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The Crux of the Cosmological Gravitational Field Equation

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Abstract

This paper reveals the core issues inherent in Einstein's cosmological field equations and their solutions, the Friedmann equations, through an in-depth analysis. The study found that to construct models of a static or accelerating expanding universe using cosmological field equations, two key assumptions must be introduced: first, the pressure in the equation set may need to be negative; second, the added cosmological constant corresponds to negative mass matter, which can exert repulsive gravity or anti-gravity on ordinary matter. Ordinary fluid pressure and radiation pressure both correspond to positive mass matter, describing states that completely align with cosmological equations built upon Newtonian mechanics and thus cannot explain the phenomenon of the universe's accelerating expansion.The research points out that introducing concepts such as negative pressure and negative mass violate fundamental principles of physics, and thus lack sufficient theoretical support.According to Newton's law of universal gravitation, even if negative mass matter truly existed, based on the positive-negative mass ratio set by current theories of cosmic acceleration, the main constituent of the universe would be negative mass matter—this would only exert attractive universal gravitation on negative mass matter, causing the universe to undergo continuous accelerated contraction rather than expansion. Even from a new perspective of *the matter in Newtonian static gravitational field*, such negative mass matter cannot be derived. Therefore, the universe cannot possibly accelerate under the repulsive gravity of negative pressure and negative energy matter. By applying Newton's law of universal gravitation, it is proven that the cosmological field equations describing a finite universe cannot accurately depict the true state of the universe and should be completely discarded. Within the framework of the infinite universe hypothesis, applying Newton's law of universal gravitation resolves the Neumann-Seeliger paradox. The established cosmological field equations are Laplace's equations, which can describe an infinite static universe and possess intrinsic harmony. This is entirely consistent with the static universe theory supported by Type Ia supernova Hubble diagrams and light curves.

Keywords: cosmological gravitational field equations, Friedmann's equations, cosmological constant, dark energy, negative mass matter, negative pressure, repulsive universal gravitation, antigravity, Laplace equation

1. Introduction

The theoretical framework of modern standard cosmology (the ΛCDM model) is built upon four core pillars: the cosmological principle, the cosmological gravitational field equations of general relativity, the hot Big Bang evolutionary scenario, and observational constraints on dark matter and dark energy.

Einstein founded the general theory of relativity in 1916 and, the following year (1917), extended the gravitational field equations to a cosmic scale, proposing the cosmological gravitational field equations[1].These equations replace Newtonian mechanics' absolute spacetime with curved spacetime described by Riemannian geometry, defining gravity as either 'the response of spacetime curvature to matter and energy' or 'the intrinsic energy density of the spacetime vacuum'. To align with the prevailing assumption of a 'static universe' at the time, Einstein intentionally introduced a cosmological constant term. In subsequent cosmological studies, he explained that this term's physical significance was precisely 'the intrinsic energy density of spacetime itself', whose repulsive effect exactly balanced the gravitational attraction between matter, thereby maintaining the universe's static nature.

In 1922, Russian physicist Alexander Friedmann, while studying Einstein's field equations, abandoned the assumption of a static universe and instead introduced the most general form of the spacetime metric based on the cosmological principle—that space is homogeneous and isotropic. He employed the Robertson-Walker metric to describe cosmic geometry. After substituting this metric into Einstein's field equations and deriving them separately for the time and space components, he ultimately obtained two core equations. Friedmann broke through the inherent notion of a static universe and first revealed mathematically the solution for a dynamic universe; these equations were used to describe the dynamics of the overall evolution of the universe. The Friedmann equations support core theories such as the Big Bang model, cosmic expansion, and dark energy, and for the first time established a systematic theoretical framework for modern cosmology [2].

After Edwin Hubble discovered the cosmological redshift and interpreted it as an expansion phenomenon in 1929 [3], Einstein admitted that introducing the Λ term was 'the biggest blunder of his life' and abandoned the constant term in 1931.

The Friedmann equations consist of 10 components, forming a second-order nonlinear partial differential equation system that is extremely difficult to solve. Exact solutions exist only for a few special cases, such as spherical symmetry. The system involves up to 15 unknown functions, but there are only 6 independent equations, requiring additional conditions for resolution. It is often oversimplified and misinterpreted as 'gravity = spacetime curvature,' but its essence reveals the geometric relationship between gravity and the distribution of matter mass—curvature is merely a manifestation of this relationship, not the nature of gravity itself. Although Einstein introduced Λ to maintain the assumption of a static universe, the equations inherently predict the dynamic evolution of the universe (expansion or contraction), which sharply conflicted with the prevailing cosmological views of the time. This also led some scholars to question the applicability of general relativity on a cosmic scale and propose modifications.

In 1998, two independent teams—the Supernova Cosmology Project and the High-Z Supernova Search Team [4][5]—observed that distant Type Ia supernovae were dimmer than theoretically expected. Based on this observation, researchers inferred that these supernovae were actually farther away than predicted, a phenomenon interpreted as evidence of an accelerating universe. To resolve this counterintuitive puzzle of cosmic acceleration, the scientific community introduced the concept of 'dark energy'—a mysterious energy that permeates cosmic space, possesses negative pressure, can counteract the binding force of gravity, and drives the universe to expand continuously. The cosmological model that incorporates dark energy and cold dark matter has now become the standard model for describing the evolution of the universe.

Why did the concept of 'dark energy' emerge in cosmology, seemingly contradicting fundamental principles of physics? Exploring its nature has thus become one of the most pressing research topics in modern physics. What exactly is dark energy? Can humans unravel its mysteries? Do existing cosmological theories contain flaws? If so, what are their root causes? Furthermore, are there issues with the cosmological equations themselves and their Friedmann solutions? Where exactly lies the crux of the problem?

2. Solutions to the Gravitational Field Equations of Cosmology

2.1 Cosmological Gravitational Field Equations

General relativity interprets gravity as the curvature of spacetime. Einstein proposed the following assumptions: the universe is finite, with matter uniformly distributed, appearing homogeneous and isotropic (i.e., without a center or special direction) on sufficiently large scales; simultaneously, there exists a pressure within the cosmic space. Based on these assumptions, he formulated the cosmological gravitational field equations. After prolonged argumentation and refinement, the generally accepted form of the cosmological gravitational field equations is as follows [1]:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

In this equation, c is the speed of light, G is the gravitational constant, $R_{\mu\nu}$ is the linearized Ricci tensor (spatial curvature tensor), $g_{\mu\nu}$ is called the metric tensor, $T_{\mu\nu}$ is called the stress-energy tensor, and Λ was later referred to as the cosmological constant. The term $\Lambda g_{\mu\nu}$ is known as the cosmological constant term, and Λ serves to balance $(-8\pi G/c^4)T_{\mu\nu}$. R refers to the Ricci scalar (also called the curvature scalar), which is a numerical measure of the overall curvature of spacetime. It is obtained by further contracting (i.e., taking the trace) of the Ricci tensor $R_{\mu\nu}$, resulting in $R=g^{\mu\nu}R_{\mu\nu}$.

The form of the energy-momentum tensor for an ideal fluid in the universe is [6]:

$$T_{\mu\nu}=(\rho c^2+p)u_\mu u_\nu+pg_{\mu\nu} \tag{2}$$

The solution to Equation (1) is not unique but takes various forms, such as the Einstein static solution, Friedmann solutions, Lemaître Solution [7], Schwarzschild-de Sitter Solution[8], Einstein-de Sitter solution [9], Gödel solution [10], the Steady-State Solution by Bondi, Gold, and Hoyle [11][12], Kerr-de Sitter Solution [13], Anti-de Sitter (AdS) solutions [14][15], and others. Among these, the Friedmann solutions, as the cornerstone of the Big Bang cosmology framework, are currently the most widely accepted. These equations reveal that the universe may be expanding, contracting, or oscillating, directly challenging Einstein's proposed static universe model. Initially, Einstein opposed this conclusion and even dismissed it as a 'mathematical error,' but he eventually publicly accepted it in 1923. However, this research remained theoretical and did not yet establish connections with specific physical mechanisms. To date, none of the solutions to these equations perfectly aligns with the cosmic evolution scenario required by the Big Bang theory.

2.2 Friedmann Solution

For a homogeneous and isotropic universe, the FLRW (Friedmann-Lemaître-Robertson-Walker) metric is adopted [16]:

$$ds^2=-c^2dt^2+a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right) \tag{3}$$

where $a(t)$ is the scale factor, and k is the spatial curvature constant (k = +1, 0, −1 corresponds to closed, flat, and open universes respectively).

The core set of equations for the Friedmann solutions to the cosmological equations has the following form:

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G\rho}{3}+\frac{\Lambda c^2}{3}-\frac{kc^2}{a^2} & (4-1) \\ \frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\left(\rho+\frac{3p}{c^2}\right)+\frac{\Lambda c^2}{3} & (4-2) \end{cases}$$

This system of equations clearly predicts the time evolution of the cosmic scale factor $a(t)$, revealing that space itself has an extensible nature. Cosmic expansion is not the motion of galaxies within a fixed space, but rather the dynamic evolution of the spatial metric over time, a fundamental property entirely determined by the geometric structure of general relativity. The dependent variable $a(t)$ in the equations is the cosmic scale factor, where ρ represents the cosmic matter density, p is the pressure, and k is the cosmic curvature, which can only take specific discrete values of k=1,0,−1.

When $\Lambda=0$, the behavior of the solution is determined by k and ρ (such as expanding, contracting, or critical universes). When$\Lambda\neq0$, the cosmological constant provides a 'repulsive gravity' that can lead to a static and accelerating universe.

The scale factor $a(t)$ describes the overall expansion and contraction of the universe: $\dot{a}>0$ corresponds to expansion, and $\dot{a}<0$ corresponds to contraction.

When solving a system of differential equations, it is necessary to conduct a classification discussion based on different scenarios of the parameters in the equations. Analysis shows that solutions to the system of equations only exist under specific parameter conditions.

In the history of cosmology, the general relativity cosmological equations and Friedmann equations have given rise to numerous solutions, among which those of key significance to the Big Bang cosmology include:

1） In 1927, Georges Lemaître independently solved Einstein's field equations without having been exposed to Friedmann's research results and arrived at a cosmological expansion solution consistent with Friedmann's. He was the first to interpret this solution as the 'primeval atom explosion' hypothesis, which was precisely the prototype of the Big Bang theory. The single-component ideal model he proposed, with curvature k = 0, ±1, offered an intuitive physical image. It not only became a widely disseminated foundational paradigm in cosmology education but also laid the groundwork for entry-level frameworks in the field [7]. However, the limitations of this model were also evident—it could not accurately depict the evolutionary trajectories of multi-component matter in the real universe. In 1931, he put forward the 'Primeval Atom' hypothesis: the universe originated from an extremely dense, highly radioactive 'super atom,' whose explosive decay initiated the evolutionary process of space-time and matter. This concept directly gave birth to the idea of the ‘Big Bang’.

2） The paper published by Boyle and Turok in October 2022 first presented an analytical solution to the most general form of the Friedmann equations, which covers the parameter space of arbitrary curvature k, cosmological constant Λ, and equation-of-state parameter w. Its core idea is to view the early universe as a non-equilibrium thermodynamic system, leveraging the principle of entropy increase and the natural tendency toward thermodynamic equilibrium to drive the universe to spontaneously evolve toward a homogeneous and isotropic state. This breakthrough represents a significant advancement in theoretical completeness and has become a benchmark solution in the field. The achievement is regarded as a key refinement of the Big Bang theory, resolving a long-standing mathematical challenge that had persisted since Friedmann proposed these equations in 1922. However, its functional form is relatively complex, and further transformation and simplification are needed for physical interpretation [17].

2.3 De Sitter solution

The de Sitter solution is a special solution for vacuum（$\rho=0$，$p=0$）with flat space（$k=0$）. Substituting $\rho=0$，$p=0$ and $k=0$ into the Friedmann equations yields:

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{\Lambda c^2}{3} \tag{5}$$

Integrating gives:

$$a(t)=a_0 e^{H_0 t} \tag{6}$$

where

$$H_0=\sqrt{\frac{\Lambda c^2}{3}} \tag{7}$$

is the Hubble constant.

The corresponding metric is:

$$ds^2=-c^2dt^2+e^{2H_0t}(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2) \tag{8}$$

The de Sitter universe is an exponentially expanding vacuum universe devoid of matter and radiation, driven solely by the cosmological constant[8] [18]. It serves as an approximate model for the current universe (the accelerated expansion phase dominated by the cosmological constant) and is also used to describe the inflationary process of the early universe. The de Sitter solution can also be considered a special case of the Friedmann solutions.

Different approaches hold foundational significance for the establishment and verification of the Big Bang cosmological model (ΛCDM model).

The following will explore solutions that differ somewhat from mainstream views, focusing on three aspects: 1) the static solution; 2) negative pressure; 3) the cosmological constant.

3. Solutions to the Friedmann Equations

3.1 Compatibility of Equations

Before solving the Friedmann equations, perform a consistency check on the equation set. Let the cosmic pressure be p, the cosmic matter density be ρ, the speed of light be c, and the matter state parameter be w. Then the cosmic matter state equation is

$$p=w\rho c^2 \tag{9}$$

The relationship between the density ρ of matter and the cosmic scale factor a is as follows

$$\rho=\frac{D}{a^n} \tag{10}$$

In the formula, D is a constant related to the mass of the universe, and n is a constant related to the state of matter.

Let Λ be related to the cosmic scale factor as

$$\frac{\Lambda c^2}{3}=\frac{\lambda}{a^{n'}} \tag{11}$$

where λ is a constant.

Substituting equations (9), (10), and (11) into (4-2) gives

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\frac{(3w+1)D}{a^n}+\frac{\lambda}{a^{n'}} \tag{12}$$

Multiplying both sides of equation (12) by a, equation (12) is transformed into

$$\ddot{a}=-\frac{4\pi G}{3}\frac{(3w+1)D}{a^{n-1}}+\frac{\lambda}{a^{n'-1}} \tag{13}$$

The left side of equation (13) can be rewritten as

$$\ddot{a}=\frac{d\dot{a}}{da}\frac{da}{dt}=\dot{a}\frac{d\dot{a}}{da}=\frac{1}{2}\frac{d\dot{a}^2}{da} \tag{14}$$

Substituting equation (14) into the left-hand side of equation (13) yields

$$\frac{d\dot{a}^2}{da}=-\frac{8\pi G}{3}\frac{(3w+1)D}{a^{n-1}}+\frac{2\lambda}{a^{n'-1}} \tag{15}$$

After integration, obtain

$$\dot{a}^2=\frac{8\pi G}{3}\frac{(3w+1)D}{(n-2)a^{n-2}}-\frac{2}{n'-2}\frac{\lambda}{a^{n'-2}}+C \tag{16}$$

Divide both sides by a^2 to get

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\frac{(3w+1)D}{(n-2)a^n}-\frac{2}{n'-2}\frac{\lambda}{a^{n'}}+\frac{C}{a^2} \tag{17}$$

Since the left side of this equation is identical to the left side of Equation (4-1), its right side must also correspondingly equal the right side of Equation (4-1), as can be obtained by comparison

$$\begin{cases} n=3(w+1) & (18-1) \\ n'=0 & (18-2) \\ C=-kc^2 & (18-3) \end{cases}$$

On the other hand, the continuity equation is

$$d(\rho a^3)+pda^3=0 \tag{19}$$

Substituting equations (8) and (9) into equation (19) yields

$$n=3(w+1) \tag{20}$$

This is consistent with Equation (18-1).

In the formula, n and w are undetermined, but n depends on the equation of state parameter w, and they are consistent with each other. Since w represents the state relationship between pressure and matter, for ordinary matter, $0\le w\le 1/3$, and for radiation pressure, $w=1/3$. Therefore, for matter, the range of n can only be $n=3\sim4$. However, the equation only constrains the relationship between n and w, without determining specific values.

In this way, due to the system of equations' compatibility requirement, the two equations take the following form:

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\frac{D}{a^n}+\frac{\Lambda c^2}{3}-\frac{kc^2}{a^2} & (21-1) \\ \frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\frac{(n-2)D}{a^n}+\frac{\Lambda c^2}{3} & (21-2) \end{cases}$$

Since n cannot be determined, D also lacks an exact expression, and thus the form of the system of equations is uncertain.

Due to compatibility requirements, only one of these two equations is actually needed.

3.2 The Case Where the Matter Density is Constant

When n=0, $w=p/\rho c^2=-1$, the gravitational term is a constant, and the form of the Friedmann equations becomes

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}-\frac{kc^2}{a^2} & (22-1) \\ \frac{\ddot{a}}{a}=\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3} & (22-2) \end{cases}$$

1) Static Solution

If the left sides of the equation system satisfy $\ddot{a}=0$ and $\dot{a}=0$, then their right sides are required to

$$\begin{cases} \frac{8\pi G\rho}{3}+\frac{\Lambda c^2}{3}-\frac{kc^2}{a^2}=0 & (23-1) \\ \frac{8\pi G\rho}{3}+\frac{\Lambda c^2}{3}=0 & (23-2) \end{cases}$$

Subtracting the left sides of the two equations yields the equation

$$\frac{kc^2}{a^2}=0 \tag{24}$$

There are two possible scenarios for its solution.

If $k = 0$, then

$$a=C_1 \tag{25}$$

where C_1 is an arbitrary constant, including infinity.

If $k\neq 0$, the solution to equation (24) is

$$a\equiv\infty \tag{26}$$

As can be seen, if k takes a non-zero value, the conclusion that $a\equiv\infty$ holds; if k=0, the conclusion that $a\equiv\infty$ still holds. Regardless of the value of k, the relationship $a\equiv\infty$ can be established without any constraints on the related parameter k — this means that an infinitely large universe is not restricted by k. However, an infinitely large universe ($a\equiv\infty$) necessarily leads to the result of zero spatial curvature (k=0), which in turn means that an infinitely large universe is a flat universe. Therefore, for the static universe model, the most reasonable assumption is that the universe is infinitely large.

According to (23-2), the parameters must meet the requirements

$$\begin{cases}\Lambda=-\dfrac{8\pi G}{c^2}\rho & (27-1)\\ p=-\rho c^2 & (27-2)\end{cases}$$

This means that to maintain the static and balanced state of the universe, a repulsive force source is needed to counteract the gravity originating from the matter density ρ .The mass density of the repulsive force source is

$$\rho_\Lambda=-\frac{\Lambda c^2}{8\pi G} \tag{28}$$

Initially, Einstein's original motivation for introducing p-related terms into the equations might have been to describe the properties of ideal fluids in the universe (including the pressure effects produced by radiation) — such fluids not only have positive mass but also gravitational effects. However, constrained by the compatibility conditions of the equations, surprisingly, this term, which was intended to serve as a source of gravity, instead manifested as a source of repulsion, and the mass density corresponding to the cosmological constant could only be negative.

As can be seen, to maintain the universe in a static state, there needs to be a form of negative mass matter that can generate negative pressure, as well as a cosmological constant with negative mass density. The role of this cosmological constant is to produce repulsive gravity, thereby counteracting universal gravitation. In contemporary physics, the cosmological constant Λ is interpreted as 'dark energy', which has negative mass and energy density, with pressure $p_\Lambda=-\rho c^2$, driving the accelerated expansion of the universe.

2) Dynamic Solution

For the dynamic solution, equation (22-2) can be expressed as

$$\ddot{a}-\left(\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}\right)a=0 \tag{29}$$

If $(8\pi G\rho+\Lambda c^2)/3>0$ in the equation, then the solution to the equation is

$$a=a_1e^{\sqrt{\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}}t}+a_2e^{-\sqrt{\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}}t} \tag{30}$$

Set the initial condition such that when$t$=0, a=0. Thus, there is a relationship $a_1+a_2=0$, which means $a_2=-a_1$. Substituting this into Equation (25) gives

$$a=a_0 sh\sqrt{\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}}t \tag{31}$$

Here $a_0=2a_1$. Substituting equation (31) into equation (22-1) gives

$$a_0=c\sqrt{\frac{-3k}{8\pi G\rho+\Lambda c^2}} \tag{32}$$

Substituting equation (32) into equation (31) gives

$$a=c\sqrt{\frac{-3k}{8\pi G\rho+\Lambda c^2}}\cdot sh\sqrt{\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}}t \tag{33}$$

Since the assumption $(8\pi G\rho+\Lambda c^2)/3>0$ holds, it must be required that $k<0$. The negative curvature of the universe implies that the universe is open.

If $(8\pi G\rho+\Lambda c^2)/3<0$, then function (31) is

$$a=a'_1 e^{i\sqrt{-\left(\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}\right)}t}+a'_2 e^{-i\sqrt{-\left(\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}\right)}t} \tag{34}$$

Set the initial condition such that when $t=0$, $a=0$. Thus, there is a relationship$a'_1+a'_2=0$, which means $a'_2=-a'_1$. Substituting this into Equation（34）gives

$$a=a'_0 sin\sqrt{-\left(\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}\right)}t \tag{35}$$

Here $a'_0=2a'_1$. Substituting equation (35) into equation (22-1) gives

$$a'_0=c\sqrt{\frac{3k}{8\pi G\rho+\Lambda c^2}} \tag{36}$$

Substituting equation (36) into equation (35) yields

$$a=c\sqrt{\frac{3k}{8\pi G\rho+\Lambda c^2}}\, sin\sqrt{-\left(\frac{8\pi G}{3}\rho+\frac{\Lambda c^2}{3}\right)}t \tag{37}$$

Since the assumption $(8\pi G\rho+\Lambda c^2)/3<0$ holds, it necessarily requires that $k<0$, meaning the universe has negative curvature, which implies the universe is open.

According to equation (33), to obtain a monotonically increasing dynamic solution, i.e., an accelerating expansion solution, both $k<0$ and $8\pi G\rho+\Lambda c^2>0$ must be satisfied simultaneously.

According to equation (37), to satisfy an oscillatory-type dynamic solution, i.e., an accelerating contraction or decelerating expansion solution, both k<0 and $8\pi G\rho+\Lambda c^2<0$ must be met at the same time.

A common feature of equations (33) and (37) is that the cosmological scale factor a depends on the curvature k. The curvature must satisfy the specific constraint k<0 and cannot take k≥0; if k=0, the coefficient in the dynamic cosmological scale factor will be zero, i.e.,$a_0=0$, so no dynamic solution exists; if k>0, the equation has no solution. Thus, the dynamic solution to the equation can only hold under open universe conditions. Since the possible values of curvature in the Friedmann equations are one of k=−1，0，+1, the only possible value for curvature is that corresponding to the open universe with k=−1.

By solving the cosmological equations under the condition that ρ, p, and Λ are all constants, it is found that:

The static cosmological solution with $k=0$ and $a=\infty$ corresponds exactly to the infinite static universe model of Newtonian mechanics. However, for Einstein to derive a static cosmological solution, he needed to satisfy the specific condition that the relevant parameters meet $p=-\rho c^2=\Lambda c^4/8\pi G$. In the full form of Einstein's field equations, pressure p is a component of the energy-momentum tensor of an ideal fluid, which, along with matter density ρ, describes the contents of the universe and is primarily used to characterize the pressure effects of ordinary matter or radiation. However, constrained by equation compatibility, adding positive ideal fluid pressure is not feasible. Only by introducing negative pressure and a cosmological constant with negative energy density can the self-consistency of the equations be guaranteed.

According to the Friedmann equations, if the universe is finite, a negative pressure is required to maintain its static and expanding state. This pressure does not come from ordinary matter but from negative mass matter. This cosmological model is indeed perplexing: if the universe is expanding, its internal matter density and pressure should decrease over time, yet they remain constant. To maintain the constancy of matter density, pressure, and the cosmological constant, it implies that as the universe expands, ordinary matter, negative-pressure matter, and negative-mass matter corresponding to the cosmological constant will spontaneously and continuously generate within it.

3.3 The Case Where the Matter Density Changes over Time

Equations (22-1) and (22-2) indicate that when n≥1 and certain conditions are met, the matter density decreases gradually as the scale factor $a(t)$ increases. However, this set of equations is not only difficult to understand but also extremely complex to solve. To clarify the physical meaning of these equations and simplify the solving process, simplifying assumptions are made.

3.3.1Solution to the Radiation Condition

When n=4 and w= 1/3 , this case corresponds exactly to the radiation state, and equations (22-1) and (22-2) can be rewritten as

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{D}{a^4} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} & (38-1) \\ \frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\frac{D}{a^4} + \frac{\Lambda c^2}{3} & (38-2) \end{cases}$$

1)Static Solution

Setting $\dot{a}=0$ in equation (38-1) and $\ddot{a}=0$ in equation (38-2) gives

$$\begin{cases} \frac{8\pi G}{3}\frac{D}{a^4} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} = 0 & (39-1) \\ -\frac{8\pi G}{3}\frac{D}{a^4} + \frac{\Lambda c^2}{3} = 0 & (39-2) \end{cases}$$

Solving this gives

$$\frac{k}{a^2} = \frac{2\Lambda}{3} \tag{40}$$

There are two cases for the solution to this equation:

The first case is when both k and Λ are finite non-zero values, and its solution is

$$a = \sqrt{\frac{3k}{2\Lambda}} \tag{41}$$

From this, it can be seen that to maintain the scale factor constant and finite, k must take the value +1; according to the equation's setup, a positive Λ is actually negative gravitational matter, i.e., negative mass matter. Moreover, the smaller the Λ, the larger the cosmic scale factor. As Λ approaches 0, the cosmic scale factor tends toward infinity. This means that without the cosmological constant, the universe must be infinite.

The second case: if Λ=0 and k≠−1, it can be directly solved from (40)

$$a = \infty \tag{42}$$

However, an infinitely large universe necessarily leads to $k=0$. This implies that such a universe is static, does not require a cosmological constant, and is independent of the universe's matter density.

The scenario with a finite scale factor is highly counterintuitive: can the radiation-dominated state maintain a static universe? Unless the universe is understood as a black hole with infinite gravity, which would tightly bind the radiation matter and prevent it from escaping gravitational confinement. If the universe is a black hole, even radiation cannot escape gravity, making cosmic expansion impossible and subsequent discussions on cosmic expansion unnecessary. Therefore, compared to this, the solution of the equations with $\Lambda=0$ and $a=\infty$ is more in line with classical physical intuition.

2)Dynamic Solution

To solve equation (38-1), transform it into the following form

$$\frac{da}{dt}=\pm\sqrt{\frac{8\pi GD}{3a^2}+\frac{\Lambda c^2}{3}a^2-kc^2} \tag{43}$$

This equation is further transformed into

$$\frac{da^2}{\sqrt{\left(a^2-\frac{3k}{2\Lambda}\right)^2+\left(\frac{8\pi GD}{\Lambda c^2}-\frac{9k^2}{4\Lambda^2}\right)}}=\pm 2\sqrt{\frac{\Lambda c^2}{3}}dt \tag{44}$$

The solution to this equation is

$$ln\left[\left(a^2-\frac{3k}{2\Lambda}\right)+\sqrt{\left(a^2-\frac{3k}{2\Lambda}\right)^2+\left(\frac{8\pi GD}{\Lambda c^2}-\frac{9k^2}{4\Lambda^2}\right)}\right]=\pm 2\sqrt{\frac{\Lambda c^2}{3}}t+C \tag{45}$$

where C is the constant of integration. Further solving gives $a(t)$ in the equation as

$$a^2=\frac{3k}{2\Lambda}-\frac{3k}{2\Lambda}ch\left(2\sqrt{\frac{\Lambda c^2}{3}}t\right)+\sqrt{\frac{8\pi GD}{\Lambda c^2}}sh\left(2\sqrt{\frac{\Lambda c^2}{3}}t\right) \tag{46}$$

Finally obtain

$$a(t)=\left[\sqrt{\frac{8\pi GD}{\Lambda c^2}}sh\left(2\sqrt{\frac{\Lambda c^2}{3}}t\right)-\frac{3k}{\Lambda}sh^2\sqrt{\frac{\Lambda c^2}{3}}t\right]^{\frac{1}{2}} \tag{47}$$

To understand this function, consider the following special cases.

If the gravitational term of the matter is neglected, by setting $GD=0$, the above equation becomes

$$a(t)=\sqrt{\frac{-3k}{\Lambda}}\cdot sh\sqrt{\frac{\Lambda c^2}{3}}t \tag{48}$$

This function indicates that the cosmic scale factor undergoes monotonically accelerating growth over time, with the cosmological constant Λ serving as the driving force behind the acceleration. This corresponds to de Sitter spacetime, a scenario of spacetime devoid of matter. In such a case, the scale factor expands at an accelerating rate driven by the cosmological constant.

If $GD\neq0$ while $\Lambda=0$ ， then let $\Lambda\to0$, and equation (47) becomes

$$a(t)=\left(2\sqrt{\frac{8\pi GD}{3}}t-kc^2t^2\right)^{\frac{1}{2}} \tag{49}$$

Equation (49) describes the evolution of the cosmic scale factor over time in the scenario where there is radiation matter but no cosmological constant. This equation indicates that the scale factor $a(t)$ is influenced by the gravitational effects of matter.

Furthermore, assuming that GD = 0 and Λ = 0, equation (48) yields

$$a(t)=\sqrt{-k}ct \tag{50}$$

Equation (50) shows that a purely radiative universe, without matter, gravity, and without a cosmological constant, expands at the speed of light in accordance with normal physical laws.

If the cosmological constant satisfies $\Lambda=0$, the universe will still expand as long as $k=-1$, but the expansion speed will reach the speed of light. Of course, $GD=0$ in the formula refers to the gravitational effect of radiation matter being neglected. Obviously, in this case, even without the cosmological constant as a driving factor, the cosmic scale factor $a(t)$ will still expand at the speed of light—this is not hard to understand, as it is exactly the speed of radiation propagation.

If $\Lambda=0$, $k=0$, and $GD\neq0$, then equation (47) can only be

$$a(t)=\left(\frac{32\pi GD}{3}\right)^{\frac{1}{4}}\sqrt{t} \tag{51}$$

The scale factor can also grow over time, but not at the speed of light, due to the gravity of radiative matter. This is because gravity slows the propagation of radiative matter, resulting in an expansion rate that is less than the speed of light—a finding that may seem counterintuitive.

Because for a radiative universe, there exists a relationship

$$\rho=\frac{D}{a^4} \tag{52}$$

Combining equations (51) and (52) yields

$$\rho=\frac{3}{32\pi G}\frac{1}{t^2} \tag{53}$$

This indicates that the density of radiative matter in the universe decreases in proportion to the inverse square of time.

Equation (47) shows that in a radiation-dominated scenario, the cosmic scale factor grows naturally with radiation, while the cosmological constant serves as the driving force for the accelerated growth of the scale factor.

As t approaches infinity, the function (47) tends to

$$a(t)\approx\left(\sqrt{\frac{2\pi GD}{\Lambda c^2}}-\frac{3k}{4\Lambda}\right)^{\frac{1}{2}} e^{\sqrt{\frac{\Lambda c^2}{3}}t} \tag{54}$$

Ultimately, the scale factor is proportional to the exponential $exp\left(\sqrt{\Lambda c^2/3}t\right)$, which is the same result obtained from the quantum redshift effect, provided that $\sqrt{\Lambda c^2/3}$ is taken as the Hubble constant. The final result of this solution tends toward the de Sitter solution. Thus, the cosmological constant is derived.

$$\Lambda=\frac{3\mathrm{H}_0^2}{\mathrm{c}^2} \tag{55}$$

In fact, the Λ in equation (4-2) generates negative gravity, and thus the matter corresponding to Λ is negative mass matter.

3.3.2 Solutions to the Equation in the Cold Matter State

When n=3 and w=0, equations (22-1) and (22-2) can be rewritten as

$$\begin{cases} \left(\frac{\dot{a}}{a}\right)^2 = \frac{2GM}{a^3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} & (56-1) \\ \frac{\ddot{a}}{a} = -\frac{GM}{a^3} + \frac{\Lambda c^2}{3} & (56-2) \end{cases}$$

1) Static Solution

Let $\ddot{a}$=0 and $\dot{a}$=0, then (56-1) and (56-2) become

$$\begin{cases} \frac{2GM}{a^3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2} = 0 & (57-1) \\ -\frac{GM}{a^3} + \frac{\Lambda c^2}{3} = 0 & (57-2) \end{cases}$$

If $\Lambda \neq 0$, then

$$a = \sqrt{\frac{k}{\Lambda}} = \frac{3GM}{kc^2} \tag{58}$$

This formula requires satisfying the following conditions

$$\begin{cases} \Lambda = \left(\frac{c^2}{3GM}\right)^2 & (59-1) \\ k = +1 & (59-2) \end{cases}$$

The static finite solution requires a cosmological constant and demands that the universe be closed. Equation (59-1) (59-2)reveals that if the universe is in a stationary state, its radius would reach 1.5 times the Schwarzschild radius of a general relativistic black hole.

If Λ=0, then k=0 and a=∞. This indicates that an infinitely large universe is flat and does not require a cosmological constant.

2) Dynamic Solution

Although the system of equations (56-1) and (56-2) has dynamic solutions, explicit expressions cannot be derived. However, the following analysis can be conducted:

Within a limited time frame, the acceleration of the scale factor $a(t)$ is always negative, resulting in an oscillatory behavior. Without the Λ term, $a(t)$ would always oscillate infinitely.

During the process as t approaches infinity, an approximate trend solution is obtained, which is that the scale factor tends to

$$a(t) \propto e^{\sqrt{\frac{\Lambda c^2}{3}}t} \tag{60}$$

Due to the presence of the Λ term, $a(t)$ will gradually tend toward an infinitely expanding state. The result is similar to equation (54). This function also conforms to the de Sitter solution.

As long as the universe is finite, static solutions require the introduction of a Λ term to generate a repulsive force that counteracts the gravitational pull of matter and maintains the universe's balanced state. Dynamic solutions, on the other hand, need the Λ term even more to produce repulsion, and the magnitude of this repulsion must exceed the gravitational pull of matter. As for Big Bang cosmology, the Λ term was introduced because solutions describing an expanding universe were needed.

3.4 Coherent Solution to the Equation

According to the Big Bang cosmological theory, the evolution of the universe can be divided into four key phases: the Planck epoch [19], the radiation-dominated era (during which the main components of the universe were high-energy photons and relativistic particles, and the radiation energy density was much greater than the matter energy density), the matter-dominated era (as radiation energy diminished, the universe expanded and cooled, with matter energy density surpassing radiation energy density, leading to the gradual formation of celestial bodies such as galaxies and stars), and the cosmological constant-dominated era (when the dark energy density corresponding to the cosmological constant becomes dominant, driving the universe to undergo exponential expansion and eventually approach a de Sitter universe).

1） Planck era: In the early stages of the universe's birth, during the Planck time t_p, when quantum effects dominated，the universe's scale was at the Planck scale l_p.

$$t_p = \sqrt{\hbar G / c^5} = 5.39 \times 10^{-44} s \tag{61}$$

$$l_p = \sqrt{\hbar G / c^3} = 1.62 \times 10^{-35} m \tag{62}$$

The energy density of cosmic matter is

$$\varepsilon = \rho(t) c^2 = \frac{c^2}{16\pi G}\left(\frac{3}{2t^2} + \frac{c^5}{G\hbar}\right) \tag{63}$$

However, an exact solution for the cosmic scale factor cannot be derived from cosmological equations.

2）Radiation-dominated era: During the radiation-dominated era (when the age of the universe $t \ll 10^4$ years), the energy of the universe primarily exists in the form of relativistic particles (such as photons and neutrinos), with the equation-of-state parameter w = 1/3. Substituting this into the energy conservation equation yields the energy density $\rho_r \propto a^{-4}$. Plugging this result into the first Friedmann equation (ignoring the cosmological constant Λ and spatial curvature k) leads to a scale factor evolution approximately given by the formula (51): $a(t) \propto t^{1/2}$ [5].

2） Matter-dominated era: Matter-dominated era: During the epoch of the universe when 10^4 years ≤ t ≤ 10^{10} years, non-relativistic matter (such as stars, galaxies, etc.) becomes the dominant component of energy density. The equation-of-state parameter w = 0. At this time, the energy density $\rho_m \propto a^{-3}$. Substituting into the first Friedmann equations yields the scale factor evolution as $a(t) \propto t^{2/3}$.

3）Cosmological constant dominated era: When the age of the universe $t \gg 10^{10}$ years, dark energy corresponding to the cosmological constant Λ becomes dominant. The equation-of-state parameter $w = -1$, and the energy density $\rho_\Lambda = \Lambda c^2 / 8\pi G$ is constant. Substituting this into the first Friedmann equation yields the scale factor evolution as $a(t) \propto e^{H_0 t}$(where H_0 is the current Hubble constant). At this stage, the universe enters a de Sitter universe state, exhibiting exponential expansion [5].

According to the Big Bang cosmology theory, the dominant energy density components vary across different expansion phases of the universe, leading to significant differences in the equation of state parameter $w = p / \rho c^2$. This causes the solutions of the standard Friedmann equations in each phase to be disconnected, preventing the formation of a unified and continuous solution.

Existing cosmological gravitational field equations exhibit a clear continuity breakdown when describing the evolution of these four phases. Specifically, during transitions between the radiation-dominated and matter-dominated phases, as well as between the matter-dominated and cosmological constant-dominated phases, the equations cannot achieve a smooth transition through a unified analytical solution. Instead, piecewise fitting is required to match observational data from different phases, which violates the principle of uniformity in physical laws. Fundamentally, this continuity defect arises from the equations' simplified treatment of cosmic energy component evolution—treating radiation, matter, and dark energy as independent components without considering interactions between components or energy conversion. This leads to issues such as discontinuous derivatives and abrupt changes in physical quantities at phase transitions. To address this deficiency, a continuous solution that encompasses all evolutionary phases and enables smooth transitions must be constructed. By introducing reasonable coupling terms for energy components, the solutions to the equations can maintain continuity and differentiability throughout the entire cosmic evolution process.

As can be seen, the solutions to the standard Friedmann equation for the three main expansion phases are respectively $a(t) \propto t^{1/2}$, $a(t) \propto t^{2/3}$, and $a(t) \propto e^{H_0 t}$. There is a clear discontinuity between these solutions, and a single unified analytical solution cannot describe the entire evolution process. For example, during the transition from the radiation-dominated phase to the matter-dominated phase, the evolution rate of the scale factor undergoes an abrupt change, and the equation cannot smoothly connect; similarly, during the transition from the matter-dominated phase to the cosmological constant-dominated phase, the evolution of the scale factor abruptly changes from a power-law form to an exponential form, contradicting the inherent continuity of cosmic evolution.

The root cause of this continuity flaw lies in the standard Friedmann equation, which treats the energy density of different cosmic phases as independent components without considering dynamic coupling and smooth transitions between them. This results in solutions that cannot span the entire cosmic expansion process, making it difficult to fully reveal the intrinsic laws of cosmic evolution.

However, many scholars have proposed solutions for continuity. Reference [20] reviews research on dark energy and cosmic accelerated expansion, providing theoretical references for the smooth transition of the dark energy-dominated phase in the continuity solution. Reference [21] supports dark energy dynamics evolution and modified gravity theory, providing observational data and theoretical basis for the energy coupling term in the continuity solution. However, all self-consistent solutions to the cosmological gravitational field equations cannot be expressed in an analytical form.

3.5 Expansion of the Universe Requires Dark Matter and Dark Energy

In the Big Bang theory, the Friedmann equation (4-1) without the cosmological constant is

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho-\frac{kc^2}{a^2} \tag{64}$$

Taking $\dot{a}/a$ as the Hubble constant and setting k=0, the cosmic mass density ρ is obtained. This mass density is referred to as the critical mass density. This implies that the universe is in a state where it is flat, expanding, and the sum of its kinetic and potential energies is exactly zero. During the expansion process, the universe will inevitably gradually approach infinity, with its expansion speed eventually approaching zero and never contracting.

The critical density is expressed as

$$\rho_c=\frac{3H^2}{8\pi G} \tag{65}$$

This formula is one of the most fundamental in cosmology, directly linking the expansion rate of the universe to its spatial geometry. Based on the Hubble constant obtained by mainstream scholars, H= $67.4km/(s\cdot Mpc)$=$2.185\times10^{-18}/s$, the calculated value is approximately

$$\rho_c=\frac{3\times(2.185\times10^{-18}/s)^2}{8\pi\times6.67\times10^{-11}\,m^3/(kg\cdot s^2)}=8.54\times10^{-27}kg/m^3 \tag{66}$$

This value corresponds to the universe's 'critical state'—neither collapsing due to excessive gravity nor infinitely diluting from too rapid expansion.

The density of matter in the universe is typically expressed as a ratio of specific matter densities, namely

$$\Omega=\frac{\rho}{\rho_c} \tag{67}$$

The mass of various components is

$$\rho=\rho_m+\rho_{DE}+\rho_r \tag{68}$$

Among them, ρ_m represents the total mass of matter, which is equal to the sum of baryonic matter mass and dark matter mass, $\rho_m=\rho_b+\rho_{DM}$. $\rho_{DE}=\rho_\Lambda$ represents the dark energy density, and ρ_r represents the radiation matter density.

The mass density ratio of each component of the substance is

$$\Omega=\Omega_m+\Omega_{DE}+\Omega_r=1 \tag{69}$$

Based on observational data and the Big Bang theory, the total matter density parameter of the universe $\Omega_m\approx0.3111$. This value is jointly constrained by cosmic microwave background radiation and large-scale structure surveys, representing the total proportion of all matter under gravitational influence. According to observations, the average density of visible (baryonic) matter is approximately $0.415\times10^{-27}\ kg/m^3$, with a baryonic matter density parameter $\Omega_b\approx0.0486$, which only includes ordinary visible matter composed of protons and neutrons. The proportion of dark energy in the universe is $\Omega_{DE}=0.6889$.

The total mass density is

$$\rho_m=\Omega_m\rho_c$$

$$=0.3111\times 8.54\times10^{-27}kg/m^3=2.657\times10^{-27}\ kg/m^3 \tag{70}$$

Compared to the observed baryonic density of $0.415\times10^{-27}\ kg/m^3$, this reveals the missing matter in the universe. Substituting the value into the dark matter density formula gives:

$$\Omega_{DM}=\Omega_m-\Omega_b=0.3111-0.0486=0.2625 \tag{71}$$

The dark matter density is

$$\rho_{DM}=\Omega_{DM}\ \rho_c$$

$$=0.2625\times 8.54\times10^{-27}\mathrm{kg/m^3}=2.242\times10^{-27}\ \mathrm{kg/m^3} \tag{72}$$

The dark energy density is

$$\varepsilon_{DE}=\Omega_{DE}\rho_c c^2$$

$$=0.6889\times 8.54\times10^{-27}kg/m^3\times(3\times10^8\ m/s)^2$$

$$=5.299\times10^{-10}\ J/m^3 \tag{73}$$

This is how the concepts of dark matter and dark energy emerged. This led to the question:

1） Although invisible matter and energy are indeed universally present, such a huge discrepancy is absolutely impossible.

2）Ordinary matter is positive mass matter, and dark energy corresponds to negative mass matter. Then Ω_{DE} should be negative. Why is it positive in Equation (68)? If dark energy is taken into account and its value is relatively large, the total mass of the universe would be negative.

These contradictions are sufficient to confirm the fatal errors in the cosmological gravitational field equations and in Big Bang cosmology.

4.Theoretical Fallacies of Cosmological Equations and Their Solutions

There are many doubts about the validity of the cosmological gravitational field equations, and only a few are listed here.

4.1The Phenomenon of the Universe Expanding Faster than the Speed of Light

The cosmological constant was initially introduced in equations to balance the universe's gravitational field, yet a finite universe could never have existed in a stable static state. Today, cosmological theorists regard this constant as the core force driving the universe's expansion.

If $\Lambda=0$, Equation (33) indicates that the scale factor $a(t)$ will also grow monotonically and accelerate over time, but this depends on the pressure $p=-\rho c^2$. Negative pressure implies negative gravity, or repulsive gravity.

Λ also has the property of repulsive gravity.

From (33), (47), and (54), it is not difficult to see that regardless of the state of matter, with the Λ term, the cosmic scale factor $a(t)$ gains a driving force for growth and eventually tends toward $exp\left(\sqrt{\Lambda c^2/3}t\right)$. If the cosmological constant Λ is sufficiently large, the universe will continuously accelerate its expansion, and given enough time, the expansion speed will exceed the speed of light. Regardless of the form of matter, that is, regardless of ρ, p, n, or w —whether it is ordinary matter or radiation—the growth of $a(t)$ is not hindered by gravitational attraction from matter and is only related to the two factors k and Λ. Finally, $a(t)$ will grow at a speed exceeding that of light.

How can physical matter move faster than the speed of light? This clearly contradicts the principles of special relativity. Even light, electromagnetic fields, and gravitational fields do not propagate at speeds exceeding that of light. Gravitational fields are also strictly limited to propagating at the speed of light as a fundamental assumption in Einstein's general theory of relativity: they cannot exceed the speed of light, nor fall below it, and their speed remains constant. However, equations (33), (47), and (54) suggest that the rate of cosmic scale expansion is not restricted by the speed of light. As time increases, the expansion rate can far exceed the speed of light and even approach infinity.

Einstein presupposed that the speed of propagation of the gravitational field is the speed of light. However, the expansion of the universe at speeds exceeding that of light contradicts the principles of special relativity. This clearly undermines the general relativistic cosmological field equations of gravity in an important aspect.

4.2 Cosmic Curvature

The Big Bang theory [4][5] explicitly states that the universe is undergoing accelerating expansion with curvature $k=0$; however, solutions to the Friedmann equations such as (33) and (47) indicate that all dynamic solutions describing accelerated expansion do not lead to the conclusion of $k=0$, instead requiring $k=-1$. On the other hand, expansion driven purely by radiation, while independent of k, is decelerating, as shown in equation (51). From this, it can be seen that an expanding universe does not align with Einstein's original intent when he formulated his equations regarding the curvature issue.

Einstein proposed the general theory of relativity in 1915 and first applied this theory to the entire universe in 1917, thereby pioneering modern cosmology. He constructed mathematical models based on the 'cosmological principle' (that the universe is homogeneous and isotropic on a large scale), deriving that the geometric curvature of cosmic space (positive/zero/negative) is determined by its total mass-energy density—this vividly embodies the core physical idea that 'the shape of the universe depends on mass.' At the time, he assumed the universe was static, finite, and unbounded (a spherical shape with positive curvature) and introduced the cosmological constant Λ to balance gravitational collapse, advocating for a finite closed universe. A finite closed universe should satisfy$k=+1$; however, an accelerating expanding universe can only conform to $k=-1$.

Furthermore, as the universe expands and the density of matter decreases accordingly, does the curvature of the universe also change accordingly?

At its core, this stems from the fallacy of equations and metric settings—the universe could never be curved with $k\neq\pm1$; instead, it must necessarily be flat with $k=0$. The cosmic space must be Euclidean space, not Riemannian space.

4.3.Conservation of Energy

Friedman's equation (4-1) is actually the law of conservation of energy in a gravitational field, while equation (4-2) represents Newton's second law in a gravitational field.

The Big Bang theory posits that the universe originated approximately 13.83 billion years ago and is currently expanding. Based on this, assuming that at some early moment in the universe's history, its radius was R_0 and its mass was M_0, with its mass-energy being $\mathscr{E}_0=M_0c^2$; and that the present-day universe has a radius of R, a mass of M, and mass-energy of $\mathscr{E}=Mc^2$, where $R\geq R_0$. Here, the Planck era of the universe (if such an era exists) is avoided, as it falls within the domain of quantum field theory. Assuming that the matter of the universe is uniformly distributed within a sphere of radius R_0, then the early gravitational potential energy is

$$\mathscr{E}_p=-\frac{3}{5}\frac{GM_0^2}{R_0} \tag{74}$$

Total energy is

$$\mathscr{E}_T=M_0c^2-\frac{3}{5}\frac{GM_0^2}{R_0}=M_0c^2\left(1-\frac{3}{5}\frac{GM_0}{R_0c^2}\right) \tag{75}$$

The gravitational potential energy of the current universe is

$$\mathscr{E}_p=-\frac{3}{5}\frac{GM^2}{R} \tag{76}$$

Total energy is

$$\mathscr{E}'_T=Mc^2-\frac{3}{5}\frac{GM^2}{R}=Mc^2\left(1-\frac{3}{5}\frac{GM}{Rc^2}\right) \tag{77}$$

If $M_0c^2\geq 0$, $Mc^2\geq 0$, and the total energy of the entire universe is conserved, it necessarily follows that $\mathscr{E}'_T=\mathscr{E}_T$, and thus there must be

$$Mc^2\left(1-\frac{3}{5}\frac{GM}{Rc^2}\right)=M_0c^2\left(1-\frac{3}{5}\frac{GM_0}{R_0c^2}\right) \tag{78}$$

Solving this gives

$$M=\frac{5}{6}\frac{Rc^2}{G}\left[1\pm\sqrt{1-\frac{12}{5}\frac{GM_0}{Rc^2}\left(1-\frac{3}{5}\frac{GM_0}{R_0c^2}\right)}\right] \tag{79}$$

M has two values. To be physically meaningful, M should be a real number. Therefore, the condition for equation (79) to hold is

$$1-\frac{12}{5}\frac{GM_0}{Rc^2}\left(1-\frac{3}{5}\frac{GM_0}{R_0c^2}\right)\geq 0 \tag{80}$$

The solution to this inequality is

$$M_0\leq\frac{5}{6}\frac{R_0c^2}{G}\left(1-\sqrt{1-\frac{R}{R_0}}\right) \tag{81}$$

or

$$M_0 \geq \frac{5}{6}\frac{R_0 c^2}{G}\left(1+\sqrt{1-\frac{R}{R_0}}\right) \tag{82}$$

Since it is assumed that $R \geq R_0$, then $1-R/R_0 \leq 0$. Because the concept of complex mass matter does not conform to physical principles, the only solution to the equation is

$$\begin{cases} R=R_0 & (83-1) \\ M_0=\dfrac{5}{6}\dfrac{Rc^2}{G} & (83-2) \end{cases}$$

Therefore, the universe is not expanding.

This mathematically proves that cosmic expansion would violate the law of conservation of energy. Therefore, the universe cannot expand, nor can it have originated from a Big Bang. Only an infinitely large universe can be eternally static.

4.4 Negative Pressure and Negative Matter State

If the universe expands while keeping its matter density constant, it requires a special equation of state condition where $\mathrm{w}=\mathrm{p}/\rho\mathrm{c}^2=-1$. This means the state equation parameter for the universe is negative—implying that matter would produce negative pressure [4][5]. Positive mass matter, even light, can only generate positive pressure, with a state equation parameter $\mathrm{w}=\mathrm{p}/\rho\mathrm{c}^2 \leq 1/3$. The concept of negative pressure contradicts the properties of normal matter and can only be understood as an anomalous pressure arising from negative mass matter. This clearly violates physical principles and is an unacceptable concept.

4.5 Negative Mass Matter and Repulsive Gravity

In Friedmann's solutions to Einstein's cosmological equations, the cosmological constant term $\Lambda c^2/3$ on the right-hand side of equation (3-2) is positive, while the gravitational term is negative—this sign characteristic precisely reveals the repulsive nature of the cosmological constant term. When viewed within the framework of Newtonian universal gravitation, this cosmological constant term corresponds exactly to negative mass-energy matter [22]. To maintain a constant matter density and achieve a static or expanding universe, negative pressure must be present; that is, the cosmic pressure must be $p=-\rho c^2$, or the cosmic state equation constant $\mathrm{w}=p/\rho c^2=-1$, which is also a manifestation of repulsive gravity. Thus, it can be seen that negative mass-energy matter is the necessary condition driving the accelerated expansion of the cosmic scale factor.

The formula for Newton's law of universal gravitation is

$$F=-G\frac{Mm}{r^2} \tag{84}$$

This formula indicates that if the masses of two objects are both positive, they must exert an attractive force. If the test particle m has a positive mass and a repulsive force is desired, M must necessarily be negative.

Einstein added the Λ term to his equations in order to establish static cosmological equations, which is actually equivalent to adding a mass factor M_- to Newton's law of universal gravitation formula, resulting in

$$F=-G\frac{(M+M_-)m}{r^2}\geq 0 \tag{85}$$

If the added mass is a negative value large enough in absolute magnitude, it can not only counteract gravity but also transform it from an attractive force into a repulsive one.Therefore, introducing the cosmological constant is essentially equivalent to adding a term for negative mass matter and is not particularly novel. The Big Bang cosmology theory states that the universe is expanding at an accelerating rate driven by dark energy, which the theory seeks to identify as negative mass matter.

However, once the variable of negative mass matter is introduced, new contradictions become apparent.

Within the framework of big bang theorists, dark energy (i.e., negative mass matter) is uniformly distributed throughout the entire universe, exerting a repulsive gravitational force on ordinary matter. More crucially, the total amount of negative mass matter far exceeds that of ordinary matter—negative mass matter (dark energy) accounts for approximately 72%, while positive mass matter (ordinary matter + dark matter) makes up about 28%. After partial cancellation between the two, roughly 44% of negative mass matter (dark energy) remains in the universe, meaning the universe as a whole exhibits negative mass properties.

So, do negative mass substances exert attractive universal gravitation on each other? If the answer is affirmative, even if positive and negative masses partially cancel out in quantity, the universe as a whole would still be dominated by negative mass matter. According to Newton's law of universal gravitation (84), the force exerted by this collective negative mass on an individual negative mass particle remains attractive and directed toward the center of the universe. In this case, negative mass matter throughout the universe would experience a gravitational pull toward the center, while positive mass matter would experience a repulsive force away from the center, leading to a flow pattern where negative mass matter converges toward the cosmic center and positive mass matter flows outward. Furthermore, the Big Bang cosmology theory states that negative mass matter (i.e., dark energy) represents vacuum energy—a connection established when Guth combined the inflationary universe model with vacuum energy in 1981 [23]. Thus, the convergence of negative mass matter toward the center implies that the universe space is contracting under gravitational influence. Galaxies composed of positive mass matter, like buoys floating serenely on the vast ocean current of negative mass matter, how could they be repelled by the repulsive gravitational force of negative mass matter and move against the current?

In summary, negative mass matter (vacuum) accelerates toward the center of the universe. In this scenario, negative mass matter in the universe exhibits an accelerating contraction. So why does ordinary matter expand against this trend at an accelerating rate? Isn't this a contradiction?

Clearly, when introducing the cosmological constant into cosmological equations to explain the universe's accelerated expansion, if the introduced term is numerically too large and exceeds the contribution of positive mass matter, it would instead lead to the universe accelerating in contraction. Only when the two values are equal and both are in a state of rest can gravity cancel out to zero, achieving static equilibrium. Einstein's initial idea was not entirely unfounded. Supporters of the Big Bang theory attempt to achieve cosmic acceleration through negative energy matter propulsion—does this not backfire?

4.6 Dark Energy Represented by the Cosmological Constant

Cosmologists who favored the Big Bang theory interpreted Einstein's cosmological field equations arbitrarily. To achieve an expanding universe, they added a cosmological constant that did not decrease with cosmic expansion, resulting in exponential growth of the universe's scale factor.

Einstein once admitted that introducing a constant term into the cosmological field equations was the biggest blunder of his life. Therefore, the cosmological constant term should be discarded.

After 1998, 'dark energy' was widely accepted as a general hypothesis describing the cause of the universe's accelerating expansion. It refers to a form of energy that permeates all space, has negative pressure characteristics, and can generate repulsive force. Physicists revived Einstein's once-discarded cosmological constant Λ and endowed it with a new physical interpretation (such as vacuum energy). Einstein initially introduced Λ as a purely mathematical patch, lacking actual physical substance; in contrast, the 'dark energy' post-1998 is an empirical concept of a cosmic component with observable effects. Observations of Type Ia supernovae were interpreted as evidence of an accelerating universe, a finding that completely contradicted all previous predictions based on gravity-induced deceleration. Michael Turner, who coined the term 'dark energy,' once stated that dark energy is the deepest mystery in all of science. Dark energy is supported by American astrophysicist Saul Perlmutter, who provided observational evidence through the Hubble diagram of Type Ia supernovae.

The nature of dark energy remains a major mystery in physics, and scientists continue to strive to explore its essence to this day. The physical properties of dark energy are reflected in its equation of state. In the ΛCDM model, dark energy is typically regarded as a vacuum energy that does not evolve over time, with its equation of state remaining constant at $w=-1$. However, as cosmological observation technologies have advanced and data precision has continuously improved, it has gradually been found that different types of observational data exhibit a certain degree of inconsistency under the ΛCDM model.

Is Λ truly equivalent to dark energy? What is its true nature—vacuum energy, a dynamic field, or a need to modify gravitational theory? These questions remain unresolved to this day.

According to the mass-energy formula of special relativity, $\mathscr{E}=Mc^2$, mass and energy are unified, and both are positive values.

The Big Bang theory posits that the universe is currently undergoing accelerated expansion, driven by a force known as dark energy—whose essence is actually negative mass matter. The cosmological equations of general relativity form the core pillar of this theory, and only by identifying dark energy (i.e., negative mass matter) can the validity of this theory be verified. However, dark energy cannot be detected through electromagnetic interactions, a characteristic that renders it unfalsifiable; and theories that are unfalsifiable are inherently pseudoscientific. The very proposal of concepts such as dark energy (i.e., negative mass-energy matter) and repulsive gravitational fields itself marks that the Big Bang theory has already reached a dead end.

However, the Big Bang theory indicates that human understanding of cosmic expansion goes far beyond such a simplistic level: 'repulsive gravity' is not the reverse form of traditional gravity—in other words, negative mass matter in the universe does not push positive mass matter outward according to Newton's law of universal gravitation; instead, it is an overall repulsive effect of spacetime caused by negative pressure (such as dark energy) allowed within the framework of general relativity. It does not act between objects but directly drives the acceleration of space itself. Galaxies and other tangible matter in the universe are merely 'drifters' in this vast expansion.

This understanding has long transcended the explanatory scope of existing physical laws. If that is the case, it seems unnecessary to rely on Newton's law of universal gravitation or the gravitational field equations of general relativity cosmology to describe it; instead, one can simply rely on boundless imagination.

Jamie Farnes proposed a bold, groundbreaking hypothesis [23]: a continuously generated fluid of negative mass particles in the universe that can both simulate the gravitational effects of dark matter and explain the repulsive expansion caused by dark energy. The core innovation of this theory lies in introducing the 'creation tensor', which allows negative mass matter to be continuously created during cosmic evolution, thereby avoiding the problem of dilution due to spatial expansion. This setup maintains a constant negative mass density, aligning with the observed behavior of dark energy. Farnes' theory challenges the standard cosmological paradigm through its elegant geometric unification. Its value not only lies in solving the mysteries of dark matter and dark energy but also in stimulating deep investigations into the nature of gravity and matter symmetry (positive-negative mass pairs). The paper opens up a crucial avenue for exploring modified gravity theories and non-particle dark matter models, standing as a bold revolution in contemporary cosmology.

However, the analytical understanding of the theory's micro-mechanisms and the realization of observational verification remain two major obstacles yet to be overcome. Farnes introduced a 'creation tensor' to explain the continuous generation of negative mass particles, aiming to maintain the density of dark fluid during cosmic expansion without dilution. However, this hypothesis lacks support from particle physics mechanisms and is considered an ad hoc theoretical patch. The repulsive gravitational interaction between negative and positive mass matter would lead to an unobservable 'perpetual motion' behavior—this mechanism not only violates the laws of energy conservation and transformation but also lacks experimental evidence. The negative mass model introduces excessive new assumptions (negative mass, creation tensor, and fluid dynamics) to unify dark matter and dark energy, making it less concise than modified gravity theories or non-standard scalar field models.

Dark energy is thought to be the cause of the universe's expansion and also constitutes its large-scale properties—but no one can clearly explain what it actually is. Now, the term 'dark energy' has come to refer to all matter that accelerates the rate of the universe's expansion, and has become a conventional label for cosmic large-scale properties that humans currently do not understand.

Clearly, the cosmological gravitational field equations cannot be made self-consistent through patchwork fixes or by invoking extraordinary theories that transcend existing physics frameworks. Instead, there are fundamental and basic errors at a foundational level.

4.7 Hubble Redshift and Hubble Constant

In the Big Bang theory, equation (60) is often regarded as a function describing the expansion of material substance throughout the entire universe. If Hubble redshift z is taken to be caused by cosmic expansion, then

$$1+z=e^{H_0 t}=e^{\sqrt{\frac{\Lambda c^2}{3}}t} \tag{86}$$

This relationship links the growth rate of the scale factor with the photon's Hubble redshift to yield the Hubble constant.

$$H_0=\sqrt{\frac{\Lambda c^2}{3}} \tag{87}$$

Thus, the relationship between the cosmological constant and the Hubble constant was derived.

$$\Lambda=\frac{3H_0^2}{c^2} \tag{88}$$

In this way, an artificial connection was established between the cosmological constant and the Hubble constant.

In fact, the Hubble constant is not the expansion rate of cosmic space, but rather the damping coefficient of photons as they propagate through cosmic space. One reason why cosmological gravitational field equations are regarded by Big Bang cosmologists as the core equations describing cosmic expansion is that the redshift formula itself has characteristics that are easily misunderstood.

4.8 The Artificial Characteristics of Metrics

As stated in Section 2.1, Einstein's cosmological field equations have at least 12 different solutions. However, these solutions do not constitute a single definitive answer; instead, each corresponds to a set of equations, and each set of equations has multiple distinct solutions. This means that there is no unique, definitive solution to the cosmological field equations. Rather, big bang theorists obtain solutions that align with their theoretical expectations by setting different metrics according to their own needs.

For example, the components of the Einstein tensor corresponding to the FLRW metric are given by Equation (3), and the metric tensor of de Sitter space is shown in Equation (8). Under both of these metric frameworks, if the relevant parameter $a(t)$ is set to a constant, the universe will exhibit a finite static state without expansion; if another setting $a(t)\equiv\infty$ is adopted, the universe will be eternally static; and if a specific parameter configuration k=0 is chosen, the universe will manifest as flat.

The anti-de Sitter metric is

$$ds^2=-c^2dt^2+e^{2\sqrt{\frac{|\Lambda|c^2}{3}}t}(dr^2+r^2d\theta^2+r^2sin^2\theta d\phi^2) \tag{89}$$

In the equation, $\Lambda<0$. Surprisingly, a negative cosmological constant can still cause the cosmological scale factor to grow exponentially.

The choice of metric is highly artificial—or, put differently, any cosmic model one wishes to construct can be realized through the corresponding metric configuration, with the ultimate goal of fitting the redshift formula (86). Yet what cosmologists favor most is the solution where the scale factor $a(t)\propto exp(\sqrt{\Lambda c^2/3}t)$, or $a(t)\propto exp(\sqrt{|\Lambda|c^2/3}t)$, which expands according to a specific law. There is only one universe, but there are infinitely many models, all arbitrarily set by human choice.

The primary observational basis for the accelerated expansion of the universe is the Hubble redshift phenomenon. However, an in-depth analysis of the Hubble diagram of Type Ia supernovae has revealed that the cause of the Hubble redshift is actually the quantum redshift effect of photons—photons lose energy by colliding with medium particles during their propagation, rather than originating from the expansion of the universe [24][25]. Further analysis of the light curves of Type Ia supernovae confirms that there is no time dilation effect for distant supernovae. Thus, it can be seen that the entire process, from the construction of cosmological gravitational field equations to the various metrics used to solve these equations, contains artificial theoretical assumptions.

In conclusion, the claim that the universe is expanding at an accelerating rate lacks solid observational evidence and should therefore be rejected.

5. Return to the Classical Mechanics System

Various criticisms have been raised regarding Einstein's cosmological gravitational field equations, but no fundamental solutions have been found. In fact, the establishment of Einstein's cosmological gravitational field equations was based on the premise of a finite universe. If the universe is finite, it cannot remain stably static; if the universe is infinite, it can remain stably static.

Why is the universe considered finite? The key lies in two famous paradoxes—the Olbers' Paradox and the Neumann-Seeliger Paradox. The former has been explained in the article 'The Origin of Cosmic Microwave Background Radiation' [26], and this paper will focus on exploring the latter.

5.1 Neumann-Seeliger Paradox

In textbooks of astrophysics and cosmology, the Neumann-Seeliger paradox has long been a central basis for deriving two key premises: "the universe is finite" and "the universe is expanding." The Neumann-Seeliger gravitational paradox is an astronomical paradox [27]: if there were infinitely many uniformly distributed and stationary stars in the universe, the resultant gravitational force exerted by all stars on any particle in the universe (such as the Earth) would be infinite. However, in reality, the Earth does not experience such infinite gravitational force.

1） In the 17th century, the theologian Bentley, a contemporary of Newton, challenged Newton's theories of an infinite and stable universe in order to argue against atheism. He posited that if the universe were infinite, the Earth would be torn apart by the infinite gravitational pull of the cosmos. In 1880, Laplace mathematically described this idea, asserting that any particle in the universe would be subject to the infinite gravitational force of the cosmos.

2） Between 1884 and 1895, German physicists Carl Neumann and Hugo von Seeliger, building on previous viewpoints, proposed that if the universe were infinitely large with matter uniformly distributed and non-zero density everywhere, the gravitational force acting at every point in space would accumulate to infinity, tearing apart stars like the Sun. This unobserved phenomenon came to be known as the Neumann-Seeliger paradox. In 1895, Seeliger published an article in which he revealed what appeared to be a flaw in Newton's law of gravitation, one that might lead to 'unsolvable contradictions'.

Cosmologists who support the Big Bang theory believe there is a contradiction in the Poisson equation for the universe. Specifically, in cosmic space with mass density ρ, the Poisson equation for the gravitational potential ϕ is expressed as

$$\nabla^2\phi=4\pi G\rho \tag{90}$$

Therefore, every particle in space is subject to the finite gravitational pull of the universe. However, in reality, such a gravitational force does not exist. All locations in the universe hold equal status, meaning that the gravitational potential is the same everywhere in the universe, and it should satisfy

$$\nabla^2\phi=0 \tag{91}$$

This is Laplace's equation.

Due to the existence of Neumann-Seeliger's paradox, people are forced to conclude that the universe must be finite and necessarily dynamic.

5.2 Gravitational Field Equations of a Finite Universe

Suppose the universe is a sphere with radius R, centered at O, where matter is uniformly distributed inside with density ρ, and there is no matter outside (i.e., density is 0). A test particle of mass m is placed at point P in the cosmic space, at a distance r from the center of the sphere. Now, perform a force analysis on P. In this model, the distribution of matter in the existing space is guaranteed to be symmetric, and the space is finite (as shown in Figure 1).

How is the force acting on P calculated?

For calculation, construct a sphere with radius r and center O. Applying Newton's law of universal gravitation gives

$$F=-\frac{Gm}{r^2}\int_0^r 4\pi r^2\rho dr \tag{92−1}$$

$$=-\frac{4\pi}{3}Gm\rho r \tag{92−2}$$

Formula (92) applies only to a finite universe and not to an infinite one. The point P under consideration is naturally a point within a finite universe, which experiences a gravitational force of finite magnitude directed toward the center of the universe. If $r<R$, the point P under consideration is inside the universe and experiences a gravitational force directed toward the universe's center, with the magnitude of this force being proportional to the distance from the point to the universe's center. If $r=R$, the point under consideration lies on the edge of the universe, where the gravitational force reaches its maximum value.

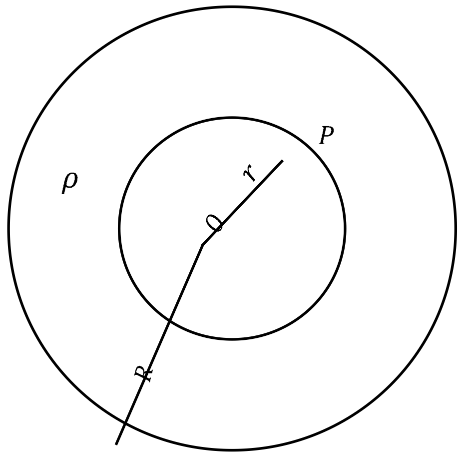

Figure 1. A spherically symmetric finite universe with uniformly distributed matter

Expressed in scalar form, the above equation is written as

$$\ddot{r}=\frac{F}{m}=-\frac{4\pi G}{3}\rho r \tag{93}$$

The above expression can be converted into vector form and expressed as

$$\ddot{\vec{r}}=\frac{\vec{F}}{m}=-\frac{4\pi G}{3}\rho\vec{r} \tag{94}$$

If the r being discussed is the cosmic radius R, then equation (93) is expressed as

$$\ddot{R}=-\frac{GM}{R^2} \tag{95}$$

Since

$$\ddot{R}=\frac{d\dot{R}}{dt}=\frac{d\dot{R}}{dR}\frac{dR}{dt}=\dot{R}\frac{d\dot{R}}{dR}=\frac{1}{2}\frac{d\dot{R}^2}{dR} \tag{96}$$

Therefore, equation (95) becomes

$$\frac{d\dot{R}^2}{dR}=-\frac{2GM}{R^2} \tag{97}$$

Integrating the above equation once gives

$$\dot{R}^2=\frac{2GM}{R}+C' \tag{98}$$

Here C' is the constant of integration. Applying the volume, mass, and density formula, $M=4\pi R^3\rho/3$, along with equations (95), (97), and (98), a system of equations is obtained.

$$\begin{cases} \left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G\rho}{3}+\frac{C'}{R^2} & (99-1) \\ \frac{\ddot{R}}{R}=-\frac{4\pi G\rho}{3} & (99-2) \end{cases}$$

This is the cosmological equation corresponding to the scenario without cosmic pressure or a cosmological constant term as a repulsive field. While the cosmic radius can change, it can never achieve accelerated expansion; even if it were to be at rest, this would only be a transient and unstable state.

In the case of a finite universe, the emergence of a gravitational field directed toward the center of the universe, while consistent with mechanical common sense, does not actually exist. This proves that the assumption of a finite universe is invalid.

To describe both static and accelerating expanding universes using this equation, adding pressure and cosmological constant terms to the equation set can achieve this without needing to establish the general relativity cosmological gravitational field equations.

$$\begin{cases} \left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G\rho}{3}+\frac{\Lambda c^2}{3}+\frac{C'}{R^2} & (100-1) \\ \frac{\ddot{R}}{R}=-\frac{4\pi G}{3}\left(\rho+\frac{3p}{c^2}\right)+\frac{\Lambda c^2}{3} & (100-2) \end{cases}$$

The integration constant C' here is not the cosmic curvature.

Clearly, a finite universe cannot be static, and there is no justification for adding a constant Λ term to the equations.

5.3 Gravitational Field Equations of the Infinite Universe

In the case of a finite universe, when applying formula (94) and taking the limit as $r\to R\to\infty$, can the force acting on a particle m at point P be obtained? The answer is clearly no. As for an infinite universe, it is naturally not possible to directly use formula (92), derived from a finite universe, and conclude that $F \to -\infty$ by taking $r \to \infty$. This is because, in an infinitely large universe, there is no definite cosmic center—put differently, every point in the universe is a center. Thus, the distance r from the so-called 'cosmic center O' to point P loses its meaning. Since every point is a cosmic center, the value of r is naturally zero everywhere. Therefore, in formula (92), $G\rho r=0$, leading to the conclusion that $F \equiv 0$.

The same result can be obtained precisely using the method of vector summation of gravitational forces. As shown in Figure 2, regardless of the position of the particle P, for an infinite universe, every point is the center of the universe. Therefore, the particle P is always located at the coordinate origin O. If the universe is uniform and assumed to be spherically symmetric, the resultant gravitational force it experiences from the entire universe is $\vec{F}$. The force in spherical coordinates is projected onto the three coordinate axes of the three-dimensional rectangular coordinate system to obtain

$$F_x = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} G \frac{m}{r^2} \rho r^2 \sin\theta \cos\phi \, d\phi \, d\theta \, dr \equiv 0 \tag{101}$$

$$F_y = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} G \frac{m}{r^2} \rho r^2 \sin\theta \sin\phi \, d\phi \, d\theta \, dr \equiv 0 \tag{102}$$

$$F_z = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} G \frac{m}{r^2} \rho r^2 \cos\theta \, d\phi \, d\theta \, dr \equiv 0 \tag{103}$$

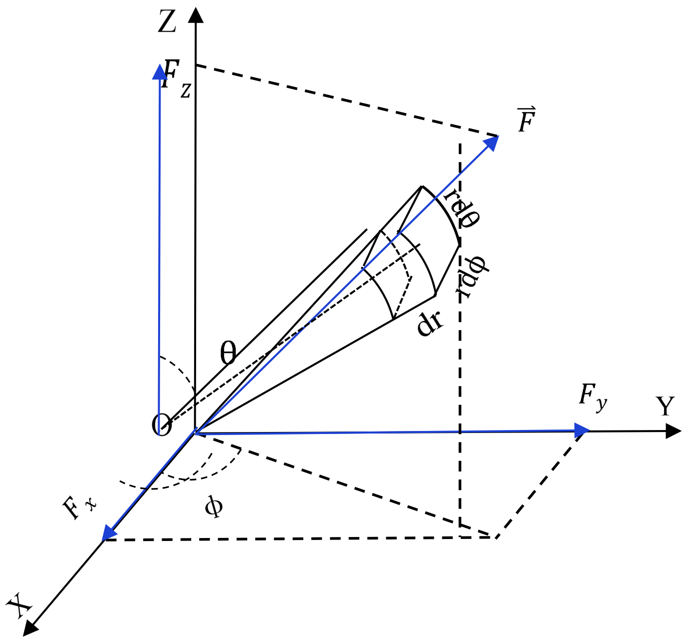

Figure 2.Components of the cosmological gravitational vector along each axis of the OXYZ coordinate system

Consequently, the force vector acting on a mass point P positioned anywhere within the infinite universe is

$$\vec{F} \equiv 0 \tag{104}$$

The acceleration of particle *P* is

$$\vec{a} = \frac{\vec{F}}{m} = 0 \tag{105}$$

Its solution is a set of equations

$$\begin{cases} \ddot{R} = 0 & (106-1) \\ \dot{R} = 0 & (106-2) \\ R = \infty & (106-3) \end{cases}$$

This result indicates that if the universe is infinite, any particle in the universe will be in a state of equilibrium; in other words, the net gravitational force exerted on the particle by the universe is zero. This implies that the cosmological gravitational field strength at any position in the universe is zero, and thus no particle will experience acceleration. As a result, the universe must be in a state of eternal rest.

Within the framework of a finite universe model, an oscillating dynamic universe can be constructed. However, to build a static or accelerating expanding universe, assumptions that violate fundamental physical principles, such as negative pressure and repulsive gravity, must be introduced. In contrast, based on an infinite universe model, a permanently static universe can be constructed without relying on special conditions like repulsive gravity.

So, how can the Neumann-Seeliger's paradox be resolved?

For a finite universe, if the gravitational potential is denoted by ϕ, applying formula (94) yields a relationship

$$\nabla\phi=-\frac{\vec{F}}{m}=\frac{4\pi G}{3}\rho\vec{r} \tag{107}$$

This equation is interpreted as the force acting on a particle of mass m in cosmic space being $F = -4\pi G\rho mr/3$. If the universe is infinitely large, i.e., $r = \infty$, this particle would experience an infinitely large force.

Taking the divergence of formula (107) yields

$$\nabla^2\phi=4\pi G\rho \tag{108}$$

This is the Poisson equation.Obviously, such a force does not exist in the universe. This is the Neumann-Seeliger's paradox paradox.

For an infinite universe, if the gravitational potential is denoted by ϕ, applying formula (107) yields the relationship

$$\nabla\phi=-\frac{\vec{F}}{m}=0 \tag{109}$$

Equation (109) means that although the universe is infinitely large and regardless of how great the matter density is, since the cosmological gravitational force acting on any particle is a balanced force, i.e., the resultant force is zero. Therefore, the gravitational potential is equal everywhere, and its gradient is zero.

Applying the divergence operation to equation (109) again yields

$$\nabla^2\phi=0 \tag{110}$$

This is the Laplace equation for the gravitational field in cosmology.

In fact, formula (108) was derived under the assumption of a finite universe and describes a dynamic universe. Formula (110), on the other hand, was derived under the assumption of an infinite universe and describes a static universe. Einstein wanted to obtain the gravitational field equations for a static universe, so he should have applied (110). There was no need to establish the general relativity cosmological equations, and even if they were established, their solutions (26) and (42) would be no different from the solutions of (110).

The hypothesis of an infinite universe not only aligns with Newton's cosmology but is also highly consistent with the cosmological principle—meaning that Newton's law of universal gravitation and his cosmological view themselves do not give rise to the Neumann-Seeliger paradox. Thus, the root of the Neumann-Seeliger paradox lies in a fundamental mathematical error; this paradox can be resolved simply by applying mathematical operations correctly.

While Newton's law of universal gravitation has its own limitations, this by no means implies that the universe must be finite, nor can it be used to negate the validity of Newtonian cosmology. In fact, the contradictions arising from the theory of a finite universe are far more severe than the problems posed by the concept of an infinite universe.

The Big Bang theory relies on the support of the finite universe theory. However, addressing the Neumann-Seeliger paradox reveals that constructing the finite universe theory requires resorting to erroneous mathematical derivations to reach misleading conclusions.

In summary, the Neumann-Seeliger Paradox essentially arises from a pure mathematical fallacy. It contains no profound physical implications and cannot serve as strong evidence for the universe's inevitable finiteness. On the contrary, the infinite universe model can completely resolve this paradox and derive a consistent cosmological equation—the Laplace equation. This equation does not require the introduction of a cosmological constant, nor does it need to rely on dark energy.

6. An Examination of Repulsive Gravity from a New Perspective

6.1 Gravitational Field inside a Spherical Celestial Body

In the study of the article '*The Matter in Newtonian Static Gravitational Field*' [28], it is pointed out that if a gravitational field is regarded as a form of matter, and its intensity is g, then the energy density of the gravitational field within the field is

$$\epsilon = \frac{g^2}{8\pi G} \tag{111}$$

According to the theory of special relativity, energy and mass are essentially two different manifestations of matter. Any object that carries energy necessarily corresponds to a certain amount of matter; the energy inherent in the gravitational field also falls within this category of matter and thus inevitably generates new gravitational fields in return. Since the gravitational field possesses mass and energy, it produces new gravitational fields. This process repeats cyclically, with iterative corrections to the gravitational mass-energy of objects and the gravitational field itself, ultimately leading to new formulas for the law of universal gravitation and mass-energy equivalence.

Suppose there is a spherical object with radius R, having a visible mass of M_0, with matter uniformly distributed and density ρ. Then the mass-energy within a sphere of radius r from the center of the sphere is

$$\mathscr{E}(\mathrm{r})=\frac{2c^4}{G}r\left[1-\left(\sqrt{\frac{2\pi G\rho}{c^2}}r\right)\mathrm{ctg}\left(\sqrt{\frac{2\pi G\rho}{c^2}}r\right)\right] \tag{112}$$

Then the gravitational mass within this volume is

$$M=\frac{\mathscr{E}(\mathrm{r})}{c^2}=\frac{2c^2}{G}r\left[1-\left(\sqrt{\frac{2\pi G\rho}{c^2}}r\right)\mathrm{ctg}\left(\sqrt{\frac{2\pi G\rho}{c^2}}r\right)\right] \tag{113}$$

According to the formula of Newton's law of universal gravitation (84), whether the force acting on a test particle of mass m placed at a distance r from the center of the sphere is an attractive or repulsive force depends on the sign of M， which is determined by the factors inside the brackets in formula (113).

To explore the function within the brackets in equation (113), let the mathematical function be

$$y=1-x\cdot ctgx \tag{114}$$

where

$$x=\sqrt{\frac{2\pi G\rho_0}{c^2}}r \tag{115}$$

To make it more intuitive, the graph of function (114) is plotted as shown in Figure 3.

From Figure 3, it can also be seen that y is negative over many ranges of x, indicating that the mass-energy of matter is negative in these ranges; in other ranges of x, y is positive, indicating that the mass-energy of matter is positive. Since $M/M_0\neq1$, dark matter and dark energy indeed exist in the internal and external spaces where the object resides. In the case where $M/M_0>1$, the additional matter and energy present in the considered space are positive, and their signs are the same; whereas in the case where $M/M_0<1$, the additional matter and energy in the considered space are negative, with their signs also remaining consistent.

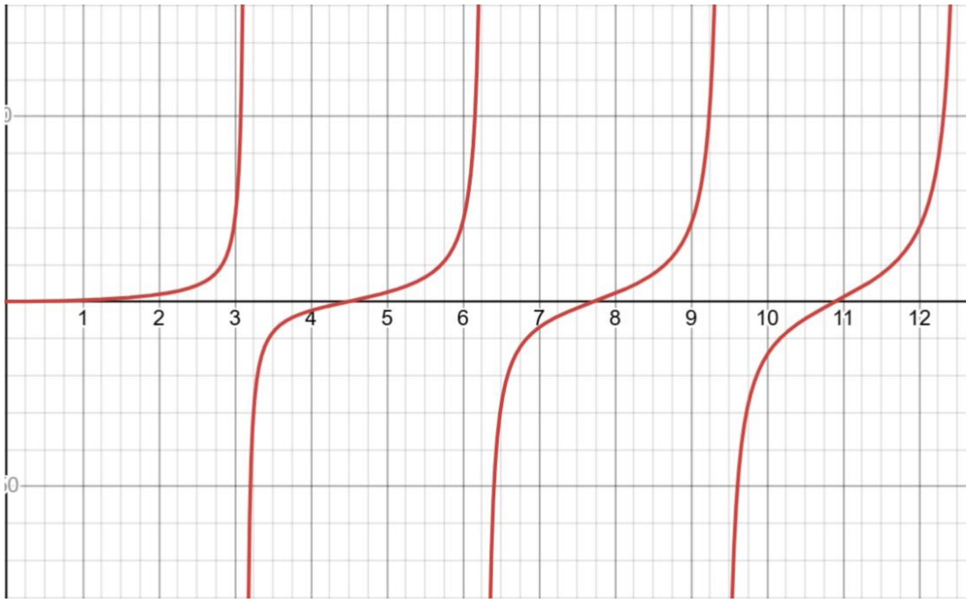

Figure 3. This is the graph of the function $y = 1 - x \cdot ctgx$. The positivity or negativity of y represents the positivity or negativity of the celestial body's matter-energy, as well as whether universal gravitation is an attractive or repulsive force.

Analysis of the function reveals that there are infinitely many alternating positive and negative intervals within the sphere, corresponding to the matter state exhibiting alternating positive and negative characteristics, as does universal gravitation. See Table 1.

Table 1. The intervals of positive and negative values of the function

$y = 1 - x \cdot ctg\, x$ and the manifestation of universal gravitation

Range of x	Sign of y	The behavior of gravity
$0<x<\pi$	+	Attraction
$\pi<x<4.49341$	−	Repulsion
$4.49341<x<2\pi$	+	Attraction
$2\pi<x<7.72525$	−	Repulsion
$7.72525<x<3\pi$	+	Attraction
$3\pi<x<10.90412$	−	Repulsion
$10.90412<x<4\pi$	+	Attraction

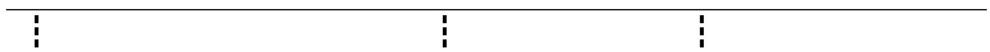

Figure 3 shows that function (114) has singularities at x=π,2π,3π,4π,⋯ ⋯ . From a traditional physics perspective, this is meaningless. Therefore, integer multiples of π represent insurmountable barriers to the variation of x. The state where $x<\pi$ is considered the normal state in physics.

During the transition from a state where x is an integer multiple of π to a state where x is a larger integer multiple of π, the positive and negative attributes of the celestial body's material state undergo a transformation. The reversal points correspond to x taking values such as x_1=4.49341，x_2=7.72525，x_3=10.90412，and so on. In regions where y is negative, the overall mass of the celestial body is negative, and gravity manifests as a repulsive force. In regions where y is positive, the overall mass of the celestial body is positive, and gravity manifests as an attractive force. Near singularities, either gravity becomes infinitely strong or repulsion becomes infinitely strong. Between two singularities, there exists a zero point where y=0. At this zero point, the mass and energy of the celestial body's matter are both zero, and the gravitational field intensity is also zero. Any particle at this point is in a state of force equilibrium, but this equilibrium is unstable. Once disturbed, the particle will deviate from this point—either experiencing increasingly stronger gravitational attraction and falling into the side where gravity is infinitely strong, or experiencing repulsion and falling into the side where repulsion is infinitely strong. Therefore, despite the existence of a balance point where gravity is zero, particles cannot remain eternally stationary there; instead, they will ultimately be driven toward the singularities.

It is not difficult to understand that under normal conditions, the mass-energy of matter is positive, while gravitational potential energy is negative. When in a negative mass-energy state, universal gravitation manifests as a repulsive force, and gravitational potential energy becomes positive. Once an object's state reverses from positive to negative (or vice versa), the system's potential energy will reverse synchronously, while the total energy remains conserved.

The celestial structure factor $GM/2c^2R$ is a key factor that triggers changes in the quantity and state of celestial matter. In certain spatial positions within a black hole, it is highly likely to reach a critical state, causing the positive and negative states of matter to invert. Due to the singularity forming an impenetrable barrier, the distribution of matter inside the black hole exhibits a shell structure. Within each shell, there are two sub-layers: one in a positive mass-energy state and the other in a negative mass-energy state. Because of the isolating effect of the singularity, matter within one shell cannot flow into another shell.

This discovery has touched upon deeper unknown realms in physics: Does negative mass-energy state exist in physics? Does repulsive universal gravitation exist? There is still no definitive conclusion. The derivation of formula (112) is based on a differential equation model constructed with the premise that the astronomical structure factor $GM/2c^2R$ can ensure the convergence of the series; if this premise is abandoned, function (112) will no longer hold, and previous inferences will also become invalid. Therefore, function (112) is only valid within a specific range and cannot be infinitely extended for deduction. When x approaches π from the left (x→π−0) and the series diverges, the above inferences no longer hold, and the related concepts proposed in this paper need to be re-examined and reconstructed.

Dark matter and dark energy undoubtedly exist. Both fall within the realm of mass-energy of the gravitational field, rather than involving negative mass-energy particles, and they share the same positive or negative state. Celestial bodies composed of negative mass-energy matter particles and subject to repulsive universal gravitation should not exist.

6.2 An Examination of the Gravitational Field in Cosmology

Assume the universe is a sphere with a finite radius. The sphere examined above represents the universe. By replacing *r* in the formula with the cosmic radius *R*, the total observable mass of the universe is $M_0=4\pi R^3\rho_0/3$. Using equation (112), the variation of mass-energy within the universe can be expressed by the following equation

$$\mathscr{E}(\mathrm{R})=\frac{2\mathrm{c}^4}{\mathrm{G}}\mathrm{R}\left[1-\left(\sqrt{\frac{3GM_0}{2c^2R}}\right)ctg\left(\sqrt{\frac{3GM_0}{c^2R}}\right)\right] \tag{116}$$

If $\mathscr{E}(\mathrm{R})<0$, meaning negative mass and negative energy are present within the universe, there will be a repulsive force on particles at the edge of the universe, resulting in the expansion of the entire universe.

Modify function (114) to

$$Y=1-XctgX \tag{117}$$

where

$$X=\sqrt{\frac{3GM_0}{2c^2R}} \tag{118}$$

When X falls within the following ranges: π~4.49341, 2π~7.72525, 3π~10.90412，…，Y takes negative values, meaning that Ɛ（R） expressed by Equation (116) also becomes negative. Negative mass-energy implies that calculating the change in the universe's radius using Newton's law of universal gravitation would lead to the conclusion that a universe composed of material substances is expanding. In this way, a theoretical basis is found for the accelerated expansion of a universe composed of material substances.

Current mainstream scholars believe that the total mass of the universe (including hypothetical dark matter and dark energy) is approximately

$$M_0 \approx 10^{53} kg \tag{119}$$

The radius of the universe is approximately R=4.4×10^{26}m. Using this value and according to formula (118), it calculated to be

$$X=\sqrt{\frac{3GM_0}{2c^2R}}=\sqrt{\frac{3\times6.67\times10^{-11}\times10^{53}}{2\times(3\times10^8)^2\times4.4\times10^{26}}}\approx0.5<\pi \tag{120}$$

By comparing this value with the image, it can be seen that the value of X lies within the range $0<X<\pi$. Therefore, $Y>0$, meaning $\mathscr{E}$（R）>0. In other words, the total mass-energy of the universe is positive, the overall matter of the universe is in an ordinary matter state, and the universal gravitational force of the universe obeys the law of universal gravitation under normal conditions. Even if the estimated value of M_0 is increased by one order of magnitude, taking $10^{54}kg$, and the radius of the universe is reduced to R = 13.7 billion light-years, this remains true; $\mathscr{E}$（R）is still a positive value.

This conclusion is verified quantitatively: if the universe is finite, its radius acceleration $\ddot{R}$ will always be negative, meaning a universe composed of material substance can never undergo accelerated expansion. The matter within the universe cannot provide the driving force for such accelerated expansion. As for whether the universe might have exhibited a state of $Y<0$ within a specific range in the past, this remains undetermined. The isolation effect of singularities restricts the range of changes in X-related physical quantities, confining them within the interval$X<\pi$, thus there is no repulsive gravitational field inside the universe. The mass-energy state of the universe cannot cross the boundary of $n\pi$, so the state of $Y>0$ has always been and will continue to be, meaning the universe has never and will never experience accelerated expansion.

7. Discussion

7.1 Does the Casimir Effect Prove the Existence of Repulsive Gravity?

The Casimir effect refers to the weak force that arises between two neutral conductor plates in a vacuum due to differences in quantum electromagnetic field zero-point energy fluctuations. This effect can generate a pressure as high as one atmosphere at sub-micron scales (e.g., a 10-nanometer gap).

The Casimir effect has experimentally confirmed the physical reality of negative energy density and negative pressure, providing a key analog mechanism for theories of dark energy driving the accelerated expansion of the universe. It not only demonstrates that quantum vacuum can generate repulsive gravity but also strengthens the theoretical plausibility of dark energy (particularly offering strong support for the cosmological constant model) while revealing potential unifying principles between microscopic and macroscopic physical laws. This provides important evidence for the hypothesis that dark energy originates from quantum vacuum fluctuations.

However, the Casimir effect differs fundamentally from the repulsive effects produced by dark energy and negative mass matter in the Big Bang theory:

Firstly, there is a difference in the physical interaction scales of these phenomena. The Casimir effect is significant only at sub-micron scales, acting as a short-range force (<1 μm) that decays according to the inverse fourth power of distance (d^{-4}). In contrast, cosmic dark energy must permeate the entire space, and current technology is unable to replicate this effect at a macroscopic scale.

Secondly, there are differences in the direction of their forces and their origins.The Casimir effect is a direct empirical manifestation of quantum vacuum electromagnetic fluctuations, belonging to vacuum zero-point energy fluctuation forces in quantum electrodynamics. Its essence remains a quantum correction form of electromagnetic interaction and has no relation to gravity. In contrast, repulsive gravity does not require a material medium and directly originates from the energy-momentum tensor structure of spacetime itself. As an electromagnetically mediated short-range force induced by quantum vacuum fluctuations, the former exhibits both attractive and repulsive properties. However, the latter is a long-range spacetime geometric effect driven by negative pressure within the framework of general relativity and only possesses repulsiveness.

7.2 Does Dark Energy Really Exist?

The origin of the concept of dark energy lies in:

1) Theoretical Aspect: The cosmological equations of general relativity based on a finite universe model have no static solutions. Only by introducing cosmological negative pressure and a cosmological constant term as repulsive gravity to counteract universal gravitation can static solutions be obtained; for expanding solutions, a cosmological constant is even more necessary to make repulsive universal gravitation exceed attractive universal gravitation. However, in an infinite universe, gravity as a vector quantity cancels out mutually, naturally reaching equilibrium and maintaining eternal static without the need for a cosmological constant. The cosmological field equations contain errors, with the root cause being the incorrect use of a finite universe model to describe an infinite and static universe.

2）Observational Aspect: Current views interpret Hubble redshift as kinematic redshift caused by cosmic expansion and regard the Hubble constant as the cosmic expansion rate. In reality, the Hubble constant is the energy loss rate of photons during propagation through cosmic space due to collisions with the medium, and it is unrelated to cosmic expansion.

Therefore, the concept of dark energy is mistaken and does not exist in nature.

7.3 What was Einstein's Biggest Blunder in His Life?

Einstein's initial view that the universe was stationary and stable was undoubtedly correct.

However, a finite universe model can keep the universe in a temporary state of rest but not an eternal one. Once a finite universe model is chosen, whether using Newtonian mechanics or general relativity, the universe cannot remain in a stable state of rest; conversely, once an infinite universe model is selected, whether using Newtonian mechanics or general relativity, the universe can be described as being in a stable state of rest. In cosmological questions, the model plays a decisive role. The differences between classical mechanics and relativity in describing the universe are merely matters of mathematical precision. General relativity itself does not reveal whether the universe is finite, nor does it indicate whether the universe is static or expanding.

The assumption of a finite universe serves as a necessary premise for establishing the gravitational field equations of a dynamic cosmology, rather than being derived from these equations as an absolute certainty. Classical cosmologists once aimed to disprove Newtonian cosmology and the theory of an infinite universe by employing Olbers' paradox and Neumann-Seeliger's paradox. However, the resolution of these two paradoxes today undermines the foundation of the finite universe theory. Within the framework of Newtonian cosmology, these paradoxes did not arise, and the theory of an infinite universe contained no inherent contradictions. In contrast, under the condition of a finite universe, a gravitational field directed toward the cosmic center would emerge within the universe, violating the cosmological principle of uniformity and isotropy.

Based on the theory of an infinite universe, Einstein's cosmological field equations would degenerate into the classical cosmological field equations, reverting to Newtonian cosmology. However, the cosmological equations Einstein established based on the theory of a finite universe violated the cosmological principle. To balance the gravitational field, he had to artificially add a negative pressure term and a cosmological constant term, which itself violated physical principles. Einstein himself considered this to be his biggest blunder in life.

However, Einstein's greatest mistake was not introducing the cosmological constant, but establishing the cosmological field equations on a finite spacetime basis, which plunged the theory of a finite universe into an even more severe crisis. Firstly, it violated Mach's cosmological principle. Secondly, there was no physical mechanism to account for the negative pressure and cosmological constant needed to maintain a stable, static, or even accelerated expanding universe. Relying on the hypothesis of dark energy introduces new difficulties, as it is an ad hoc assumption.

Conclusion

The cosmological equations of Einstein's general theory of relativity and their solutions have fundamental flaws: to construct a model of the universe's accelerated expansion based on these equations, one must introduce a key assumption—that the added pressure is negative pressure, with the cosmological constant corresponding to repulsive gravity generated by negative mass density matter. However, the pressures of ordinary fluids and radiation both originate from positive energy density matter, producing attractive universal gravitation, which fundamentally cannot explain the phenomenon of cosmic acceleration. Even if negative mass matter truly exists, according to the ratio of positive and negative mass matter set by current theories of cosmic acceleration expansion, the main constituent of the universe would be negative mass matter—but in accordance with Newton's law of universal gravitation, the universe should then be in an accelerating contraction state instead. This paradox pushes physics into new difficulties and casts doubt on the credibility of this theoretical framework. Therefore, the universe could never expand at an accelerated rate due to the repulsive gravity of negative mass matter.

The cosmological gravitational field equations describing a finite universe are fundamentally unable to accurately characterize the true state of the physical universe. By utilizing Newtonian mechanics and straightforward mathematical reasoning, the Neumann-Seeliger paradox can be completely resolved: in an infinite universe, the vector superposition of gravitational forces cancels out, resulting in no cosmological gravitational field. This resolution directly refutes the assertion that 'the universe must be finite,' while confirming that the theory of an infinite universe does not give rise to any cosmological contradictions—infinite universes are inherently harmonious and self-consistent systems. Cosmological equations based on Newtonian mechanics are essentially Laplace's equation; if Einstein had wished to construct a reasonable cosmological equation, he would have had to acknowledge the infinity of the universe.

In the real universe, the cosmological gravitational potential is uniform everywhere, with neither a cosmological gravitational field nor a cosmological repulsive field. The universe as a whole does not expand, does not contract, and has not undergone a Big Bang; instead, it remains in a stable and stationary state—this is the inevitable result of gravitational balance in an infinite universe, without the need for ad hoc assumptions such as negative mass matter. The crux of Einstein's general relativity cosmological field equations lies in their foundation on the concept of a finite universe; once the finite universe model is revised to an infinite model, the cosmological equations only require Laplace's equation, i.e., $\nabla^2\phi=0$.

Based on comprehensive theoretical analysis from multiple perspectives, the universe is infinite and eternally static, requiring no support from dark energy at all.

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