi(1020)$]	0.35	input	0.03	0.13	0.145	0.1	1.25
E [J/ψ]	0.35	0,02	input	0.13	0.12	0.1	1.24
F [$Y(1S)$]	0.37	1.1	−2.03	0.13	0.28	0.06	input
G [$\phi, J/\psi$]	0.35	input	input	0.13	0.15	0.06	1.0

These results show that the pure strong interaction meson masses from (2) largely agree with data. Errors >1% are all associated with the heaviest Y(1S). However, errors for the remaining charged 1^- vector mesons, $K^{*\pm}$, $D^{*\pm}$, $D_s^{*\pm}$, and B^* in Table 4 are not bigger except for the +1.7% for $K^{*\pm}$ again connected to Y(1S). Therefore, the choice of the neutral meson masses as inputs to (2) in Cases D-G appears to be acceptable. This supported by the fact that Case C, using 3 charged 0^- mesons as inputs, also works well for both neutral and charged mesons.

Case G differs from all other cases in that no pion mass enters as input. This leads to that the predicted pion masses are extremely small, −40%, in Tables 2 and 3.

In Table 5, errors in Cases C and D are all small and < 1%. But the quarks masses and d_{m0} in Table 1 for these both cases are not so close. The *u, d* and *s* quark masses differ by about +10% and d_{m0} by +14%. Comparison of Cases C and F with Y(1S) as an input gives on the other hand −5% and −7%, respectively.

In addition to the five sets of input mesons for Cases B to G in Table 1, there are many other possible sets for new Cases H, I, J… using other combinations of input mesons that can be added to Table 1. Each such set may be specialized to reproduce the input meson masses exactly, by definition, and other meson masses approximately to varying degree of accuracies.

This is seen more clearly in Case G. Tables 2-4 show that the *K, D* and *B* meson masses have very small errors because the *s, c* and *b* quarks are amply present in the input mesons according to Table 1. But the errors increase appreciably, up to −41%, for mesons consisting of *u* or *d* quark, the π's. For the vector mesons in Table 4, the error for ρ is also larger but to a less degree due to the much larger ρ mass. Similarly, error for the heavy double *b* quark $Y(1S)$ is also larger.

Thus, Case G may be of interest for charmed baryons containing s and c quarks but no u or d quark. The doublet Ω_c^0 (ssc) and Ω_{cc}^+ (scc) belongs to this category. Analogously, Case E may find applications for the charmed baryons Σ_c^0 (ddc) and Ξ_c^0 (dsc). Case D may be applied to the strange baryons Ξ^0 (uss) in [2 Table 11.1] and Ω^- (sss). Case F may find applications involving bottom baryons, the singlet Λ_b^0 (udb), the triplet Σ_b′s (uub, ddb, ssb), the isodoublet Ξ_b (usb, dsb) and the singlet Ω_b^- (ssb).

Case B has proven to be applicable for both mesons and nucleons, as was mentioned beneath (1). Its upgraded version, the nominal Case C, yields improved agreement with meson data, as are shown in Table 5. But the differences are small so that the results of nucleon applications using Case B are expected to hold if Case C were used instead.

Such variable quark masses and d_{m0} values are inherent in (2a) which gives the same meson mass E as long as

$$M_d = \left(m_p + m_r\right)^2 - 4d_{m0} = \left(m_{p1} + m_{r1}\right)^2 - 4d_{m01} \text{ GeV}^2 = \text{constant} \tag{4}$$

where $m_{p1} \neq m_p$, $m_{r1} \neq m_r$ and $d_{m01} \neq d_{m0}$.

The relatively large d_{m0} values are barely overcome by the mass term for the u and d quarks. This is why $\pi^\pm$, π^0 masses are small. As the quarks get heavier, the meson masses increase.

M_d values in Cases C-G have been displayed in Table 1 and are nearly a constant, deviating from their mean value by < 1 %. This is because the quark masses and d_{m0} values in Cases B-G, though different, are still not too far from each other, indicating that their mean values represent natural constants. The actual values in each Case differ from them by some % for the light u, d and s quarks depending upon the composition of the input mesons. This can be seen from the Δ values in Table 1. Similar deviations relative to Case C are seen in (3).

These results show that the quark mass and d_{m0} values are not natural constants with fixed values like the masses of observable leptons and hadrons. Such values for each Case can deviate from some mean value in Table 1 by up to ~±10% and still yield approximate meson masses. If the deviations become much larger, then the predicted masses will deviate more from data.

In SSI, therefore, quark mass and d_{m0} values are indefinite and variable and can assume values within certain limits. They lead to good predictions for mesons having similar quark content as the input mesons that give rise to these values. For other mesons, the predictions are more approximate.

Nature is economical in providing natural constants. To give out many precise quark masses which cannot be measured and confirmed is uneconomical. Nature has found ways to comply to this principle via indefinite, *variable quark masses* and d_{m0} values utilizing the invisibility of quarks.

5. SPECIAL CASE FOR PIONS

Table 5 shows that the maximal deviations from data occur mostly for π^0 and $\pi^\pm$. Masses of mesons having the same isospin I are split due their differences in charges. All such mesons are 0^- isodoublets whose members have different quark contents. The strong interaction contributions to meson mass from such quark masses are different, as is seen in (2). An exception is the isotriplet $I = 1$ $\pi^\pm$ and π^0 which have the same quark content, the u and d quarks, and the same strong interaction. In this case, the strong interaction part of the masses of $\pi^\pm$ and π^0 is the same (see the line beneath (9)). This is supported by the weak pion beta decay $\pi^- \to e + \bar{\nu}_e$. The mass contribution from the charge of $\pi^\pm$ can now be separated off the main contribution from strong interaction in (2). This not possible for the isodoublets like $K^\pm$ and K^0 because they have different quark contents, hence different strong interaction masses.

For Case B in Table 1, the classical value 2.1 MeV was put in by hand into the quantum mechanical SSI, as was mentioned above (2a) and shown in Table 1. This value as well as the e^2/r_m =2.18 MeV mentioned above (2a) are much smaller than the $\pi^\pm$ - π^0 mass difference 4.6 MeV. This was the subject of [3] where the effect of $\pi^\pm$ charge is introduced as a perturbation in the laboratory frame $\underline{X}$ into the strong interaction meson equation in the hidden space $\underline{x}$ that led to (1). A "marble" model (2b) was used.

[3 (6.5)] gives the mass differences between $\pi^\pm$ and π^0

$$\Delta m_\pi = E_1 = -\frac{e}{R_m^3}\left(\frac{q_u+q_d}{8(M_m^2-\Phi_{mav})} + \frac{q_u-q_d}{4}R^2\right) \tag{5a}$$

$$\Delta m_\pi = E_1 = -\frac{e}{R_m^3}\left(-\frac{q_u+q_d}{8(M_m^2-\Phi_{mav})} + \frac{q_u-q_d}{4}R^2\right) \tag{5b}$$

where q_u =2e/3, q_d =−e/3 and R=|$\underline{X}$| [3 (5.1)]. $\underline{X}$ is a point in the marble measured from its center limited by the radius of the marble R_m (2b). In [3],

$$M_m^2 = \frac{1}{4}(m_1+m_2)^2 \text{ for } \pi^\pm,\ \ \frac{1}{2}\left(m_1^2+m_2^2\right)^2 \text{ for } \pi^0, \tag{6}$$

$$\Phi_{mav} = \int d\underline{x}^3\, \chi_0^2(r)\Phi_m(r)/\int d\underline{x}^3\, \chi_0^2(r) = 0.5361\ \ \text{GeV}^2 \tag{7}$$

$$\Phi_m(r) = d_{m0} - d_h^2 r^2, \qquad d_{m0} = 0.64113\text{GeV}^2, \qquad d_h = 0.07\text{GeV}^2 \tag{8}$$

$$\chi_0(r) = \frac{1}{\sqrt{\Omega}}\alpha_{00}\, exp\left(-\frac{d_h}{2}r^2\right),\ \alpha_{00} = \left(\frac{d_h}{\pi}\right)^{3/4} = 0.0577\ \text{GeV}^{3/2} \tag{9}$$

The values of the two forms in (6) differ only by 0.0064 %. $r=|\underline{x}|$ is the quark-antiquark distance in the hidden space $\underline{x}$. $\Phi_m(r)$ is the strong quark-antiquark interaction potential and χ_0 the 0^- meson wave function.

As was pointed out in [3], (5) is an approximate solution of the starting equations [3 (5.8)] which depends upon both the hidden relative coordinate $\underline{x}$ and the laboratory frame $\underline{X}$. The large number of dependent variables prevents an exact solution presently. In the approximate solution (5), the constant E_I depends upon $R=(|\underline{X}|)$. The R^2 terms in (5) are small and R^2 has been assumed to be half its maximal value or $R_m^2/2$ in [3] so that (5) yields 4.306 MeV and −4.922 MeV. Rejecting the latter, the predicted 4.306 MeV is closer to the measured value of 4.5936 MeV [1], differing by −6.7%, than does the 2.1 MeV used for Case B of Table 1.

Here, another interpretation is given. $R=|\underline{X}|$ above is interpreted to be an average of all $\underline{X}$ points in the marble which is 0. E_I in (5) now becomes a mathematically consistent constant. The values in (7) and (8) are now altered for Cases B and C. Using the values in Case C,

$$d_{m0} = 0.639932 \ \text{GeV}^2, \quad \Phi_{mav} = 0.45859 \ \text{GeV}^2 \tag{10}$$

(5) now yields $E_I=\pm 4.6018$ MeV. The upper sign value is now only +0.018% greater than 4.5936 MeV [1]. In terms of $\pi^\pm$ mass, (2) with Case C in Table 1 now yields 134.876 MeV. Adding it by 4.6018 MeV gives 139.4776 MeV which is −0.066% from 139.57 MeV [1].

Applying (5) to $K^\pm$ leads to $E_I = \pm 16.2$ MeV which is far too big. As was mentioned above, there is no neutral kaon with the same strong interaction content as $K^\pm$; E_I in (5) cannot be separated off from the much greater strong interaction mass contributions to the masses of $K^\pm$ and K^0. Thus, (5) does not apply to kaons, as was pointed out in [3 end of Sec. 7].

6. COMNCLUSION

Anticlimax: This work set out to improve the accuracy of the quark masses to the precision level of the pseudoscalar and vector meson masses that give rise to them. But it ends up with opposite results; quark masses are indefinite and variable.

In SSI, quark masses are not natural constants with fixed values, like the lepton masses. Their magnitudes are natural constants, but each quark mass can vary within certain ranges, ~±10% here, dependent upon its environment and the input mesons that gave rise to them.

Nature is economical in dealing out constants. It is wasteful to provide many precise quark masses which cannot be measured and confirmed. Nature handles this problem via indefinite, *variable quark masses* and d_{m0} values allowed by the quarks´ invisibility.

A mathematically consistent interpretation of a formula for the mass difference of $\pi^{\pm}$ and π^{0} derived earlier now yields a value only 0.02% off data.

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