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REMARKS ABOUT THE EULER'S THEORY REGARDING A BODY WITH A FIXED POINT

The Chandler's periods of latitude variation

Monica Zoe Ciobanu, retired astronomer
Astronomical Institute of Romanian Academy
Email: mzciobanu@yahoo.com

Abstract. The paper presents the necessity to assume that the Earth is a rigid body when using the Euler's theory regarding the solid-rigid body with a fixed point, in the nutation problems. Also, the paper suggests which may be those external forces causing the two Chandler's periods regarding the terrestrial variation of latitude. An opinion about the Chandler's period of 426 days is presented.

Key words: nutation, the Euler's mechanical moment related to the Earth's center, Chandler's periods.

1. Introduction

After Bradley (1693 – 1762) detected a wobble in the ERA (Earth's Rotation Axis) having an 18.6 years period, a great difficulty appeared for the astronomers. Thus, after Copernic, Kepler and Newton, in problems regarding the dynamic of celestial bodies, the Earth was considered as being a simple point, having no volume but only mass and, therefore, the variation in the direction of its ERA could not be explained; this difficulty was solved by Leonard Euler (1707 – 1783) in his theory of a solid-rigid body with a fixed point, published in 1790 (1).

Leonard Euler presented his theory regarding a solid-rigid body with a fixed point, in a very abstract form. Certainly, Euler understood very well that the nutation phenomenon discovered by Bradley is connected to the retrogradation of the lunar nodal point, but he conceived his fundamental law as to be an universal one - in fact, that means not only for the Earth-Moon case. Now, about three centuries after Bradley, the amount of the observational data and the related precision, increased; then, even more now, using the Euler's theory related to solid-rigid body with a fixed point, each term is important and must be respected.

Naturally, the only possible motion of a solid rotating body with a fixed point is a rotation around an axis inside of its body, which passes through that fixed point. For the case when the body is a rigid one, the axis must be considered as being fixed in the body itself and usually is chosen as being one of the reference axes in the mobile frame of axes, as the ERA is itself. In fact, due to the Euler's theory, the only variation of ERA is related to an inertial frame. For instance, a simple school geographical globe, permanently rotating, after moving it or balancing it in a room, it changes the position of its axis related to the room's objects but not inside the globe itself.

The fundamental equation of the classical mechanics in this case is:

$$D\overrightarrow{K}/Dt = \overrightarrow{M}Fex$$

where $\overrightarrow{K}$ is the kinetic momentum of the solid-rigid body with a fixed point, $D\overrightarrow{K}/Dt$ is the differential of $\overrightarrow{K}$ relative to the fixed frame of coordinates and $\overrightarrow{M}Fex$ is the Euler's mechanical momentum related to the fixed point of the resultant of those external forces which do not pass through the fixed point.

Obviously, if the Euler's mechanical momentum $\overrightarrow{M}Fex$ is null, the direction of the Earth's kinetic momentum is the same as that of the ERA and a free nutation (a nutation with no external forces) cannot exist.

In fact, a mechanical momentum of an external force around a point is defined as the vectorial product between the vector of the external force and its vector distance to the fixed point, the resultant vector being perpendicular to both.

From a simple vectorial point of view, a wobble of the ERA vector may be caused by a new vector component in the Earth's equatorial plane; if this wobble becomes a nutation whose periodicity is similar to a phenomenon which appears in a dynamic of two or three bodies, then the equatorial component of $\overrightarrow{M}Fex$ is due to an external force.

Therefore:

- a nutation may imply the existence of a component of the Euler's mechanical momentum $\overrightarrow{M}Fex$ in the equatorial plane;

- if no external force exists (as in the Euler-Poinsot case), the kinetic momentum vector remains located on the ERA direction inside the rigid body, continuing its permanent position, and a free nutation cannot exist (2).

2. A particular case, supposing a celestial body having a circular orbit situated in the ecliptic plane

Let us consider that the origin of the mobile references axes Oxyz and the origin of the ecliptic axes OXYZ are in the Earth's gravity center located in the ecliptic plane, the angle between the axes OZ and Oz being noted „obl" (23^o,5).

It is well known that a celestial body running on an orbit around the Earth is not only subject of a Newtonian force but it is also subject of the tangential forces on its orbit.

In the beginning, we suppose a fictive celestial body which, under the Earth's attraction, runs in a simple circular orbit plane located in the ecliptic plane and having a yearly constant orbital period. In accordance with the Euler's theory, the tangential component of this celestial body's force could cause a mechanical momentum related to the fixed point (the Earth's center); then, the Euler's mechanical momentum $\overrightarrow{M}Fex$ will be located on the positive ecliptic axis OZ.

It is well known that, during a year, the Euler's $\overrightarrow{M}Fex$ components on the mobile frame of axes depends on the declination of the celestial body related to the celestial equatorial plan.

When the celestial body crosses for an instant the equinox line, the projection of the Euler's momentum $\overrightarrow{M}Fex$ on the celestial equatorial plan is null, and then no wobble exists.

If the declination of the celestial body is $+23^o$,5 or -23^o,5, then the equatorial component of the Euler's momentum becomes -MFex sin23^o,5 and respective +MFex sin23^o,5; these two equatorial components may cause the most important wobble of ERA. Naturally, in this case, the Earth's kinetic momentum will be located between ERA and $\overrightarrow{MFex}$, these three vectors being coplanar. Therefore, in this case,

- if a wobble does not exist, the Earth, supposed as being a rigid body, could have only its own daily rotation around ERA, as in the equinoctial instants,

- if a wobble exists, when the equatorial component of the Euler's momentum $\overrightarrow{MFex}$ is not null, the Earth will rotate together with its ERA around the new kinetic momentum position, describing a yearly nutation.

3. Simple remarks about the yearly nutation

During a single orbital period, the tangential forces acting on ERA may cause in each moment only one instant wobble; if the orbital parameters remain the same during a great period of time (like a millennium), their action on ERA grows and a periodical nutation can be detected.

Naturally, if the celestial body could also be supposed as having a great mass, it may cause a yearly variation of terrestrial latitude.

In 1892, Chandler announced the discovery of the annual variation of terrestrial latitude (3).

Obviously, the yearly nutation discovered by Chandler is due to the tangential forces of the apparent Sun's orbit.

From the astrometric data, it is known until now that the greatest value for a nutation is 9'',21 (due to the retrogradation of the lunar nodal point). The yearly latitude variation announced by Chandler is around 0'',1.

From a cosmological point of view, it is interesting to remark that, when a celestial body is orbiting around the Earth, then:

- as the inclination of the orbit of the celestial body, related to the celestial equatorial plan, is greater, the wobble of ERA will be greater;

- if the orbit's inclination is null, then no wobble exists and that means that the orbit of the celestial body is situated only in the celestial equatorial plane;

- each instant wobble of ERA depends on the component of the Euler's mechanical moment, $\overline{M}Fex$, on the equatorial plane;

- the Earth must be supposed as being a solid body, to be able to explain the Earth's tides, in accordance with the Newton's laws;

- the Earth must be supposed as being a rigid body, to be able to explain a variation of ERA related to an inertial plane, in accordance with the Euler's theory.

4. Some details about the Moon's orbit inclination

The well known periodical and beautiful astronomical phenomena are the phases of the Moon; however, a terrestrial latitude variation of around four weeks period has not been detected.

Naturally, supposing the Moon as being ejected in the Precambrian geological time, the Earth diminishing its Newtonian attraction (4), the Moon's orbit began to constantly grow under the Sun's attraction and, step by step (in an infinitesimal manner), could have intersected later the ecliptic plane and could have caused the well known phenomenon of lunar nodal point retrogradation.

Bradley, looking for an annual star parallax to prove the Copernic's heliocentric theory, remarked a variation of stars coordinates related to the celestial equatorial plane, but not a yearly period; by continuing long time his astronomical observations, he remarked a period of 18,6 years in stars declinations, similar to that of the retrogradation period of the lunar nodal points.

Supposing the Earth as being a rigid body, each periodic variation of the ERA's direction is connected to the oscillation of its equatorial plane with the same period which, in turn, implies the same variation in the stars declinations – in this way, the first nutation phenomenon was detected.

Astronomers know very well that, when the longitude of the nodal point is 360^o, the Moon's declination related to the celestial equatorial plane has the variation between $+28^o,5$ and $-28^o,5$, in a monthly period of around four weeks; when the longitude of the nodal point decreases to 180^o degrees, the Moon's declination has a variation between $+18^o,5$ and $-18^o,5$. Therefore, during 18,6 years, the Moon's orbit inclination related to the celestial equatorial plane changes from $28^o,5$ to $18^o,5$.

In fact, during 18,6 years the Moon runs in its about four weeks orbit and changes continuously its orbit inclination related to the celestial equatorial plane. Therefore, in each instant the Euler's mechanical moment $\overrightarrow{M} Fex$, due to the tangential forces of the Moon's orbit, is unique in position and acts only one instant on ERA, being not in the same position in its periodical orbit during 18,6 years; and more, the great eccentricity of the Moon's orbit imposes variation even inside the value of the Euler's moment $\overrightarrow{M} Fex$, because the distance Earth – Moon changes between 360 000 km to 405 000 km in its monthly period.

In fact, during 18,6 years the Moon changes continuously its orbit's inclination related to the celestial equatorial plane. Therefore, each wobble is different and acts only one instant on ERA, so that it is too weak to cause a detectable periodical nutation.

5. **The Chandler's period of 14 months. A simple opinion**.

Naturally, if a period of around 14 months in the latitude variation was detected by Chandler, this must be caused by a periodic change in the ERA direction. Corresponding to the Euler's theory, this must be caused by some periodical external forces having a 14 months period. A question arises: which is that astronomical phenomenon having an around 14 months period?

Supposing that, for a short time, the Moon's orbit is situated in the ecliptic plane, the component of the Euler's moment mechanic $\overrightarrow{M} Fex$ on the equatorial plane could be null when the Moon crosses the equinox line, and this component increases when the Moon's declination starts to increase.

Naturally, changes in the ERA direction may be caused not only by the tangential forces of the Moon's orbit, but also by those strong forces which cause the yearly nutation. But during a month period, there are moments when the yearly wobbles and the wobbles caused by the tangential forces of the Moon's orbit may act simultaneously together - this is happening in the syzygy short moments; the wobble resultant may then have extreme different values due to the great eccentricity of the Moon's orbit.

But there are some syzygy moments when the wobble resultant has approximately the same value - this is happening in the Sun-Moon conjunction moments, when the New Moon is situated in the perigee position. These syzygy moments happen with a period of 814 days.

When the New Moon is in a perigee position, it needs 814 days to be again a New Moon in a perigee position. Indeed, 29,54 days (the synodic period) x 27,55 days (the anomalistic period) = 814 days.

For simplicity, supposing that the Full Moon in a perigee position, it also needs 814 days to be again a Full Moon in a perigee position, too. But the Full Mon is situated around at half a time and half a way between two New Moon's position, on the Moon's orbit. Then, between those two moments when the Full Moon or the New Moon are being in a syzygy position and in a perigee position, the period could be 407 days (around 13,5 months).

Is the time interval of 13,5 months between two syzygy moments when the Moon is in a perigee position, causing the Chandler's 14 months variation of latitude? Is this why this specific wobble, having the same value, is acting on ERA with a period of 13-14 months?

Due to the great advance of perigee (+40^{o} yearly) and to the retrogradation of the lunar nodal point (-10^{o} yearly), the wobbles, caused by the Moon, change continuously in direction and magnitude in a 18,6 years period. But there are some moments, with around 14 months period, in the Earth–Sun-Moon syzygy position, when the Moon is in a perigee position; then, acting together with the wobbles caused by the yearly nutation, the resultant wobble on ERA has approximately the same value and could cause a latitude variation.

6. **Conclusion**

The paper's purpose is to present (a) some details regarding the Euler's theory about a solid-rigid body with a fixed point, and (b) some remarks about those external forces which may change the Earth's Rotation Axis (ERA) direction, acting periodically on the Earth's latitudes.

Indeed, the main external forces which could act on ERA direction are (a) the tangential forces of the Moon's orbit and (b) those around the apparent Sun's orbit (in reality, around the Earth's orbit).

Summarizing, the yearly latitude variation is caused by the yearly ERA periodical variation in direction related to the inertial ecliptic frame; then, the kinetic moment represents the instantaneous North Pole being (in that moment) not situated on the same direction as ERA. The wobbles caused by the tangential forces of the apparent moving of the Sun, cause the yearly periodical nutation. These wobbles are important because they keep the same periodic intensity values from millennia ages ago.

But, naturally, the ERA is continuously submitted to both the yearly permanent wobble and those variables caused by the tangential forces of the Moon.

A special moment happens when the Sun's apparent tangential forces act simultaneously together with the Moon's tangential forces; this may happen when the Earth, the Moon and the Sun are in a syzygy moment. In these syzygy moments, due to the great eccentricity of the Moon's orbit, the wobble may have a variable value.

But there are some special syzygy moments when the wobble has approximately the same value and this happens only when the Moon, being in that moment of syzygy, is also in a perigee position. These special syzygy instants may have a period of 13,5 months, like the period found by Chandler in the latitude variation of 14 months.

To conclude: the Chandler's periods regarding the Earth's latitude variation may be due to the tangential forces of Moon's orbit and, also, to the tangential forces of the orbit of the apparent Sun movement. When these external forces act simultaneously together in some moments of the syzygy astronomical phenomena, it results the same value for the wobble only if the Moon is situated in a special position of its orbit, namely in a perigee position. Is the periodical apparition of that same wobble, with a 13,5 month's period, the phenomenon which causes the Chandler's well known latitude variation of 14 months?

And more, the Chandler's periods can be explained only assuming that the Earth is a rigid body, as in the Euler's theory of a solid-rigid body with a fixed point.

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