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Principles of a Gravitational Field Quantization

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Abstract

The problems connected to propagation of a gravitational field are considered. The law of change of an electromagnetic radiation frequency in a gravitational field is shown. On the basis use of a quantum gravitational eikonal the energy of a single graviton is found. Refusal from a stresses tensor in structure of an energy-impulse tensor has allowed the quantum form of the energy-impulse tensor in Einstein's equation is found. It is shown that the solution of the Einstein's equation for the certain direction in this case represents the sum of a gravitational wave and a graviton. It is noticed that the deep understanding of process of the gravitational waves with massive body interaction can be only at the quantum philosophy. It is shown that at approach of a graviton to the massive bodies (double stars) radiating gravitational waves there is a resonant pumping of the gravitational field energy of these bodies to the gravitons. It enables registration of the gravitons with the help of the detector located near to massive bodies.

Keywords: a gravitational eikonal, metric tensor, Einstein's equation, energy flux, gravitational waves, energy-impulse tensor, registration of gravitons.

1. Introduction

The modern theory of gravitation - the theory general relativity of Einstein - is a basis for calculation of the various phenomena. It is generalization Newtonian dynamics, including the law of universal gravitation. As well as Newtonian dynamics the theory general relativity is not the quantum theory. The Einstein's equation for a gravitational field does not have stochastic nature.

Obviously such situation is unacceptable however the problem of a gravitational field quantization till now is not solved though for the solving of this problem many efforts have been applied [1-4].

Recently the problem of gravitational waves detecting [5] which description is possible with the help of the Einstein's equation [6] of general relativity is solved. This is one more experimental confirmation of validity of the theory general relativity.

From the physical point of view the general relativity theory assumes that the mass curvatures a space-time. This curvature of space-time influences all particles moving in space, including and what create a curvature. Influence is carried out and on massless particles, for example - photons. It is connected to a curvature of geodetic lines of space-time on which photons move. Photons change the frequency in a gravitational field. Space-time curvature in the general relativity theory identify with occurrence of some gravitational field due to which there is an interaction of mass particles.

Gravitational waves are propagating oscillations of the curved space-time as similar the waves on a water surface are propagating oscillations of the water particles.

However in the submitted physical picture of gravitation there is no the major element - quantization of a gravitational field.

Attempts to solve a problem of the gravitational waves quantization with the help of 5-dimensional space-time use [7] can hardly lead to success. Apparently, the theory of 5-dimensional space-time now has only a historical value. By comparison of distance from the source of gravitational waves calculated by the attenuation of experimentally registered gravitational waves and by the red displacement of electromagnetic radiation it has been established [8] that dimension of our space-time is equal $\sim 4 \pm 0{,}1$. Thus our space-time is described by four coordinates: time and three spatial coordinates.

The quantums of a gravitational field refer to gravitons. Gravitons represent a local wrinkling of the space-time which is propagated on more smooth space with a light velocity. Assuming as a whole correctness of the Einstein's equation for a gravitational field we research some features of the gravitational waves quantization.

2. Photon in Constant Homogeneous Gravitational Field

For the beginning to a gravitational field quantization we shall consider how frequency of a electromagnetic radiation quantum (photon) in a constant homogeneous gravitational field changes. Research we shall carry out in flat space-time the name Minkowski's space. The interval in inertial reference system looks like [6]:

$$ds^2 = c^2 d\tau^2 - dX^2 - dY^2 - dZ^2 = \\ = g_{00}\left(dX^0\right)^2 - g_{11}\left(dX^1\right)^2 - g_{22}\left(dX^2\right)^2 - g_{33}\left(dX^3\right)^2, \tag{1}$$

where designations there are $X = X^1, Y = X^2, Z = X^3$ - the Cartesian coordinates, c – a light velocity in vacuum, $d\tau$ - an interval proper time between events, so $cd\tau = dX^0$, $g_{00} = 1,\ g_{11} = g_{22} = g_{33} = -1$ - the metric tensor components which signature $(+,-,-,-)$.

From (1) follows:

$$d\tau = \frac{1}{c}\sqrt{g_{00}}\,dX^0. \tag{2}$$

In a gravitational field $g_{00} < 1$. Therefore, proper (or own) time flows that more slowly than is less g_{00} in the given point of space ($\Delta\tau = \tau_2 - \tau_1$ decreases; τ_2 in field it is less τ_2 outside of field). The clock per a gravitational field is slow.

The component of metric tensor g_{00} in a gravitational field decreases:

$$g_{00} = \left(1 + \frac{\varphi_g}{c^2}\right)^2 \qquad \text{or} \qquad \sqrt{g_{00}} = 1 + \frac{\varphi_g}{c^2}, \tag{3}$$

where is φ_g gravitational potential of a field, negative size so that acceleration $\dot{\boldsymbol{V}} = -\mathrm{grad}\varphi_g$.

For the further analysis we use concept of an eikonal. Eikonal there is a phase of the periodic function describing a field of electromagnetic wave:

$$\phi = \boldsymbol{kq} - \delta\tau, \tag{4}$$

where $\boldsymbol{k}$ there is a wave vector of an eikonal, $\boldsymbol{q}$ - a coordinate vector of an eykonal (it is optional Cartesian), δ - cyclic frequency of the eikonal.

Taking into account (2) and (4) it is possible to find the eikonal frequency (a photon frequency in the given point in a proper time):

$$\delta = -\frac{\partial \phi}{\partial \tau} = -\frac{\partial \phi}{\partial X^0}\frac{\partial X^0}{\partial \tau} = -\frac{c}{\sqrt{g_{00}}}\frac{\partial \phi}{\partial X^0}. \tag{5}$$

If to use the world time t (outside of a gravitational field), so that $t = \frac{X^0}{c}$ the photon cyclic frequency measured in world time is equal $\delta_0 = -\frac{\partial \phi}{\partial t} = -c\frac{\partial \phi}{\partial X^0}$. Hence, according to (5) with the account (3) we have:

$$\delta = \frac{\delta_0}{\sqrt{g_{00}}} = \frac{\delta_0}{1+\frac{\varphi_g}{c^2}}. \tag{6}$$

where δ_0 there is photon frequency at absence of a gravitational field.

Thus photon frequency depends on size of a gravitational field potential. As the gravitational field potential it is negative size at approach to the creating a field bodies, the photon frequency δ grows, and at removal falls (red displacement). For example, for a body in mass M the potential of a field depends on radius r under the formula $\varphi_g = -k\frac{M}{r}$ where $k = 6.67 \cdot 10^{-8}\ \frac{cm^3}{g \cdot s}$ there is gravitational constant.

If the concept of eikonal to a gravitational field apply the formula (6) should be correct and for gravitational waves.

The eikonal wave vector (or a photon wave vector) $\boldsymbol{k} = \frac{\partial \phi}{\partial \boldsymbol{q}}$, and 4-impulse in the Cartesian coordinates is equal $k_i = -\frac{\partial \phi}{\partial X_i}$. But for the 4-impulse the formula $k_i k^i = k^0 k^0 - \boldsymbol{kk} = 0$ is correct. Hence, $\frac{\partial \phi}{\partial X_i}\frac{\partial \phi}{\partial X^i} = 0$ there is the eikonal equation.

Let's note that the eikonal is a quantized size. The eikonal quantum is equal:

$$S_0 = \hbar\phi, \tag{7}$$

where $\hbar$ there is reduced Planck's constant.

3. The Einstein's Equation for a Gravitational Field

The gravitational field is described by the Einstein's equation. For writing of the Einstein's equation it is necessary first of all by the mathematical to describe a curvature of space-time. For the description of the mathematical curvature of space-time it is used a tensor of curvature (Riemann's tensor) [6]:

$$R_{iklm} = \left(\frac{\partial \Gamma^i_{km}}{\partial X^l} - \frac{\partial \Gamma^i_{kl}}{\partial X^m} \right) + \left(\Gamma^i_{nl}\Gamma^n_{km} - \Gamma^i_{nm}\Gamma^n_{kl} \right), \tag{8}$$

where Γ^i_{kl} there is a projection of a derivative unit vector $\boldsymbol{e}_k$ on coordinate X^l on a coordinate axis X^i - Cristoffel's symbol $\Gamma^i_{kl} e_i = \frac{\partial e_k}{\partial X^l}$. Cristoffel's symbols it is the functions of coordinates characterizing change a component of a vector at its parallel displacement. All indexes, bottom (covariant, usually functional sizes) and top (contravariant, usually coordinate sizes) accept values 0 (a time index), 1, 2, 3 (coordinate indexes). As it is usual summation is carried out on indexes repeating in products.

Cristoffel's symbols can be expressed through metric tensor under the formula $\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial X^l} + \frac{\partial g_{ml}}{\partial X^k} - \frac{\partial g_{kl}}{\partial X^m} \right)$ [6]. Thus curvature tensor of a space-time it is determined by a velocity and rapidly of a metric tensor g_{ik} change in the space - time. Generally there are 10 components of a metric tensor: 4 - with identical indexes (00, 11, 22, 33), and 6 with different indexes (01, 02, 03, 12, 13, 23, $C_4^2 = \frac{4 \cdot 3}{1 \cdot 2} = 6$).

The curvature tensor is a fourth rank. Physically the curvature of space-time can be described only the second rank tensor since energy–impulse tensor creating this curvature is the second rank tensor. Therefore we shall pass with the help of a curtailing operation in (7) to the second rank tensor (Ricci's tensor):

$$R_{ik} = g^{lm} R_{limk} = \left(\frac{\partial \Gamma^l_{ik}}{\partial X^l} - \frac{\partial \Gamma^l_{il}}{\partial X^k} \right) + \left(\Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{km} \right). \tag{9}$$

The Ricci's tensor it is symmetrical $R_{ik} = R_{ki}$.

Further we shall introduce a scalar curvature of a space-time under the formula:

$$R = g^{ik} R_{ik}\,, \tag{10}$$

where g^{ik} there is contravariant metric tensor.

Using the Ricci's tensor (9), a scalar curvature of a space-time (10), and also a metric tensor g_{ik}, the Einstein has written down the basic equation for a gravitational field:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi k}{c^4} T_{ik}\,. \tag{11}$$

The left part of this equation refers to Einstein's tensor $E_{ik} = R_{ik} - \frac{1}{2} g_{ik} R$. It characterizes geometrical properties of a space-time in particular its curvature. The right part of the equation includes an energy-impulse tensor of the second rank describing the source which creates the curvature of a space-time:

$$T_{ik} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} \tag{12}$$

Einstein's equation (11) can be written down in the other kind. We shall multiply at left the equation (11) on the contravariant metric tensor g^{ik}:

$$g^{ik} R_{ik} - \frac{1}{2} g^{ik} g_{ik} R = \frac{8\pi k}{c^4} g^{ik} T_{ik}\,. \tag{13}$$

Taking into account $R = g^{ik} R_{ik}$ and $g^{ik} g_{ik} = 4$ (in a four-dimensional space-time) we shall find:

$$-R = \frac{8\pi k}{c^4} \mathrm{Sp} T_{ik} = \frac{8\pi k}{c^4} T\,, \tag{14}$$

where it is designated $\mathrm{Sp} T_{ik} = T$.

Having substituted (14) in the Einstein's equation (11) we shall find:

$$R_{ik} = \frac{8\pi k}{c^4} \left(T_{ik} - \frac{1}{2} g_{ik} T \right). \tag{15}$$

Outside of a spherical symmetric body in mass M creating a gravitational field the energy-impulse tensor is equal to zero $T_{ik} = T = 0$. In this case according to (15) $R_{ik} = 0$.

Zero size of the Ricci's tensor according to the equation does not mean that there is no curvature of a space-time. The space–time curvature which is characterized by the Riemann's tensor (7) is kept in distance from the bodies creating this curvature.

4. Action of the Systems Gravitational Field - Particle

Quantization of the gravitational waves we shall carry out by quantization of the volumetric density of action in space of the generalized coordinates:

$$s = \int l\sqrt{-g}dt = \int (T - U)\sqrt{-g}dt = \int T\sqrt{-g}dt - \int U\sqrt{-g}dt, \qquad (16)$$

where $l = T - U$ there is total Lagrangian of a system a gravitational field - particle and their interaction [9], $\sqrt{-g}$ - defines the dependence of an volume normalizing element on space-time curvature, g - the determinant of a metric tensor.

Let's consider Lagrangian l of a systems a gravitational field - particle. It looks like:

$$l = T - U = (\rho - \rho_0)c^2 - (\rho\varphi_g - l_g), \qquad (17)$$

where $T = (\rho - \rho_0)c^2$ there is the volumetric density of a kinetic energy of a particle in a gravitational field, $U = (\rho\varphi_g - l_g)$ - volumetric density of a potential energy of a particle in a field (energy of interaction of a particle and field) including Lagrangian of the field [6]:

$$l_g = \frac{c^4}{16\pi k}R, \qquad (18)$$

where R there is a scalar curvature of a space-time, ρ - mass density of a particle in a field, ρ_0 - mass density of a rest particle, φ_g - gravitational potential of a field.

Lagrangian to the Lagrange's equation submits which looks like:

$$\frac{d}{dt}\left(\frac{\partial l}{\partial \dot{\boldsymbol{q}}}\right) = \frac{\partial l}{\partial \boldsymbol{q}}. \qquad (19)$$

For the generalized velocity we use size:

$$\dot{\boldsymbol{q}} = \sqrt{2T}. \qquad (20)$$

In this case the known formula for volumetric density of energy is correct:

$$w = \dot{\boldsymbol{q}}\frac{\partial l}{\partial \dot{\boldsymbol{q}}} - l = \sqrt{2T}\frac{\partial l}{\partial \sqrt{2T}} - T + U = \sqrt{T}\frac{\partial l}{\partial T}\frac{\partial T}{\partial \sqrt{T}} - T + U = T + U \ , \qquad (21)$$

owing to $\frac{\partial l}{\partial T} = 1$.

From formulas (17) and (20) follows that volumetric density of a particle kinetic and potential energy in a gravitational field is equal:

$$T = \frac{\dot{\boldsymbol{q}}^2}{2} = (\rho - \rho_0)c^2 , \qquad U = \rho\varphi_g - l_g \ . \qquad (22)$$

Thus both components of volumetric density of action depend on a body density which creates curvature of a space -time.

Therefore in the Einstein's equation (11) the right part of the equation dependent on mass creating a gravitational field should be subject to quantization only. The time is not quantized value. Apparently are not quantized all parameters of a space-time (scalar curvature, metric tensor, Ricci's tensor, etc.). To quantization can be subject to only energy-impulse tensor.

5. Graviton Energy and Quantum Gravitational Eikonal

The quantum of a gravitational field – a graviton as the quantum effect of gravitational radiation can be propagated far from massive bodies. It creates a curvature of the Riemann's spaces-time. Therefore a graviton has mass. The graviton mass is equal:

$$m = \frac{E}{c^2} , \qquad (23)$$

where *E* there is a graviton energy, *c* – a light velocity.

Trace *T* an energy–impulse tensor in Einstein's equation is connected to scalar curvature of space-time *R* a ratio (14).

If the graviton is propagated in a direction of an axis X_1 the diagonal components of a metric tensor wave fluctuations owing to the cross-section of gravitational waves following: $h_{11} = 0$, $h_{22} = -h_{33}$ [10]. The same ratio of components should be in the energy-impulse tensor of a graviton (12). Therefore a trace of a graviton energy-impulse tensor is equal:

$$T = T_{00} = \frac{m}{\mathrm{V}} c^2 = \frac{E}{\mathrm{V}}, \tag{24}$$

where V there is normalizing volume. According to (14) the scalar curvature of space-time is equal:

$$R = -\frac{8\pi k}{c^4}\frac{E}{\mathrm{V}}, \tag{25}$$

The action of a gravitational field is equal [6]:

$$S = \int l_g \sqrt{-g}\, d\Omega = \int l_g \sqrt{-g}\, d\mathrm{V} d(ct), \tag{26}$$

where $d\Omega = d\mathrm{V} d(ct)$ there is an element of a space-time volume. The size $\sqrt{-g}$ owing to its small size due to a graviton we believe to equal unit. At use of the space-time volume as $d\Omega = d\mathrm{V} d(ct)$ the Lagrangian of a gravitational field as against (18) it is necessary to use as $l_g = \frac{c^3}{16\pi k} R$.

Substituting this formula in (26) and assuming approximately constancy of scalar curvature in area of a graviton we shall find:

$$S = \int l_g d\mathrm{V} d(ct) = \frac{c^4}{16\pi k} \mathrm{V} R t + C, \tag{27}$$

where *C* there is a constant of integration which can depend from X_1.

Substituting (25) in (27) we have:

$$S = -\frac{1}{2} E t + C. \tag{28}$$

Further to similarly electromagnetic field, see (7), we shall enter into consideration the concept of a quantum gravitational eikonal $S = \hbar \phi$ where $\hbar$ there is Planck's reduced constant, $\phi = r X_1 - \omega t$ - its phase, *r* - wave number of a graviton, ω - its own frequency. We assume function of the quantum gravitational eikonal $S(X_1, t)$ approximately linear in a weak gravitational field [6, 11]. We shall note equivalence of quantum gravitational eikonal and actions of system a gravitational field - particle. Both sizes submit to a principle of a minimum (to Maupertuis' principle for action or Fermat's for eikonal) [6].

Equating a quantum gravitational eikonal and action we find:

$$S = \hbar\phi = \hbar(rX_1 - \omega t) = -\frac{1}{2}Et + C. \quad (29)$$

From the formula (29) follows that graviton energy is equal:

$$E = 2\hbar\omega, \quad (30)$$

and size $C = 2\hbar rX_1$. The spin of graviton it is $\pm 2\hbar$.

Energy of a relativistic mass particle is equal $E = \sqrt{p^2c^2 + m_0^2c^4}$. Therefore the formula (30) allows to assume that as against an ordinary particle the graviton rest mass is equal to zero $m_0 = 0$, and the impulse of the graviton is equal $p = \frac{E}{c} = \frac{2\hbar\omega}{c} = 2\hbar r$. One of differences of a graviton from a photon consists that the photon mass is always equal to zero, and the graviton mass is equal to zero only rest. However as well as the photon the graviton cannot are in a condition of rest.

Having substituted (30) in (23) we find $m = \frac{E}{c^2} = \frac{2\hbar\omega}{c^2}$ hence the mass of a graviton is proportional to its frequency. If for a graviton (by analogy to a photon) the rule of "red displacement" operates then at removal from massive bodies the graviton frequency, and hence its mass should decrease down to disappearance of the graviton (gravitonic darkness[1]). At approach a graviton to the massive bodies the graviton frequency and its mass should increase.

If to accept for example the frequency of background thermal gravitational radiation $\omega = 1.26 \cdot 10^{12} s^{-1}$ [10] then graviton energy is $E = 2\hbar\omega = 2.66 \cdot 10^{-15} erg = 0{,}00166\, eV$.

A graviton mass (23) of the background thermal gravitational radiations is $m = 2{,}96 \cdot 10^{-26}\ g$.

Propagation of a graviton passes in a direction of a normal to constant eikonal surface. Thus we have average radius of curvature of a constant eikonal surface (curvature of the Riemann's spaces) in the given approximation (a weak gravitational field) are much greater of a graviton wave lengths $\lambda = \frac{2\pi c}{\omega}$ [11].

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[1]It is possible that in the Universe there are the vivid individuals seeing with the help of gravitons as well as Earth's individuals see with the help of photons.

6. The Quantum Form of an Energy-Impulse Tensor

We shall consider the energy-impulse tensor (12) in more detail. It is supposed that into the energy-impulse tensor enters as making the stresses tensor:

$$\sigma_{ik} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} = \begin{pmatrix} \sigma_X & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_Y & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_Z \end{pmatrix}. \tag{31}$$

where T_{11}, T_{22}, T_{33} there are normal stresses, other components $\sigma_{ik} = T_{ik}$ for $i \neq k$ - tangential stresses. A component $T_{00} = \rho c^2$ - volumetric density of energy of a mass particle or graviton, T_{10}, T_{20}, T_{30} - components of the impulse density multiplied on light velocity *c*, T_{01}, T_{02}, T_{03} - components of the energy flux density divided on *c*.

In a basis of the further research we shall assume absence of a stresses state birth of empty space owing to its possible curvature.

In [12] it has been shown that in a fluid or gas there is an uncertainty of a sign on tangential stresses. Moreover the stresses tensor only approximately describes a stressed state of a fluid and gas. In a fluid and gas the stresses tensor is absent. For calculation of a fluid or gas flux it is necessary to use the vector formula connecting force $d\boldsymbol{F}$ and velocity $\boldsymbol{V}$ as [12]:

$$d\boldsymbol{F} = \vec{\eta}\, d\boldsymbol{S} \times \mathrm{rot}\boldsymbol{V}, \tag{32}$$

where $d\boldsymbol{S}$ there is area of contacting layers in a fluid or gas, $\vec{\eta}$ - viscosity tensor of the second rank which diagonal components there are molecular viscosity and not diagonal components - turbulent viscosity.

If a flux of fluid or gas passes for example in a direction axes *X* the directions of vectors in the formula (32) are shown on fig. 1.

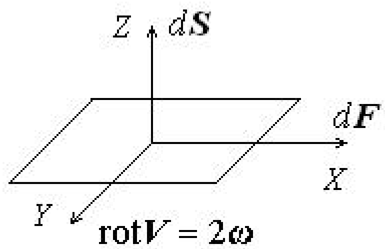

Figure 1. The directions of vectors in the formula (32)

For our purposes in the formula (32) using cyclic frequency of a medium rotation $\boldsymbol{\omega} = \frac{1}{2}\mathrm{rot}\boldsymbol{V}$ [13] therefore it is convenient to write down as:

$$d\boldsymbol{F} = 2\vec{\eta}\, d\boldsymbol{S} \times \boldsymbol{\omega}, \tag{33}$$

The scalar variant of the formula (33) looks like:

$$\begin{gathered} dF_X\boldsymbol{i} + dF_Y\boldsymbol{j} + dF_Z\boldsymbol{k} = 2\eta\{(dS_Y\omega_Z - dS_Z\omega_Y)\boldsymbol{i} + (dS_Z\omega_X - dS_X\omega_Z)\boldsymbol{j} + \\ + (dS_X\omega_Y - dS_Y\omega_X)\boldsymbol{k}\} \end{gathered}, \tag{34}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ there are in this case unit vectors of 3-dimensional space. Assuming space homogeneous we use a scalar variant viscosity tensor η.

At use of the formula (32) the uncertainty of the tangential stresses sign and some other problems connected with stresses tensor [12] on which we shall not stop disappear.

We shall write down the energy-impulse tensor (12) in the following kind:

$$T_{ik} = \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & \dfrac{dF_1}{dS_1} & \dfrac{dF_2}{dS_1} & \dfrac{dF_3}{dS_1} \\ T_{20} & \dfrac{dF_1}{dS_2} & \dfrac{dF_2}{dS_2} & \dfrac{dF_3}{dS_2} \\ T_{30} & \dfrac{dF_1}{dS_3} & \dfrac{dF_2}{dS_3} & \dfrac{dF_3}{dS_3} \end{pmatrix}, \tag{35}$$

where transition to digital indexes is carried out.

Taking into account (34), we have:

$$T_{ik} = \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & \dfrac{2\eta(dS_2\omega_3 - dS_3\omega_2)}{dS_1} & \dfrac{2\eta(dS_3\omega_1 - dS_1\omega_3)}{dS_1} & \dfrac{2\eta(dS_1\omega_2 - dS_2\omega_1)}{dS_1} \\ T_{20} & \dfrac{2\eta(dS_2\omega_3 - dS_3\omega_2)}{dS_2} & \dfrac{2\eta(dS_3\omega_1 - dS_1\omega_3)}{dS_2} & \dfrac{2\eta(dS_1\omega_2 - dS_2\omega_1)}{dS_2} \\ T_{30} & \dfrac{2\eta(dS_2\omega_3 - dS_3\omega_2)}{dS_3} & \dfrac{2\eta(dS_3\omega_1 - dS_1\omega_3)}{dS_3} & \dfrac{2\eta(dS_1\omega_2 - dS_2\omega_1)}{dS_3} \end{pmatrix}. \tag{36}$$

The tensor (36) is an energy-impulse tensor creating a curvature of a space - time. For quantization of the energy-impulse tensor first of all the size η should be expressed through the Planck's sizes: Planck's length $\left(\frac{\hbar k}{c^3}\right)^{\frac{1}{2}}$, Planck's time $\left(\frac{\hbar k}{c^5}\right)^{\frac{1}{2}}$ and Planck's mass $\left(\frac{\hbar c}{k}\right)^{\frac{1}{2}}$.

Instead of size η we use it Planck's value $\eta = \frac{\hbar}{V}$ where V there is normalizing volume. Besides there are no reasons to assume the areas in (36) various therefore shall accept $dS_1 = dS_2 = dS_3$. In this case the tensor (36) it will be transformed to a kind:

$$T_{ik} = \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & \frac{2\hbar(\omega_3 - \omega_2)}{V} & \frac{2\hbar(\omega_1 - \omega_3)}{V} & \frac{2\hbar(\omega_2 - \omega_1)}{V} \\ T_{20} & \frac{2\hbar(\omega_3 - \omega_2)}{V} & \frac{2\hbar(\omega_1 - \omega_3)}{V} & \frac{2\hbar(\omega_2 - \omega_1)}{V} \\ T_{30} & \frac{2\hbar(\omega_3 - \omega_2)}{V} & \frac{2\hbar(\omega_1 - \omega_3)}{V} & \frac{2\hbar(\omega_2 - \omega_1)}{V} \end{pmatrix}. \quad (37)$$

Let's consider for example the energy-impulse tensor for propagation of gravitational radiation to a direction of axis X_1. Gravitational waves are cross-section hence the vector of frequency is directed along an axis X_1. In this case the formula (37) looks like:

$$T_{ik} = \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & 0 & \frac{2\hbar\omega_1}{V} & -\frac{2\hbar\omega_1}{V} \\ T_{20} & 0 & \frac{2\hbar\omega_1}{V} & -\frac{2\hbar\omega_1}{V} \\ T_{30} & 0 & \frac{2\hbar\omega_1}{V} & -\frac{2\hbar\omega_1}{V} \end{pmatrix}. \quad (38)$$

Asymmetry of the energy-impulse tensor T_{ik} is connected with basic refusal from use symmetric stress tensor [12].

In the tensor of energy-impulse the components $\pm\frac{2\hbar\omega_1}{V} = \pm\varepsilon$ enter which characterize volumetric density of energy of gravitational radiation quantum - graviton. Two signs of the spin are reflect two directions of polarization: (plus) a vector of cyclic frequency ω_1 is directed along direction of a graviton propagation and (minus) against direction of a graviton propagation.

In spite of the fact that we have entered the graviton energy in energy-impulse tensor it does not mean that tensor began to have quantum character. Further it is necessary to take into account ideas of the quantum mechanics matrix form and to add at least to tensor components with graviton energy a factor $\exp\left(\frac{i}{\hbar}S\right) = \exp i(rX_1 - \omega_1 t)$ [14] where $S = \hbar\phi$ is a quantum of gravitational eikonal (29), $\phi = rX_1 - \omega_1 t$ - its phase. We assume function of a gravitational eikonal quantum $S(X_1, t)$ approximately linear in a weak gravitational field.

In the matrix form of quantum mechanics ω_1 carries the name of spectral frequency and characterizes transition of system from one quantum state in another. The factor $\exp\left(\frac{i}{\hbar}S\right)$ characterizes transition of a gravitational field quantum (graviton) on quantum states in a space-time. We shall note that at record of matrixes in the matrix form of quantum mechanics the exponential factors at a component of matrix frequently omit [14]. Thus the energy-impulse tensor will receive a kind:

$$T_{ik} = \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & 0 & \varepsilon \exp\left(\frac{i}{\hbar}S\right) & -\varepsilon \exp\left(\frac{i}{\hbar}S\right) \\ T_{20} & 0 & \varepsilon \exp\left(\frac{i}{\hbar}S\right) & -\varepsilon \exp\left(\frac{i}{\hbar}S\right) \\ T_{30} & 0 & \varepsilon \exp\left(\frac{i}{\hbar}S\right) & -\varepsilon \exp\left(\frac{i}{\hbar}S\right) \end{pmatrix} =$$

$$= \begin{pmatrix} \rho c^2 & T_{01} & T_{02} & T_{03} \\ T_{10} & 0 & \frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) & -\frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) \\ T_{20} & 0 & \frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) & -\frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) \\ T_{30} & 0 & \frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) & -\frac{2\hbar\omega_1}{V}\exp i(rX_1 - \omega_1 t) \end{pmatrix} . \qquad (39)$$

The energy-impulse tensor (39) has quantum character.

7. The Graviton Equation

For a finding of the graviton equation we shall substitute an energy-impulse tensor (39) in Einstein's equation as (15). The tensor energy-impulse trace (39) looks like $T = \rho c^2$. Hence the equation (15) - the graviton equation will be transformed to a kind:

$$R_{ik} = \frac{8\pi k}{c^4}\left(T_{ik} - \frac{1}{2}g_{ik}\rho c^2\right). \qquad (40)$$

Let's consider the graviton equation in almost flat space at absence of a mass component in energy-impulse tensor (39) i.e. at $T = \rho c^2 = 0$. The gravitons practically do not bend space-time. Therefore it is possible to use the formula for wave fluctuations of metric tensor h_{ik} as [6]:

$$\frac{\partial^2 h_{ik}}{\partial X_\alpha \partial X_\alpha} - \frac{1}{c^2}\frac{\partial^2 h_{ik}}{\partial t^2} = \frac{16\pi k}{c^4} T_{ik} . \quad (41)$$

The graviton radiation cross-section therefore all tensor components h_{ik} in the equation (41) with indexes 1 at the propagation of graviton in direction X_1 are excluded. Tensor components h_{ik} remain only: h_{23} and $h_{22} = -h_{33}$ [10]. We shall notice also that in the energy-impulse tensor (39) $T_{22} = -T_{33}$.

For a component h_{23} the wave equation (41) looks like:

$$\frac{\partial^2 h_{23}}{\partial X_1^2} - \frac{1}{c^2}\frac{\partial^2 h_{23}}{\partial t^2} = -\frac{16\pi k}{c^4}\frac{2\hbar\omega}{\mathrm{V}} \exp i(rX_1 - \omega t). \quad (42)$$

For component h_{22} a sign in the right part of the equation is positive, and for component h_{33} is negative. The index 1 at frequency is omitted. Therefore designating $\chi = h_{ik}\mathrm{V}$ where $ik = 22, 23, 33$ we shall find:

$$\frac{\partial^2 \chi}{\partial X_1^2} - \frac{1}{c^2}\frac{\partial^2 \chi}{\partial t^2} = \pm\frac{32\pi k\hbar\omega}{c^4} \exp i(rX_1 - \omega t). \quad (43)$$

The factor before an exponent in the right part (43) is proportional to the double scalar curvature of space-time due to presence graviton, (25) $R = \frac{8\pi k}{c^4}\frac{E}{\mathrm{V}} = \frac{8\pi k}{c^4}\frac{2\hbar\omega}{\mathrm{V}}$. This factor is extremely small $\frac{32\pi k\hbar}{c^4} = 0{,}87 \cdot 10^{-74}\, cm \cdot s$ that specifies almost flat space-time.

The equation (43) describes a gravitational wave and graviton propagating from left to right therefore the general solution of this equation we shall search as:

$$\chi = C_1 f_1(rX_1 - \delta t) + C_2 f_2(t) \exp irX_1 , \quad (44)$$

where C_1 and C_2 there are constants, $f_1(rX_1 - \delta t)$ and $f_2(t)$ - any functions, δ - gravitational wave frequency connected with a velocity of its propagation $r = \frac{\delta}{c}$. Frequency δ is not equal to own graviton frequency ω which can belong to other wave. The first term (44) describes the gravitational wave the second term - graviton.

Let's substitute (44) in the equation (43) which have been written down as:

$$\frac{\partial^2 \chi}{\partial X_1^2} - \frac{1}{c^2}\frac{\partial^2 \chi}{\partial t^2} = \pm\gamma\omega \exp i(rX_1 - \omega t), \quad (45)$$

where it is designated $\gamma = \frac{32\pi k\hbar}{c^4}$ - a constant.

The first term (44) satisfies to the equation (45) without the right part. Therefore after substitution we shall find:

$$\frac{d^2 f_2}{dt^2} + \delta^2 f_2 \pm \frac{\gamma\omega c^2}{C_2}\exp(-i\omega t) = 0 . \quad (46)$$

The particular solution of the equation (46) dependent on own graviton frequency ω we shall search as:

$$f_2 = A\exp(-i\omega t). \quad (47)$$

Substituting (47) in (46) we shall find:

$$A = \pm\frac{\gamma\omega c^2}{C_2(\omega^2 - \delta^2)}. \quad (48)$$

Hence according to (44) the solution of the equation (45) looks like:

$$\chi = C_1 f(rX_1 - \delta t) \pm \frac{\gamma\omega c^2}{(\omega^2 - \delta^2)}\exp i(rX_1 - \omega t). \quad (49)$$

Taking into account a designation $\chi = h_{ik}\mathrm{V}$ where $ik = 22, 23, 33$, and also $\gamma = \frac{32\pi k\hbar}{c^4}$, and (25) we shall find:

$$\begin{aligned} h_{ik} &= ae_{ik}\cos(rX_1 - \delta t) \mp \frac{32\pi k\hbar\omega\, e_{ik}}{c^2(\delta^2 - \omega^2)\mathrm{V}}\exp i(rX_1 - \omega t) = \\ &= ae_{ik}\cos(rX_1 - \delta t) \mp \frac{2Rc^2 e_{ik}}{(\delta^2 - \omega^2)}\exp i(rX_1 - \omega t) \end{aligned}, \quad (50)$$

where a there is an amplitude of a wave, $\boldsymbol{r}$ - a wave vector (of a wave and graviton) in a direction of a wave propagation. The value e_{ik} there is a unit polarization tensor (of a wave and graviton) obeying the conditions [15]:

$$e_{ik} = e_{ki}, \qquad e_{ii} = 0, \qquad k_i e_{ik} = 0, \qquad e_{ik}e_{ik} = 1. \quad (51)$$

The first term (50) characterizes propagation of the gravitational wave the second term - graviton. As against an electromagnetic wave which polarization is determined by a vector of the electric field oscillations, the polarization of a gravitational wave (and graviton) has essentially tensor character. The third condition (51) is a condition of the gravitational waves (and graviton) cross-section.

The top sign (49) and (50) corresponds to size h_{22} , and the bottom sign h_{23} and h_{33} . The size h_{ik} naturally does not depend on normalizing volume V.

8. Registration of Graviton

The kind of function (50) allows draw the some conclusions. Far from massive bodies a graviton as the quantum effect is practically imperceptible. It practically has not influence on registration of the gravitational waves.

However function (50) has interesting feature. At $\omega \to \delta$ i.e. at aspiration of a graviton own frequency to the gravitational wave frequency fluctuation components of the metric tensor h_{ik} aspire to infinity. The resonance of the gravitational wave frequency and graviton own frequency which is in structure of other wave is observed. This phenomenon can be used for registration of gravitons.

Let's assume that two massive cosmic bodies (for example a double star) rotate around of the common mass centre. This system radiates gravitational waves $h_{ik} = ae_{ik} \cos(rX_1 - \delta t)$ with constant frequency δ . If graviton as quantum phenomenon of another wave gets in area of such cosmic bodies its own frequency ω grows according to the formula similar (6) similar photon frequency. According to the formula (50) growth of a graviton energy occurs much faster $2\hbar\omega$ that specifies resonant pumping of energy of these massive bodies gravitational field in a graviton. Energy of a graviton гравитона become very big and graviton can be registered. Probably for this purpose it is necessary to install the detector on massive bodies or near to them (on their planet or the artificial satellite). With the help of such detector it is possible to register abnormal splash in gravitation at graviton passage.

At distance from the bodies a graviton return the energy to a gravitational field of the bodies; its frequency falls (as red displacement in gravitation). Therefore energy of a gravitational field of the massive bodies does not change at flight near them of gravitons. We shall notice also that Riemann's curvature of space-time is positive therefore in the considered effect the fluctuation components of metric tensor h_{23} and h_{33} take part only. They resonantly grow at flight of a graviton near the massive bodies.

9. Conclusion

On the basis of Einstein's equation for gravitation the gravitational radiation of the massive bodies as a double star is investigated. By refusal from a stresses tensor into energy-impulse tensor, and replacement corresponding a component in power sizes also introductions of a gravitational eikonal quantum the quantization of gravitational radiation is lead.

The solution of the quantum equation of Einstein in the certain direction shows that this solution represents the sum of two composed, first of which characterizes gravitational waves, and the second - graviton.

At approach of the gravitons to a double star there is a resonant pumping of the gravitational field energy to gravitons. It allows register the gravitons. At distance of the gravitons from a double star their energy comes back in a gravitational field of the stars. Frequency of the gravitons decreases as red displacement for gravitation. Therefore as a whole the gravitons as quantum phenomenon do not influence on a gravitational field of stars.

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