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How the invariance of *c*, the *time dilation* and the H atom parameters, are in full accordance with Classical Physics

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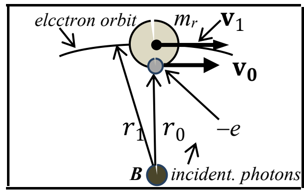
Abstract

It is shown that the speed of light corresponds to the escape speed (u) from all the masses in space, in accordance with $c = u = \sqrt{-2U}$, where U is the *total* gravitational potential energy (due to all the masses in space) whose value depends on the location of the masses with respect to a given point. Moreover, the equality $c = u$ implies the massiveness of the light, and because of its two parameters, λ and ν, the light turns out to be composed of longitudinal particles, (*photons*), where λ is the length of one photon and where their number flowing in 1s along a given direction (ray of light), corresponds to their frequency, while the mass of one photon, related to its transit time $T\,(= 1/\nu)$, becomes $\gamma = 2h/c^2 = 1.474499 \times 10^{-50}$ kg s, being h the Planck's constant**;** indeed, each photon can be represented as an electrical (massive) dipole, with a positive charge on its front and an equal negative charge on its tail. On these bases, and in accordance with Classical Physics, as for photons emitted by a source and re-emitted by an Observer, the speed of the re-emitted photons turns out to be invariant nevertheless of any relative speed "source-Observer"**:** in short, during the impact photons-circling electron (forcing the electron to move toward higher orbit, hence having a radial velocity) the frequency of the photons re-emitted by the impacted electron will decrease, their length will increase (and vice versa) yielding, (in the same location, where $c = \sqrt{-2U}$ has a certain value), the invariance of c which, as then shown, has a cosmological reason. Moreover, on our results, the electric charge of electrons is not uniformly distributed, but it turns out to be a point-particle, fixed on the electron surface, which, during the electron revolution, (see the figure hereafter), is always facing the electron-proton common center of

gravity B (and therefore forcing the electron to make one rotation every revolution). This electron structure implies the impacts photons-electron (being the electron charge their impact-point) to move the impacted electron toward outer orbits, avoiding the electron from its possible fall into the atom nucleus.



Then, still in accordance with Classical Physics, considering, on H atom, the electron reduced mass $m_r(=\frac{m_e}{1.0005446})$, circling around the electron-proton common centre B, we found n = 1,2,..137, (instead of the claimed n = 1,2,..∞), as the consecutive number of progressive circular orbits of its electron, while n^2 turns out to be the number of incident (or released) photons along the nth circular orbit and therefore, in particular, we obtained $\frac{m_r}{\gamma c R_\infty}=\frac{m_r c}{2hR_\infty}=\frac{cm_e/1.0005446}{2hR_\infty}=18769=137^2$ as the number of impacting (or released) photons along the 137th circular (ionization) orbit; then, being α the fine-structure constant, we found $r_1=\alpha/4\pi R_\infty = 5.291772\times10^{-11}$ m, (equal to Bohr radius), as the radius of the electron ground-state (g-s) orbit; then, $r_0=137\alpha r_1=5.290381\times10^{-11}$ m, as the electron charge orbital radius; $r_e=r_0(\frac{1}{137\alpha}-1)=1.390550\times10^{-14}$ m as the electron radius, and therefore the Bohr radius can also be given by $r_1=r_0+r_e=5.291772\times10^{\mathbf{-11}}$ m; moreover, along the electron g-s orbit r_1, the ratio $\delta_1=\lambda_1/2\cdot2\pi r_1$, between the length of the admitted photon $\lambda_1(=\frac{1}{R_\infty})$ and two (2) electron orbits, gives $\delta_1=\frac{1}{R_\infty4\pi r_1}=1/\alpha\cong137.036$ while, being $r_1^{'}\equiv r_1+r_e=5.293162\times10^{-11}$m the electron orbit after the impact of the admitted photon λ_1, we found $\delta'=\frac{\lambda_1}{4\pi\, r_1^{'}}=\frac{1/R_\infty}{4\pi\, r_1^{'}}=(1/10973731)/\ 4\pi\cdot 5.293162\times10^{-11}=137.0000$ showing that, along the electron g-s orbit, the length of the admitted photon λ_1 has to be 137 times the length of two electron orbits; we also found $v_1=c/137$ as the g-s orbital speed of the electron while, as for the g-s orbital speed of the electron charge, we obtained $v_0=\alpha c$, complying the electron charge location.

Keywords: Escape speed, Time dilation, Bohr radius, Electron reduced mass

1. The speed of light as function of the gravitational potential energy.

The gravitational potential energy in a point at distance s from a mass M, is $U=-\frac{MG}{s}$, while the escape speed is $u=\sqrt{-2U}=\sqrt{2MG/s}$; both u and U are scalar quantities hence, as for two masses, M_1, M_2, being s_1 and s_2 the distances between each mass and the considered point, the escape speed is

$$u_{1,2}=\sqrt{-2U_{1,2}}\equiv\sqrt{-2(U_1+U_2)}=[\left(\frac{2M_1G}{s_1}\right)+\left(\frac{2M_2G}{s_2}\right)]^{1/2}\implies u_{1,2}=\sqrt{u_1^2+u_2^2}\ ; \quad (1)$$

then, as for one only mass $M_{1,2}\equiv(M_1+\ M_2)$ located in their center of mass, and being $\alpha_n\equiv M_n/M_{1,2}$ we have

$$U_{1,2} = (U_1 + U_2) =- \left(\frac{M_1G}{s_1} + \frac{M_2G}{s_2}\right) \equiv - M_{1,2}G/s_{1,2} \implies \quad 1/s_{1,2} = [(\alpha_1/s_1) + (\alpha_2/s_2)] \qquad (2)$$

thus, since $U_{1,2} \equiv - M_{1,2}G/s_{1,2}$, we may write

$$u_{1,2} = \sqrt{-2U_{1,2}} = \sqrt{-2M_{1,2}G/s_{1,2}} \qquad (3)$$

that is the escape speed from one mass only, sum of two masses located in their center of mass.

Therefore, being $M_\mathrm{u} \equiv \Sigma M_n$ one mass located in the center of all the masses M_n, the potential due to M_u only, is

$$U \equiv \Sigma U_n =- \Sigma \frac{M_nG}{s_n} \equiv -\frac{M_\mathrm{u}G}{s_M} \qquad \Rightarrow \qquad s_M =- M_\mathrm{u}G/U \qquad (4)$$

being s_M the distance, from the considered point, to the only mass M_u, giving the same potential as all the masses.

According to NASA, WMAP spacecraft observations[1], the universe has a mass density equal to the critical density $\rho_c = 9.9 \times 10^{-27}$ kg/m^3, while many authors[2,3] give its mass as $M_\mathrm{u} \cong 10^{53}$ kg. The finite value of M_u implies $U_\infty = 0$, hence we infer that the mass density is decreasing toward the infinity like a function $\rho = \rho_c \mathrm{e}^{-as}$, and therefore

$$M_\mathrm{u} \cong \int_0^\infty 4\pi\, s^2 \rho_c \mathrm{e}^{-as} \mathrm{d}s = \frac{8\pi\rho_c}{a^3} \cong 10^{53}\ \mathrm{kg} \implies a \cong (8\pi\rho_c/M_\mathrm{u})^{1/3} \approx 1.3 \times 10^{-26}\ \mathrm{m}^{-1} \qquad (5)$$

where, as then shown, $s_M = 2/a \approx 1.5 \times 10^{26}$m ($\cong 1.5 \times 10^{10}$ ly). We may now find the value U_0 on Earth**:** toward the infinity, the variation of potential, due to an increase of the distance ds, can be written as d$U = -$ dm G/s, where d$m = \rho\, 4\pi s^2$ ds and $\rho = \rho_c \mathrm{e}^{-as}$**;** therefore the potential U_0 on Earth becomes

$$U_0 =- \int_0^\infty (4\pi s^2/ s) G \rho_c \mathrm{e}^{-as} \mathrm{d}s =- 4\pi\rho_c G/a^2 \cong -4.5 \times 10^{16}\mathrm{J} \implies u_0 = (-2U_0)^{1/2} \cong 3 \times 10^8\ \mathrm{m/s} \cong c_0. \qquad (6)$$

Thus, since $\frac{U_0}{M_\mathrm{u}} =- (4\pi\rho_c G/a^2)/\frac{8\pi\rho_c}{a^3} =- Ga/2$, comparing to Eq. (4) written, on Earth, as $\frac{U_0}{M_\mathrm{u}} =- G/s_M$, one gets $s_M = 2/a$; then, since from Eq. (6), $u_0 = (-2U_0)^{1/2} \equiv c_0$, for each point in space we can write

$$c = u = \sqrt{-2U} \qquad \implies \qquad U_0 =- c_0^2/2 \quad \text{(on Earth)} \qquad (7)$$

where U_0 is practically constant: indeed, between the ground and the height h, with $h << r_\mathrm{E}$ (Earth radius), we found

$$c_h = \sqrt{-2U_h} = \sqrt{-2(U_0 + U_{0h})} = \sqrt{c_0^2 - 2gh} \cong c_0\left(1 - \frac{gh}{c_0^2}\right) \qquad (8)$$

while its variation between Aphelion ($a = 1.52 \times 10^{11}$m) and Perihelion ($p = 1.47 \times 10^{11}$m), being M_S the Sun mass, is

$$\Delta c_{AP} =- \frac{\Delta U_{AP}}{c_0} =- \frac{U_P - U_A}{c_0} = \left[\left(\frac{M_SG}{p}\right) - \left(\frac{M_SG}{a}\right)\right]/c_0 \cong 0.10\ \mathrm{m/s}\,. \qquad (9)$$

[Other variations of c: From Earth toward Mercury (mass M_e), being $U_0 =- c_0^2/2$ the potential on Earth, and taking into consideration the variation of U due to the Sun only, we can write $U_{Me} =- \frac{1}{2}c_0^2 + \left(\frac{M_SG}{d_{ES}}\right) - \left(\frac{M_SG}{d_{Me\,S}}\right)$, where d_{ES} is the distance Sun-Earth, and d_{M_eS} is one Sun-Mercury; in SI units, $M_S = 2 \times 10^{30}$, $G = 6.7 \times 10^{-11}$, $d_{ES} = 1.5 \times 10^{11}$, $d_{M_eS} = 0.6 \times 10^{11}$, we find $c_{M_e} = (-2U_{M_e})^{1/2} = [c_0^2 + 2M_SG(d_{ES} - d_{M_eS})/d_{ES}d_{M_eS}]^{1/2} \cong c_0 + 2.2$ m/s.

On the contrary, from Earth toward Mars (M_a), still taking in consideration the variation of U due to the Sun only, we can write $U_M =- \frac{1}{2}c_0^2 + M_SG\left(\frac{1}{d_{M_aS}} - \frac{1}{d_{ES}}\right)$; being $M_S = 2 \times 10^{30}$, $G = 6.7 \times 10^{-11}$, $d_{ES} = 1.5 \times 10^{11}$, $d_{M_aS} = 2.3 \times 10^{11}$, we find $c_{M_a} = (-2U_M)^{1/2} = [c_0^2 + 2M_SG(d_{ES} - d_{M_aS})/d_{ES}d_{M_aS}]^{1/2}$, giving $c_{Ma} \cong c_0 - 3$ m/s.]

The equality $c = u$ implies the massiveness of the light, hence the light has a structure.

2 - Photon physical characteristics, electron structure, photon-electron *Impact Point*

During the interaction light-matter, to move an impacting electron toward outer orbits, and avoiding its fall into the atom nucleus, the impacts photons-*circling electron* shall happen, see Fig.1(**b**), in a specific point of the electron surface, we named *Impact Point*, corresponding to the electron charge which, *during the electron revolution,* is always facing the atom nucleus, see also Fig.1(**c**).

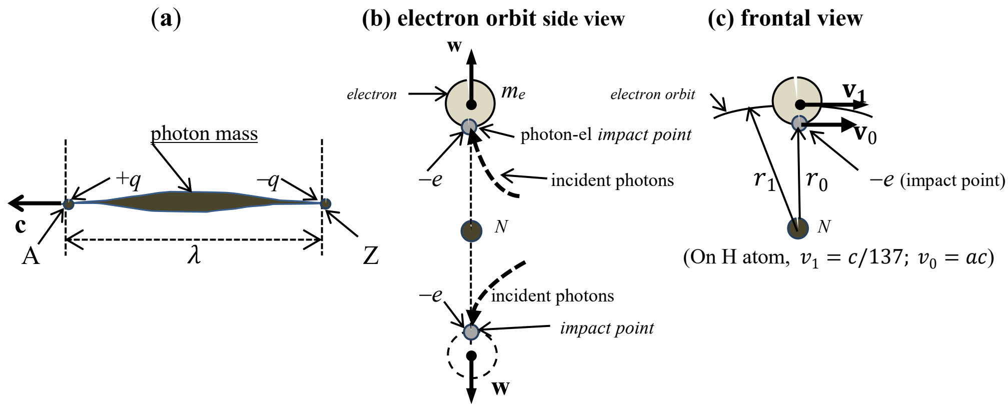


Fig. 1- (a) Photons representation; (b) impact photons-electron and electron radial velocity w.

These impacts (contrary to *neutrinos* usually crossing the matter without interaction) may happen if each photon *front* is provided, see Fig.1(**a**), with a positive charge $(+q)$ while its *tail* with an equal negative one, $(-q)$, like an electrical dipole. Moreover, the kinetic energy of *one ray* of light has to be $K_c = \frac{1}{2}mc^2$ and therefore equating K_c to the empirical relation $E = h\nu$, we have

$$\tfrac{1}{2}mc^2 = h\nu \quad \Longrightarrow \quad m = 2h\nu/c^2 \equiv \gamma\nu \tag{10}$$

where m is the mass of photons flowing along one ray in 1 s, while the constant

$$\gamma = 2h/c^2 = m/\nu = mT = 1.474499 \times 10^{-50} \text{ kg s} \tag{11}$$

is the mass of *one* photon related to its transit time T. From Eq. (11), the Planck's constant h becomes

$$h = \frac{1}{2}\gamma c^2 \quad \Longrightarrow \quad E = h\nu = \frac{1}{2}\gamma\nu c^2 = \frac{1}{2}mc^2 \tag{12}$$

where h *turns out to be the kinetic energy of one photon*, showing that h is not a real constant since it depends on c; then, being n_r the number of rays emitted by a source of photons S, the term

$$P = \frac{n_r E}{t_0} = n_r \tfrac{1}{2}mc^2/t_0, \tag{13}$$

being $t_0 = 1$ s, is the power emitted by S**;** hence, writing $n_r m \equiv m_{\text{tot}}$, Eq. (13) yields

$$m_{\text{tot}} = 2P/c^2, \tag{14}$$

that is the mass lost per second by a source of light**;** for a 1W source, $m_{\text{tot}} = 2P/c^2 \cong 2.2 \times 10^{-17}$ kg s^{-1}, while the number n_r of rays emitted by a source, since $\gamma = 2h/c^2$, becomes

$$n_r = m_{\text{tot}}/m = m_{\text{tot}}/\gamma\nu = 2P/c^2\gamma\nu = P/h\nu. \tag{15}$$

Then, the number n of photons emitted in 1 s by a source of unitary power, becomes

$$n = (n_r\nu) = P/h; \quad \text{for } P = 1\text{W} \quad \Longrightarrow \quad n = h^{-1} \ (\cong 1.5\times10^{33} \text{ photons/s}) \tag{16}$$

so, the ***inverse* of Planck's *constant*** turns out to be the photons number emitted in 1 s by a source of unitary power.

Then, in accordance with the massiveness of the light, the momentum of a ray of photons becomes

$$\mathbf{p} = m\mathbf{c} = \gamma\nu\,\mathbf{c} = \gamma\,\mathbf{c}/T \tag{17}$$

and therefore, during the impact *photons-circling electron*, the conservation of momentum $m\mathbf{c} = m_e\mathbf{w}$ gives the electron the *radial* velocity **w**, having, see also Fig. 1(**b**), the value

$$w = mc/m_e = \gamma\nu c/m_e \implies w/c\ (\equiv \beta_e) = m/m_e = \gamma\nu/m_e\,. \tag{18}$$

3 - Invariance of *c* during the interaction light-matter: let *S* be a source of photons impacting a structure *R* from where, see Fig. 2(**a**), the incident light ($\nu, \lambda,\ c$) is re-emitted (ν', λ', c') and let $\mathbf{v}_{RS} = \mathbf{v} = 0$**;** the re-emission, if photons are not absorbed, has to start at the same time of impact and therefore, the interaction time (absorption + re-emission), becomes $T' = 1/\nu'$.

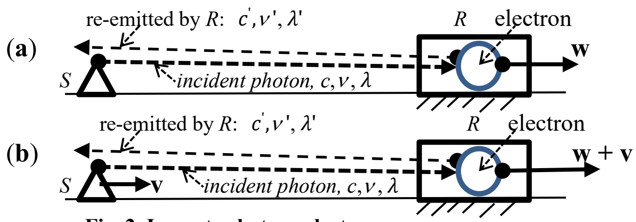


Fig. 2. Impacts photons-electron

During the impact, meanwhile the electron is circling around its nucleus (at rest with respect to *R*), since the electron radial velocity **w** has same direction, see Fig. 1(**b**) and also Fig. 2(**a**), as the incident photons, it turns out that, due to Doppler effect, the re-emitted photons frequency becomes $\nu' = \nu(1 - w/c)$, where *w* is given by Eq.(18), hence

$$\nu' = \nu(1 - w/c) = \nu(1 - \frac{\gamma\nu c}{m_e c}) = \nu(1 - \frac{\gamma\nu}{m_e}) = \nu(1 - \frac{m}{m_e}) = \nu(1 - \beta_e),\ [\text{where } \frac{m}{m_e} \equiv \beta_e]. \tag{19}$$

The speed *w* of the electron is acting during the whole *interaction time* $T' = 1/\nu'$, and therefore the increase of the re-emitted photon length becomes

$$\Delta\lambda = wT' = T'\left(\gamma\nu\frac{c}{m_e}\right) = \frac{cT'm}{m_e} = cT'\beta_e = \beta_e\lambda' \implies \lambda' = \lambda + \Delta\lambda = \lambda + \beta_e\lambda' \implies \lambda' = \lambda/(1 - \beta_e) \tag{20}$$

and finally $c' = \nu'\lambda' = \nu(1 - \beta_e)\lambda/(1 - \beta_e) = c$; in short, ν' decreases, λ' increases, yielding $c' = c$.

Let now the source *S* move toward *R*, see Fig. 2(**b**) with speed $v = |\mathbf{v}|$**;** the total electron radial speed w_t becomes $w_t = w + v$, hence, the previous Eq. (19) $\nu' = \nu(1 - \beta_e) \rightarrow \nu'_t = \nu[1 - (\beta_e + \beta_S)]$ where $\beta_S = v/c$ yielding

$$\nu'_t = \nu(1 - \beta), \quad \text{where } \beta \equiv \beta_e + \beta_S. \tag{21}$$

Therefore, during $T' = 1/\nu'$, the increase of the length of the re-emitted photon corresponds now to Eq. (20) plus the additional value given by the motion of the source *S* toward *R*, hence the total increase of λ becomes

$$\Delta\lambda_t = (w + v)T' = (\gamma\nu\frac{c}{m_e} + \beta_S c)T' = \beta_e\lambda' + \beta_S\lambda' = \beta\lambda' \tag{22}$$

and since $\lambda' = \lambda + \Delta\lambda_t = \lambda + \beta\lambda' \implies \lambda' = \lambda/(1 - \beta)$ we get

$$\nu'_t\lambda' = \nu(1 - \beta)\lambda/(1 - \beta) = c \tag{23}$$

showing that, **nevertheless of any relative motion source-observer**, after any interaction light-matter, *c* **is invariant**, as it happens during the measurements of *c* which, requiring a double path, implies an interaction. On the Appendix, we also obtain the same result ($\nu'_t\lambda' = c' = c$) throughout the conservation of energy of the incident photons.

This everlasting equality $c = u$ has a ***Cosmological Reason***: Indeed, since both photons and neutrinos have same speed c and, as then shown, same mass, should their speed be $c > u$, all the ordinary masses (being attracted by these particles) will also move toward the infinity, and therefore they would be finally dispersed toward the infinity; on the contrary, if $c < u$ all the ordinary masses would collapse, while, for $c = u$, both photons and neutrinos, see also § 9, will ensure an *endless* balance between dispersion and collapse.
Moreover, at different potential, the variation of c can also be inferred by the variation of the counted time of atomic clocks (ACs) in altitude (*claimed as Time Dilation*) with respect to equal ACs on the ground .

4. The *"Time Dilation"* observed on atomic clocks (ACs) in altitude

ACs in altitude are increasing their ticking time T_h (their photons specific emission time) with respect to identical clocks on the ground, that is $T_h > T_0$; this increase, on our results, because of the variation of c in altitude, has ***a different meaning with respect to* the *so-called time dilation***. At this regard, let us consider the orbital speed of GPS satellites[4], where the daily counted time of the on-board ACs clocks has to be decreased by $t_{1d} \cong 38$ μs/day, necessary to obtain synchronism between the on-ground ACs clocks and the on-board ACs clocks. In details, the orbit of GPS satellites, being r_0 the Earth radius and h their height from the Earth surface, is $r_h = (r_0 + h)$ hence, between the ground and the orbit r_h, the variation of gravitational potential

$$\Delta U = U_h - U_0 = (-M_E G/r_h) + (M_E G/r_0) = M_E G h/r_h r_0 \quad (24)$$

with $M_E = 5.96 \times 10^{24}$ kg, $r_0 = 6.37 \times 10^6$ m, $h = 20{,}200$ km, $r_h \cong 26{,}600$ km, $G = 6.67 \times 10^{-11}$ m^3/kg s^2, gives

$$\Delta U = 5.96 \cdot 10^{24} \times 6.67 \cdot 10^{-11} \times 20.2 \cdot 10^6/(26.6 \cdot 10^6 \times 6.37 \cdot 10^6) = 4.73 \cdot 10^7 \text{ m}^2/\text{s}^2 \quad (25)$$

hence

$$c_h = \sqrt{-2U_h} = \sqrt{-2(U_0 + \Delta U)} = \sqrt{c_0^2 - 2\Delta U} = c_0\sqrt{1 - 2\Delta U/c_0^2} \cong c_0(1 - \Delta U/c_0^2) = c_0(1 - 5.2 \cdot 10^{-10}), \quad (26)$$

a decrease of c. Indeed, in altitude, the observed increase of the ticking time T_h of atomic clocks, with respect to their value (T_0) on the ground, corresponds to a decrease, from ν_0 to ν_h $(= 1/T_h)$, of their emitted photons frequency, and therefore we inferred that this decrease of ν implies the same decrease of c (from c_0 to c_h), hence

$$\nu_h = \nu_0(1 - \Delta U/c_0^2) = \nu_0(1 - 5.2 \cdot 10^{-10}) \rightarrow T_h = T_0/(1 - 5.2 \cdot 10^{-10}) \cong T_0(1 + 5.2 \cdot 10^{-10}), \quad (27)$$

that is an increase of T, from T_0 to T_h, obtaining, from Eqs. (26), (27),

$$\lambda_h = \frac{c_h}{\nu_h} = \frac{c_0(1-\Delta U/c_0^2)}{\nu_0(1-\Delta U/c_0^2)} = \frac{c_0}{\nu_0} \Rightarrow \lambda_h = \lambda_0 \quad (28)$$

showing the invariance of the length of photons emitted, by equal ACs, either on ground or in altitude (see left part of Fig 3); then, since the increase of their ticking time during the unit time is

$$(T_h - T_0) = \Delta T_{1\text{sec}} = \Delta U/c_0^2 = 4.73 \cdot 10^7/c_0^2 = 5.2 \cdot 10^{-10} \text{ s}, \quad (29)$$

it turns out that the increase of the counted time of the h-clocks (in altitude), in one day, $\Delta T_{(h)\text{-1day}}$, will become

$$\Delta T_{(h)\text{-1day}} = 86400 \text{ s/day} \times 5.2 \cdot 10^{-10} \text{ s} = 45.5 \text{ μs}. \quad (30)$$

Then, the satellites orbital speed (corresponding to two orbits/day) is $v = 3{,}874$ m/s, hence the photons emitted on-board by the ACs, when reaching the ground, being $\beta \cong v/c_0 = 1.29 \cdot 10^{-5}$, will have a speed equal to

$$c_0^{'} = \sqrt{c_0^2 + v^2} = c_0\sqrt{1+\beta^2} \cong c_0(1+\frac{\beta^2}{2}) \tag{31}$$

so that, the transit time $T_0^{'}$ of one photon reaching the ground is

$$T_0^{'} = T_0/(1+\frac{\beta^2}{2}) \cong T_0(1-\frac{\beta^2}{2}) \tag{32}$$

hence the *daily* counted time, (due to the satellites speed v), because of the ***increased*** *c* during the path *h*-ground, is

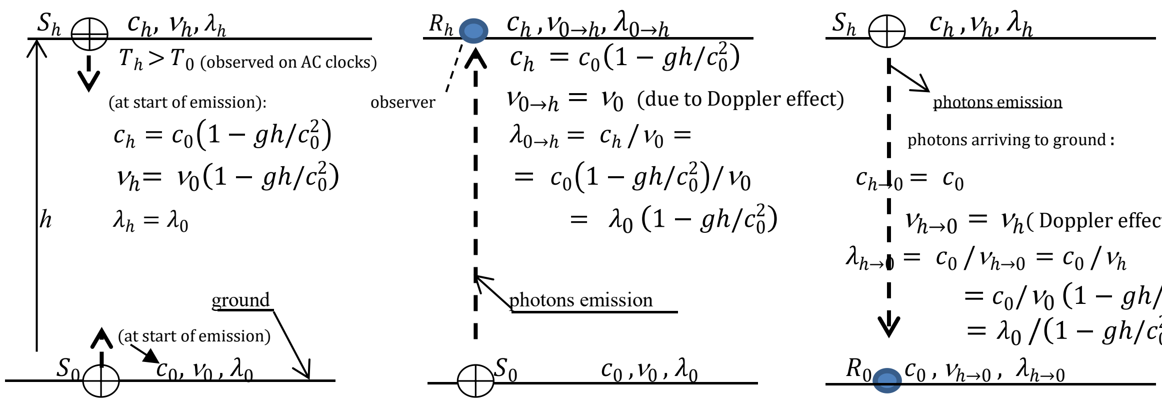
$$\Delta T_{(v)\text{-1day}} = T_0^{'} - T_0 = -T_0\frac{\beta^2}{2} = -86400 \times \frac{(1.29\cdot 10^{-5})^2}{2}\text{s} = -7.2\ \mu\text{s} \tag{33}$$

and finally

$$\Delta T_{\text{1day}} = \Delta T_{\text{(h)-1day}} + \Delta T_{\text{(v)-1day}} = 45.5 - 7.2 = 38.3\ \mu\text{s} \tag{34}$$

as measured.

In short: In altitude, the increase of the ACs ticking time, (hence the decrease of their frequency), as well as the invariance, (see Eq.(28), $\lambda_h = \lambda_0$***, of the photons length, yields*** $c_h = \lambda_h \nu_h$ ***and since*** $c_0 = \lambda_0 \nu_0$***, we have*** $\frac{c_h}{c_0} = \frac{\nu_h}{\nu_0} = T_0/T_h \Rightarrow c_h = \frac{c_0 T_0}{T_h} = \lambda_0/T_h$ ***showing that the increase of ACs ticking time in altitude, is a clear evidence of the decrease of c;*** moreover, Eq.(31) implies a variation of *c*. Fig. 3 shows the variations of the parameters of the light emitted by two sources, S_0 and S_h both at rest from the Observers R_0, R_h, at different potential.



Source S_0 on the ground ($c = c_0$) Source S_h at the altitude *h*.

$c_h, \nu_{0\to h}, \lambda_{0\to h}$ are the parameters of the photons when reaching *h*.

$c_0, \nu_{h\to 0}, \lambda_{h\to 0}$ are the parameters of photons reaching the ground.

Fig. 3 – Parameters of photons emitted either from S_0 or S_h. Sources *S* and observers *R*, at reciprocal rest.

5. H atom parameters

Along each electron circular orbit r_n, the H atom spectrum, being ν_0 its highest frequency, satisfy the relation

$$\nu_n = \nu_0/n^2 \quad (n = 1,2,3,\ldots) \tag{35}$$

and being the photons frequency their number absorbed/emitted, along one direction during the unit time, it turns out that n^2 is the number of photons absorbed (or released) along the electron nth circular orbit; (indeed, photons are admitted, or released, during two (2) turns of the same electron orbit, as then shown).

On Fig. 4(**a**), the electron is circling around the electron-proton common center of gravity *B* along the orbit r_B, while the proton (mass m_p) is circling along r_P. In accordance with the two-body problem, see Fig. 4(**b**), the mass $(m_p + m_e) \equiv M$ is centered in *B*, while the electron, still orbiting around *B*, has a reduced mass[5] $m_r = m_e/(1+\frac{m_e}{m_P})$; on these bases, we can write

$\frac{m_e}{m_P} \equiv \varepsilon_m \Rightarrow \left(1+\frac{m_e}{m_P}\right) = (1+\varepsilon_m) = 1.000544; \quad \frac{m_r}{m_p} = \frac{m_e}{(1+\varepsilon_m)}\frac{1}{m_p} = \frac{m_e}{m_p(1+\varepsilon_m)} = \frac{\varepsilon_m}{(1+\varepsilon_m)}; \; \frac{m_r}{m_e} = \frac{1}{(1+\varepsilon_m)}$. (36)

The *electron charge*, like the electron mass, has also to be reduced, on configuration (**b**), to $-e_r =- e/(1+\frac{m_e}{m_P}) = - e/(1+\varepsilon_m)$. Anyhow, according to our electron structure, the reduced electron charge (still facing the atom nucleus during the electron revolution), is also still circling, see Fig. 4(**b**), along the orbit $r_0 = r_c$; therefore, applying the equality between the *electron centrifugal force (F_c)* and the *Coulomb force* F_C, (both of them can be considered applied, see Fig. 4(**b**), in the electron center), we may write

$$\frac{m_r v_1^2}{r_1} = \frac{e_r \, . \, e}{4\pi\varepsilon_0 r_0^2} = \frac{e^2/(1+\varepsilon_m)}{4\pi\varepsilon_0 r_0^2} \; . \tag{37}$$

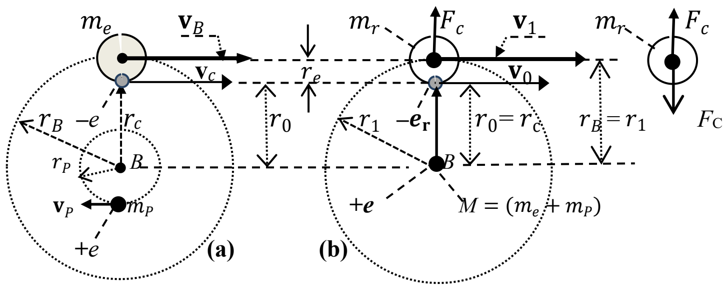


Figure 4. H atom representations

(**a**) - observed from the electron-proton common center of gravity B.
(**b**) **-** observed from B with the electron having a reduced mass m_r, while its charge is also reduced to $-\, e_r =- e/(1+\varepsilon_m)$.

Then, see 4(**b**), since $r_1 = r_0 + r_e = r_0(1 + r_e/r_0) \equiv r_0(1+\varepsilon_r)$, where $r_e/r_0 \equiv \varepsilon_r$, Eq. (37) becomes

$$\frac{m_r v_1^2}{r_1} = \frac{e^2/(1+\varepsilon_m)}{4\pi\varepsilon_0 \, r_0^2} \Rightarrow m_r v_1^2 = \frac{e^2 r_1}{4\pi\varepsilon_0 \, r_0^2(1+\varepsilon_m)} \tag{38}$$

and since $m_r = m_e/(1+\varepsilon_m)$ and being $r_1 = r_0(1+\varepsilon_r)$, one gets $m_e v_1^2 = \frac{e^2 r_0\,(1+\varepsilon_r)}{4\pi\varepsilon_0\, r_0^2} = \frac{e^2(1+\varepsilon_r)}{4\pi\varepsilon_0\, r_0}$; then, being $v_1 = v_0(1+\varepsilon_r)$, we have

$$m_e v_0^2(1+\varepsilon_r)^2 = \frac{e^2(1+\varepsilon_r)}{4\pi\varepsilon_0\, r_0} \;\Rightarrow\; m_e v_0^2(1+\varepsilon_r) = \frac{e^2}{4\pi\varepsilon_0\, r_0} \;\Rightarrow\; \frac{1}{2} m_e v_0^2 = \frac{e^2}{8\pi\varepsilon_0\, r_0(1+\varepsilon_r)} = \frac{e^2}{8\pi\varepsilon_0\, r_1} \tag{39}$$

where $½\, m_e v_0^2 \;\hat{=}\; ½\, m_r v_1^2$: indeed, $m_e v_0^2 = m_r(1+\varepsilon_m)v_1^2/(1+\varepsilon_r)^2$ and since, see next Eq.(70), $\frac{1}{(1+\varepsilon_r)} = 137\alpha$, we have $m_e v_0^2 = m_r(1+\varepsilon_m)137\alpha\, v_1^2 = 1.000017\, m_r v_1^2 \;\hat{=}\; m_r v_1^2$ corresponding to the electron binding energy $W_0 = hcR_\infty = h\nu'_0$, being ν'_0 the *incident* photons frequency along the electron g-s orbit, yielding

$$W_0 = ½\, m_e v_0^2 = \frac{e^2}{8\pi\varepsilon_0\, r_1} = hcR_\infty = h\nu'_0 \;\Rightarrow\; v_0 = (2hcR_\infty/m_e)^{1/2} = 2\,187\,691 \text{ m/s} \tag{40}$$

which, from the relation $R_\infty = R_H \frac{(m_e+m_p)}{m_p} = R_H(1+\varepsilon_m)$, even obtained on Eq. (56), can also be written as

$$v_0 = (2hcR_\infty/m_e)^{½} = (2hcR_H(1+\varepsilon_m)/m_e)^{½} = (2hcR_H/m_r)^{½} = (2h\nu_0/m_r)^{½} = 2\,187\,691 \text{ m/s} \tag{41}$$

whose value equals (*exactly*) αc; moreover, on H atom, according to the CODATA evaluated value[6] of the electron binding energy $W_b =$ 13.605691 eV, we find, from 1st part of Eq.(40), $v_0 = (2W_b/m_e)^{1/2} = 2\,187\,691$ m/s, same value of Eq. (41), confirming both $W_b = W_0$ and also the above value $v_0 = ac$; hence we can write

$$v_0 = ac. \tag{42}$$

Then, still from Eq. (40), we have

$$r_1 = \frac{e^2}{8\pi\varepsilon_0\ hcR_\infty} = \frac{e^2}{2\varepsilon_0\, hc}\ \frac{1}{4\pi R_\infty} = \frac{a}{4\pi R_\infty} = 5.291772 \times 10^{-11}\ \mathrm{m} \tag{43}$$

which, see Fig 4 where $r_1 = r_B$, equals the Bohr radius value. Then, still referring to Fig. 4(**b**), the ratio ν'_0/f_1, (where $f_1 = \frac{v_1}{2\pi r_1} = f_0 = \frac{v_0}{2\pi r_0}$ is the electron frequency along its g-s orbit), becomes $\nu'_0/f_1 = \nu'_0/(\frac{v_1}{2\pi r_1})$, and since $r_1 = r_0(1+\varepsilon_r)$, and being $\varepsilon_r \ll 1$ (hence $r_0 \cong r_1$), just for the ratio ν'_0/f_1, we may write

$$\nu'_0/f_1 = \frac{\nu'_0}{\frac{v_1}{2\pi r_1}} = cR_\infty/\frac{v_1}{2\pi r_1} = cR_\infty/\frac{v_0}{2\pi r_0} = cR_\infty/\frac{ac}{2\pi r_0} \cong cR_\infty/\frac{ac}{2\pi r_1} \cong 2\pi r_1 R_\infty/a \cong 0.499999 \tag{44}$$

practically corresponding ($\nu'_0/f_1 = 0.5$) to the absorption of *half* (incident) photon during one (1) electron orbit and since the ***number of photons has to be an integer***, *we have to infer*

$$\delta_0 = 2\nu'_0/f_0 \equiv 1 \tag{45}$$

meaning that, along the electron g-s orbit, the absorption time of one (1) incident photon lasts **two** electron equal orbits; (between different circular orbits, the electron moves along spiral orbits, as per Rydberg formula). We also point out that, according to Eq. (44), we can write $4\pi r_1 R_\infty/a = 1$ exact, confirming Eq. (43), $r_1 = \frac{\alpha}{4\pi R_\infty}$.

6. Absorption of admitted photons and their emission

Let first examine the influence of the circling proton during the photons *absorption/emission*: on Fig. 4(**b**), the electron kinetic energy $K_1 = ½\, m_r v_1^2$ must be equal, (conservation of energy), see Fig.4(**a**), to both the proton kinetic energy $K_p(= ½\, m_p v_p^2)$ and the electron kinetic energy $K_e\ (= ½\, m_e v_B^2)$, hence we can write

$$K_1 = K_e + K_p \quad\Rightarrow\quad ½\, m_r v_1^2 = ½\, m_e v_B^2 + ½\, m_p v_p^2 \tag{46}$$

which, due to the conservation of momentum $m_p v_p = m_e v_B \ \Rightarrow\ v_p = m_e v_B/m_p$, gives

$$m_r v_1^2 = m_e v_B^2 + m_p(\frac{m_e v_B}{m_p})^2 = m_e v_B^2\left(1+\frac{m_e}{m_p}\right) \quad\Rightarrow\quad v_1^2 = (m_e/m_r)v_B^2(1+\varepsilon_m) \tag{47}$$

and since, see Eq.(36), $(1+\varepsilon_m) = m_e/m_r$, we have $v_1^2 = v_B^2\ (m_e/m_r)^2$ yielding

$$v_1 = v_B\, m_e/m_r \quad\Rightarrow\quad v_B = v_1\, m_r/m_e = v_1/(1+\varepsilon_m)\ . \tag{48}$$

Then, as $R_\infty = R_H(1+\varepsilon_m)$, writing Eq.(43) as $r_1 = \frac{e^2}{8\pi\varepsilon_0\, h\nu'_0} = \frac{e^2}{8\pi\varepsilon_0\, h\nu_0(1+\varepsilon_m)}$ and since from Eq. (35), $\nu_n = \nu_0/n^2$ or even $\nu'_n = \nu'_0/n^2$, we get

$$r_1 = \frac{e^2}{8\pi\varepsilon_0\, h\nu_n n^2(1+\varepsilon_m)} \quad\Rightarrow\quad r_1 n^2 = \frac{e^2}{8\pi\varepsilon_0\, h\nu_n\,(1+\varepsilon_m)} \equiv r_n \tag{49}$$

where r_n is the electron nth circular orbit; then, generalizing Eq. (41), along r_n we may write

$$v_n^2 = 2h\nu_n/m_r = 2h\nu_0/n^2 m_r = v_0^2/n^2 \quad\Rightarrow\quad v_n = v_0/n \tag{50}$$

where n is the progressive number of the orbit r_n.

Moreover, Eq. (46) means that the electron kinetic energy $K_1 = ½\, m_r v_1^2$ equals the electron binding energy W_1, so we may write

$$K_1 = ½\, m_r v_1^2 \;=\; W_0 = \; W_1 \equiv h\nu'_0\,, \tag{51}$$

therefore, see Fig. 5 (left side, absorption), ν'_0 turns out to be the *incident* photons frequency moving the electron from r_0 to r_i; moreover, the energy $h\nu'_0$ has to be equal, see Eq.(46), to both the emitted photons energy $K_e = h\nu_0$

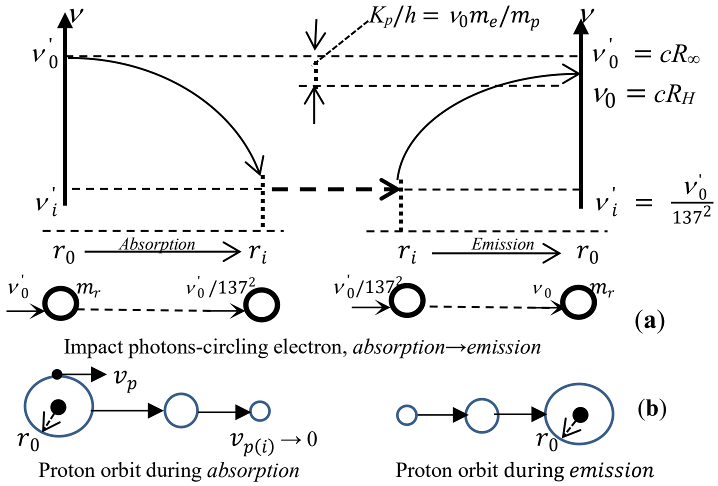


Figure 5 – H atom (photons absorption/emission).

and the related proton kinetic energy K_p, yielding $h\nu'_0 = h\nu_0 + K_p$; hence, referring to Fig. 4(**a**), and being $v_p = \frac{m_e v_B}{m_p}$, we get $K_p = ½\, m_p \left(\frac{m_e v_B}{m_p}\right)^2 = ½\, m_e v_B^2 \frac{m_e}{m_p}$ which, see (48), $v_B = v_1\, m_r/m_e$, can be written as:

$$K_p = ½\, m_e\,(v_1 m_r/\, m_e)^2\, \frac{m_e}{m_p} = ½\, \frac{m_e^2}{m_p} v_1^2 m_r^2/m_e^2 \;=\; ½\, m_r v_1^2 m_r/m_p = \frac{K_1\, m_r}{m_p} \tag{52}$$

which, see Eq. (36), $\frac{m_r}{m_p} = \frac{\varepsilon_m}{1+\varepsilon_m}$, and since, as also shown, $R_\infty = R_H(1+\varepsilon_m) \;\Rightarrow\; \nu_0 = \nu'_0/(1+\varepsilon_m)$, gives

$$K_p = K_1\,\varepsilon_m/(1+\varepsilon_m) = \;\varepsilon_m\, h\nu'_0\,/(1+\varepsilon_m) = \varepsilon_m\, h\nu_0 = h\nu_0 \frac{m_e}{m_p} \tag{53}$$

hence the previous equality $h\nu'_0 = h\nu_0 + K_p$, becomes $h\nu'_0 = h\nu_0 + h\nu_0 \frac{m_e}{m_p}$ and finally to

$$\nu'_0 = \nu_0 + \;\nu_0\, \frac{m_e}{m_p} = \nu_0 \left(1 + \frac{m_e}{m_p}\right) = \nu_0\,(1+\varepsilon_m) \quad\Rightarrow\quad cR_\infty = cR_H(1+\varepsilon_m). \tag{54}$$

[We can also get the same above results as follows: from CODATA[7], the Rydberg *frequency* $cR_\infty = ½\frac{\alpha^2 c^2}{h}\, m_e$, being our Eq.(42) $v_0 = \alpha c$, and being, see Eq.(36), $m_e = m_r(1+\varepsilon_m)$, becomes

$$cR_\infty = ½\frac{v_0^2}{h}\, m_e = ½\frac{v_0^2}{h} m_r(1+\varepsilon_m) \quad\Rightarrow\quad cR_\infty/(1+\varepsilon_m) = ½\frac{v_0^2}{h} m_r \tag{55}$$

and since, from Eq.(41), $v_0^2 = \frac{2h\,\nu_0}{m_r} \;\Rightarrow ½\frac{v_0^2}{h} m_r = \nu_0 (= cR_H)$, the Eq.(55) gives $\frac{cR_\infty}{1+\varepsilon_m} = \nu_0 = cR_H$ yielding

$$R_\infty = R_H(1+\varepsilon_m) \tag{56}$$

and then

$$\nu_0 = cR_H = cR_\infty/(1+\varepsilon_m) \quad\Rightarrow\quad \nu_0(1+m_e/m_p) = cR_\infty \equiv \;\nu'_0 \tag{57}$$

where ν'_0 turns out to be the *incident* photons frequency, showing the difference between *emission* (ν_0) and *absorption* (ν'_0) frequencies along the electron ground-state orbit.]

7. H atom: electron radial velocity, number of electron circular orbits, electron radius

Referring to previous Fig. 4(**b**), during *absorption*, the electron radial speed along the generic orbit r_n, due to the impact of *one* (1) admitted photon, see Eq. (18), is

$$w'_n = \gamma\nu'_n c/m_r = \nu'_n 2\gamma c^2/2cm_r = 2h\,\nu'_n/cm_r \tag{58}$$

and since, along the orbit r_n, the photons admitted frequency is $\nu'_n = \nu'_0/n^2$, meaning that the number of admitted incident photons is n^2, the electron total radial speed (due to n^2 photons) becomes

$$w'_{n^2} = \frac{n^2 2h}{cm_r}\frac{\nu'_0}{n^2} = \frac{2h\,\nu'_0}{cm_r} \tag{59}$$

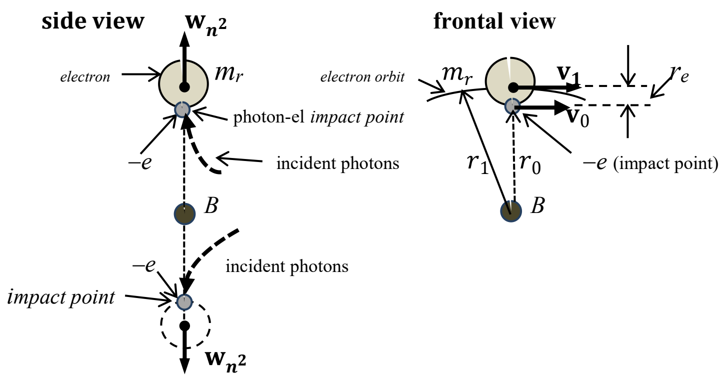


Fig. 6. H atom, photons absorption

constant along each nth orbit. Indeed, because of the impacts photons-electron, the electron moves toward the ionization orbit r_i, hence the ionization condition, along the orbit r_i where, see Eq. (50), $v_i = v_1/n_i$, becomes

$$w_{n^2}\left(= \frac{2h\nu'_0}{cm_r}\right) = v_i(= v_1/n_i) \tag{60}$$

where n_i is the progressive number of the circular ionization orbits, hence we have

$$\frac{2h\nu'_0}{cm_r} = v_1/n_i \quad \Rightarrow n_i = v_1 cm_r/2h\nu'_0 \tag{61}$$

and since, see Eq. (42), $c = v_0/\alpha$, and being, see Fig. 6, $v_0 = \frac{v_1}{(1+\varepsilon_r)}$, we get

$$n_i = \frac{v_1 v_0 m_r}{\alpha 2h\nu'_0} = \frac{v_1 v_1\, m_r}{(1+\varepsilon_r)\alpha 2h\nu'_0} = m_r v_1^2/2h\nu'_0(1+\varepsilon_r)\alpha \tag{62}$$

and being, see Eq. (51), $h\nu'_0 = ½\, m_r v_1^2$, we have

$$n_i = \frac{1}{(1+\varepsilon_r)\alpha} \cong 137 \tag{63}$$

but n_i **has to be an integer**, so we can infer n_i = 137 **exact**, that is the progressive number of the widest circular orbit (r_{137}). Thus, as $(1+\varepsilon_r) = 1/137\alpha$ and since $v_0 = \alpha c$, and being $v_1 = v_0(1+\varepsilon_r)$, we have

$$\boxed{v_1 = v_0(1+\varepsilon_r) = \frac{v_0}{137\,\alpha} = \frac{\alpha c}{137\alpha} = \frac{\mathbf{c}}{\mathbf{137}} = 2\,188\,266 \text{ m/s}} \tag{64}$$

while from Eq. (48),

$$v_B = \frac{v_1}{(1+\varepsilon_m)}\,\frac{\frac{c}{137}}{(1+\varepsilon_m)} = \frac{\frac{c}{137}}{1.0005446} = 2\,187\,075 \text{ m/s} \tag{65}$$

then,

$$v_c = \frac{v_B}{(1+\varepsilon_r)} = v_B 137\,\alpha = \frac{\frac{c}{137}}{1.0005446}137\alpha = \frac{\alpha c}{1.0005446} = 2\,186\,500 \text{ m/s} \tag{66}$$

while, as showed,

$$v_0 = \alpha c = 2\,187\,691 \text{ m/s.} \tag{67}$$

Then, see Fig.4, being $r_1 = r_0(1+\varepsilon_r) = r_0/137\alpha$, the electron charge g-s orbital speed r_0 can be written as

$$r_0 = 137\alpha\, r_1 = 137\,\alpha\frac{\alpha}{4\pi R_\infty} = \frac{137\alpha^2}{4\pi R_\infty} = 5.2903815 \times 10^{-11} \text{ m} \tag{68}$$

while the Bohr radius may also be written as

$$r_1 = r_0(1+\varepsilon_r) = \frac{r_0}{137\alpha} = \frac{a}{4\pi R_\infty} = 5.291772 \times 10^{-11} \text{ m.} \tag{69}$$

Then, from Eq. (63), we have

$$(1+\varepsilon_r) = \frac{1}{137\alpha} \;\Rightarrow\; \varepsilon_r = \frac{1}{137\alpha} - 1 = 0.000262777 \tag{70}$$

and being $r_e = \varepsilon_r r_0$ we find

$$r_e = \varepsilon_r r_0 = r_0\left(\frac{1}{137\alpha} - 1\right) = 1.3901905 \times 10^{-14} \text{ m} \tag{71}$$

rather different from the value claimed by Codata, $r_e = \alpha^2 a_0 = 2.82 \times 10^{-15}$ m; then, see Fig. 6, we may also get the Bohr radius as follows: $r_1 = r_0 + r_e = 5.2903815 \times 10^{-11} + 1.3901905 \times 10^{-14} = 5.291772 \times 10^{-11}$ m.

8. Number of incident photons admitted on H atom along the electron circular orbits

Along the ionization orbit # 137, where the electron radial speed due to the impact of *one incident* photon, as shown by Eq.(58), is $w'_{137} = \frac{\left(\gamma\, \mathcal{V}'_{137}\, c\right)}{m_r}$, and where, see Eq. (35), $\mathcal{V}'_{137} = \mathcal{V}'_0/137^2$, it turns out that, along $r_{137,}$ the electron total radial speed will become $N_{137}\, w'_{137}$ being N_{137} the total number of photons, admitted (or released) along $r_{137,}$ which depends on both the number *n* of the circular orbits (*n* = 137) and on the same number (n) of admitted photons along $1/\text{n}$th of each circular orbit; (hence along the nth circular orbit, the number of photons necessary to cover, two times, the orbit r_n, is therefore n^2); hence, the ionization condition can be written as $N_{137}\, w'_{137} = v_{137}$ and since the electron orbital speed along r_{137} is equals to $v_{137} = \frac{v_1}{137} = \frac{c}{137^2}$, we obtain

$$N_{137}\, w'_{137} = v_{137} \;\Rightarrow\; N_{137}\, w'_{137} = \frac{c}{137^2} \;\Rightarrow\; N_{137} = \frac{c}{137^2\, w'_{137}} = \frac{c}{137^2\left(\gamma\, \mathcal{V}'_{137}\, c\right)/m_r} =$$

$$= \frac{1}{137^2\left(\gamma\, \mathcal{V}'_0/137^2\right)/m_r} = \frac{m_r}{\gamma\, \mathcal{V}'_0} = \frac{m_r}{\gamma c R_\infty} = \frac{m_r c}{2hR_\infty} = 18769 \;\Rightarrow\; N_{137} = n^2 = 137^2 \textbf{ exact.} \tag{72}$$

Then, since from Eq.(10), $\gamma\mathcal{V} = m$, the Eq.(72) can also be written as $N_{137} = \frac{m_r}{\gamma\,\mathcal{V}'_0} = \frac{m_r}{m'_0} = 137^2$ being m'_0 the mass of the incident photons impacting the circling electron along its g-s orbit. Then, as $N_{137} = 137^2$, and because of Eq. (35) $\mathcal{V}_n = \frac{\mathcal{V}_0}{n^2}$ (*n* = 1,2,3,…137), the number of emitted photons along the electron circular orbits turns out to be n^2 while, regarding the *incident* admitted photons frequency, we have $\nu'_n = \frac{\nu'_0}{n^2} = \frac{cR_\infty}{n^2}$, same number (n^2) as for incident/emitted photons. Hereafter we summarize the H atom main parameters.

Table 1 - H atom parameters, where the electron has a reduced mass m_r, circling around the electron-proton common center of gravity B, see Fig 4.

El. circ. orbits n	Spectrum frequency ν	Incid. photons admitted freq. ν'	El. orbital speed v	El. charge orb. speed v_0	El. orbital radius r_1	El. charge orb. radius r_0	Nmb of photons admitted/released along circ. orbits N
1	$\nu_0 = cR_H$	$\nu'_0 = cR_\infty$	$v_1 = c/137$	$v_0 = \alpha c$	$r_1 = \alpha/4\pi R_\infty$	$r_0 = r_1 137\,\alpha$	1
2	$\nu_2 = \nu_0/4$	$\nu'_0/4$	$v_1/2$	$v_0/2$	$4r_1$	$4\ r_0$	4
nth	ν_0/n^2	ν'_0/n^2	v_1/n	v_0/n	$n^2\, r_1$	$n^2\, r_0$	n^2
137	$\nu_0/137^2$	$\nu'_0/137^2$	$v_1/137$	$v_0/137$	$r_{137} = 137^2\, r_1$	$137^2\, r_0$	137^2

9. Gravitational red/blue shifts

As for the Relativity, the only way to explain high cosmological redshifts, is the Doppler effect (which implies an *incredible* universe expansion at speed $v_u \cong c$) whereas, on our results, red/blue shifts have, mainly, a gravitational reason**:** in fact, as for the light emitted by a Galaxy (having, at its emission, parameters c, λ, ν, where c depends on the value of its local potential U), when its light reaches the Earth, its speed, according to U_0, becomes c_0 and therefore, (disregarding the velocity Galaxy-Earth), which implies $\nu \cong \nu_0$, the shifts can be expressed as

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\Delta c}{c} = \frac{c_0 - c}{c} = \frac{c_0}{c} - 1 = \sqrt{U_0/U} - 1 \tag{73}$$

with U_0 the potential on Earth, U on the source. Thus, apart from Doppler effects (which can give some variation to the values obtained through the above equation), z turns out to be the relative variation of c (as well as λ) during the path of light toward different potentials. In particular, see Table # 2, being s the distance Earth-source, for $s < \cong 45$ Mpc (roughly corresponding to $-0.01 < z < +0.01$), if U (potential on the source) is, in *absolute value*, higher than the potential on Earth U_0, the (73) gives, on Earth, $z < 0$ (blue shift), and vice versa for $|U| < |U_0|$ while, in the range $\cong 0.01 < z < \cong 0.20$, the Eq.(73), written as

$$U = U_0/(1+z)^2 \cong U_0/(1+2z) \cong U_0(1-2z) \qquad \text{(valid for } z \ll 1) \tag{74}$$

shows that, for $z \ll 1$, U depends linearly on z, as the Hubble's law; then, for $s \to \infty$, $U \to 0$, hence $z \to \infty$.

[For $s > \cong 45$ Mpc, the observed values of z are always positive, hence we may argue that our galaxy is rather close to the centre of the masses of universe (where $|U|$ has the max value)]. Hereafter we show some values related to the variation of U, given by NASA[8], on the bases of the observed shifts related to the galaxies estimated distances from the Earth.

Table # 2. **Calculated values of both U and c related to the observed shifts on Earth.**

blue/redshift	z	s (Mpc)	$U/U_0 = 1/(z+1)^2$	$U/U_0 \cong 1-2z$ valid for z << 1	$c/c_0 \cong 1/(z+1)$
blue/red shift	− 0.01 → 0.01	< ≅ 45	0.98-1.02	0.98 - 1.02	0.99 - 1.01
red shift	≅0.01	≅45	0.98	0.98	0.99
red shift	0.20	900	0.69	0.60	0.83
red shift	1		0.25		0.50
red shift	→∞	→∞	→0		→0

10. Photons, neutrinos and their contemporary nuclear emission

Finally, the massiveness of light gives also a different meaning to the famous $E = mc^2$**:** during the nuclear reaction n + p → d + γ where γ is a detected photon, there is a *claimed*, (as photons have been claimed to be massless), loss of a mass m (which is measured) in accordance with the relation $E = mc^2$ regarded as an equivalence mass-energy whereas, on our results, because of the massiveness of the light (photons), during the nuclear reactions (where photons are emitted), the conservation of momentum also implies the release of a corresponding particle: This *elusive* massive particle, whose speed has been found to be equal to c, is the neutrino, and since its speed c is equal to the total escape speed, the neutrinos, like the photons, must also have a mass and, because of the conservation of energy during their emission, they must have the same mass as the contemporary emitted photons, but *contrary direction.* Therefore, as for *nuclear* emissions of light we should write

$$E = mc^2 = ½\, mc^2 + ½\, mc^2 = K_c + K_\nu \tag{75}$$

where K_c regards the light, while $K_\nu = ½\, mc^2$ the neutrinos, whose existence, on our opinion, has, as reason, the same fundamental support (like photons) to the "***cosmological reason***": indeed, being $\mathbf{u}_S$ the *relative* escape velocity (referred to their source S) of the photons emitted during nuclear reactions, should S have a velocity $\mathbf{v}_{BS}$ with respect to the universe masses center B, (where the *effective* escape velocity has to be referred to surely tend to infinity), and since (referred to B) the *effective* escape velocity is $\mathbf{u} = \mathbf{u}_S + \mathbf{v}_{BS}$, it turns out that $\mathbf{u}_S$ could not be equal to $\mathbf{u}$; hence, during the primary emission of photons, there is also a contemporary emission of the same number of neutrinos whose direction is contrary to each emitted photon, and therefore, one of the two particles will tend to the infinity (complying with the "cosmological reason").

Appendix - *Equality $c' = c$ during the interaction light-matter, through the conservation of energy*

Referring to previous Fig. 2, here recalled, let S be a source of photons (parameters c, ν, λ,) impacting a structure R from where the incident light, during the interaction time t_i is re-emitted (c', ν', λ'). Let be, first, $\mathbf{v}_{RS} = 0$**.** Now, the conservation of energy of the incident photons, gives

$$h\nu - h'\nu' = m_e w^2/2 \tag{A1}$$

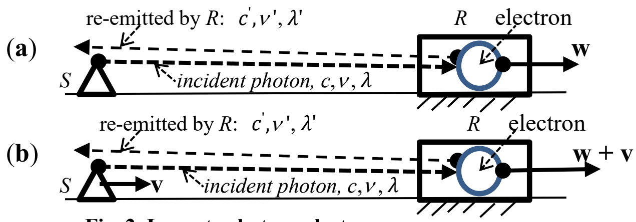


Fig. 2. Impacts photons-electron

being, see Eq.(12), $h\nu = \frac{1}{2}\gamma c^2 \nu$ the incident photons energy, $h'\nu' = \frac{1}{2}\gamma c'^2 \nu'$ the re-emitted one, $m_e w^2/2$ the electron radial kinetic energy (due to the impact photons-*circling electron*) and where the electron radial speed w, due to the photons impact, see Eq. (18), is

$$w = \gamma\nu c/m_e = mc/m_e \equiv \beta_e c \implies w/c = \beta_e . \tag{A2}$$

Therefore, during the interaction time $t_i = T' = 1/\nu'$, still referring to Fig 2(**a**) where $\mathbf{v}_{RS} = 0$, the increase of the photons length, see Eq.(20), gives $\Delta\lambda = wT'$ while, due to Doppler effect, their re-emitted frequency see Eq. (19), becomes $\nu' = \nu(1 - \frac{w}{c}) \equiv \nu(1 - \beta_e)$; then, being $w^2 = \beta_e^2 c^2$, Eq.(A1) can be written as

$$\frac{1}{2}\gamma c^2 \nu = \frac{1}{2}\gamma c'^2 \nu' + \frac{1}{2} m_e \beta_e^2 c^2 \tag{A3}$$

which, as $m_e = \frac{m}{\beta_e}$ and since $m = \gamma\nu$, becomes

$$\gamma c'^2 \nu' = \gamma c^2 \nu - m_e \beta_e^2 c^2 \implies \gamma c'^2 \nu' = \gamma c^2 \nu - \frac{m}{\beta_e}\beta_e^2 c^2 \implies \gamma c'^2 \nu' = \gamma c^2 \nu - \gamma\nu\beta_e c^2 . \tag{A4}$$

Then, as $\nu' = \nu(1 - \beta_e)$, we find $c'^2 \nu(1 - \beta_e) = c^2\nu - \nu\beta_e c^2$ yielding

$$c'^2 (1 - \beta_e) = c^2(1 - \beta_e) \implies c' = c. \tag{A5}$$

Let now S move toward R with speed $v = |\mathbf{v}|$, see Fig. 2(**b**); the electron radial speed referred to R becomes $w_t = w + v$, while the photons frequency, from the above relation $\nu' = \nu(1 - \beta_e)$, will become now $\nu'_{tot} = \nu[1 - (\beta_e + \beta_S)]$ where $\beta_S = v/c$ and calling $(\beta_e + \beta_S) \equiv \beta$ we can write

$$\nu'_{tot} = \nu(1 - \beta). \tag{A6}$$

Therefore, during $t_i = T' = 1/\nu'$, the total increase of the length of the re-emitted photon corresponds now to the above $\Delta\lambda = wT'$ plus the additional value given by the motion *S-R*, hence

$$\Delta\lambda_{tot} = (w + v)T' = T'\left(\gamma\nu\frac{c}{m_e}\right) + \beta_S c T' = \beta_e \lambda' + \beta_S \lambda' \equiv \beta\lambda' \implies \lambda' = \lambda + \beta\lambda' \implies$$

$$\lambda' = \lambda/(1 - \beta) \tag{A7}$$

and finally

$$c' = \lambda' \nu'_{tot} = \nu(1 - \beta)\lambda/(1 - \beta) = c. \tag{A8}$$

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NGC 0063, z ≅ + 0.004 with $s \cong 20$ Mpc; VCC0815, z ≅ – 0.0025 with $s \cong 20$ Mpc.