SCIREA Journal of Physics
ISSN: 2706-8862
http://www.scirea.org/journal/Physics
July 7, 2022
Volume 7, Issue 3, June 2022
https://doi.org/10.54647/physics14465

31% (30.65%)- Limiting Highest Efficiencies obtained in $n^+(p^+) - p(n)$ Crystalline Silicon Junction Solar Cells at T=300 K, Due to The Effects of Heavy (Low) Doping and Impurity Size

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Abstract:

In our recent two works [1, 2], by basing on: **(1)** the effects of heavy(light) doping and donor (acceptor), d(a), size , which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{En(p)o}(J_{Bp(n)o})$, being injected respectively into the heavily doped donor (acceptor)-Si emitter-lightly doped acceptor (donor)-Si base regions, HD[d(a)-Si]ER-LD[a(d)-Si]BR, of $n^+(p^+) - p(n)$ junction solar cells, respectively, **(2)** an effective Gaussian donor-density profile to determine $J_{En(p)o}$, and **(3)** the use of two experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, short circuit current density $J_{scI(II)}$, fill factor $F_{I(II)}$, and finally efficiency $\eta_{I(II)}$. Further, we obtained the highest maximal values of $\eta_{I(II)}$,

$\eta_{I(II)-max.} = 31.55\% \ (27.56\%)$, being due to the taken large values of d(a)-radius, $r_{d(a)}$=0.163 (0.141) nm, which do not correspond to the of $r_{S(Tl)}$-radius, $r_{S(Tl)} = 0.10(0.19)$ nm [8], for the emitter thickness $W = 85\ \mu m$ and surface recombination velocity $S = 10^{-50}$ cm/s, for example, corresponding to the completely opaque COER, given in the COHD[d(a)-Si]ER, and for a low Tl(S)-acceptor(donor) density $N_{a(d)} = 10^{16} cm^{-3}$, taken in the LD[a(d)-Si]BR, respectively.

In the present work, by basing on such a treatment method, but using now the usual physical conditions such as: $W = 15\ \mu m$, $N_{Bi(In)} = 5 \times 10^{20}\ cm^{-3}$ and $S = 100\ (cm/s)$, according to the highly transparent HD[Bi(In)-Si]ER-case, and then $N_{In(Bi)} = 5 \times 10^{18} cm^{-3}$ for LD[In(Bi)-Si]BR, with $r_{Bi(In)} = 0.160(0.135)$ nm [8], we now get: $\eta_{I(II)-max.}$ = **31% (30.65%)**, respectively, which can be compared with the result **η =31%** for $W = 15\ \mu m$ and $S = 100\ (cm/s)$, obtained recently by Bhattacharya and John, using the numerical simulation method [3, 4].

Keywords: donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; photovoltaic conversion efficiency

1. Introduction

In our recent works [1, 2], which will be henceforth referred to as I and II, by basing on: **(i)** the heavy doping and impurity size effects, which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, where those $J_{En(p)o}$ ($J_{Bp(n)o}$) are injected respectively into the heavily doped donor (acceptor)-Si emitter-lightly doped acceptor (donor)-Si base-regions, HD[d(a)-Si]ER-LD[a(d)-Si]BR, of $n^+(p^+) - p(n)$ junction solar cells, **(ii)** an effective Gaussian donor (acceptor)-density profile $\rho_{d(a)}$ to determine $J_{En(p)o}$[1, 2, 13, 18-20, 22] and **(iii)** the use of two fixed experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, the short circuit current density $J_{scI(II)}$, the fill factor $F_{I(II)}$, and finally the efficiency $\eta_{I(II)}$ [1, 45]. These physical quantities were expressed as functions of the open circuit voltage V_{oc}, and various parameters such as: the emitter thickness W, high donor (acceptor) density $N_{d(a)}$, surface recombination velocity S, given in the HD[d(a)-Si]ER, and low acceptor (donor) density $N_{a(d)}$, in the LD[a(d)-Si]BR. Further, in I and II, we remark that: (a) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations and strongly affect other ($J_{scI(II)}$, $F_{I(II)}$, $\eta_{I(II)}$)-

results, and (b) for a given V_{oc}, and with decreasing S and increasing W, while both $n_{I(II)}$ and $J_{oI(II)}$ decrease from the completely transparent emitter region (CTER)-case, as $S \to \infty$, to the completely opaque emitter-region (COER)-case, as $S \to 0$, $J_{scI(II)}$, $F_{I(II)}$, and $\eta_{I(II)}$ therefore increase from the CTER-case to the COER-case. Here, in the COER-case: $J_{oI(II)} = J_{Bp(n)o}$. So, our important results, obtained in I and II, are reported in the following.

In the CTHD[d(a)-Si]ER-LD[a(d)-Si]BR, in which $r_{d(a)}$ =0.163 (0.141) nm, which do not correspond to the of $r_{S(Tl)}$-radius, $r_{S(Tl)} = 0.10(0.19)$ nm, for $W = 0.000206$ μm, $N_{d(a)} = 5 \times 10^{20}$ cm^{-3}, $S(\to \infty) = 10^{50}$ (cm/s), and $N_{a(d)} = 10^{16}$cm^{-3}, we obtained the maximal values of $\eta_{I(II)}$ as: $\eta_{I(II)-max.} = \mathbf{31.51}\%$ (26.52%) at $V_{ocI(II)} = 703(743)$ mV, respectively.

Then, in the COHD[d(a)-Si]ER-LD[a(d)-Si]BR, in which $r_{d(a)}$=0.163 (0.141) nm, which do not correspond to the of $r_{S(Tl)}$-radius, $r_{S(Tl)} = 0.10(0.19)$ nm [8], for the physical conditions: $W = 85(136)$ μm, $N_{d(a)} = 5 \times 10^{20}$ cm^{-3}, $S(\to 0) = 10^{-50}$ (cm/s), and $N_{Tl(S)} = 10^{16}$cm^{-3} , we achieved: $\eta_{max.I(II)} = \mathbf{31.55}\%$ (27.56%) at $V_{ocI(II)} = 703(739)$ mV.

Then, in our present work, by basing on such a treatment method developed in I (II), we will use other usual physical conditions, given in the highly transparent HD[Bi(In)-Si]ER-LD[In(Bi)-Si]BR-case, in which $r_{Bi(In)}$ =0.160 (0.135) nm, respectively, as: $W = 15$ μm, $N_{Bi(In)} = 5 \times 10^{20}$ cm^{-3}, $S = 100$ (cm/s), and $N_{In(Bi)} = 5 \times 10^{18}$cm^{-3}, we achieve: $\eta_{I(II)-max.} =$ **31% (30.65%)** at $V_{oc} = 703$ (733) mV, as those given respectively in Tables 4 and 5. Those results can be compared with the result **η =31%** for $W = 15$ μm and $S = 100$ (cm/s), obtained recently by Bhattacharya and John, using the numerical simulation method [3, 4].

In Section 2, as developed in I and II, all the results energy-band-structure parameters for d(a)-Si systems are reported in Table 1, and the expressions for $J_{En(p)o}$ are also reported, so that we can determine the total (or dark) carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, where $J_{Bp(n)o}$ is determined in Eq. (C1) of the Appendix C. In Section 3, the photovoltaic effect is presented. Finally, some concluding remarks are given and discussed in Section 4.

2. Energy-Band-Structure Parameters and dark minority-carrier saturation current density, due to impurity-size and heavy doping effects

Here, as investigated in I and II, we now present the effects of donor (acceptor) [d(a)]-size and heavy doping, taken on the energy-band-structure parameters and minority-carrier saturation current density, as follows.

2.1. Effect of d(a)-size

In d(a)-Si-systems at T=0 K, since the d(a)-radius $r_{d(a)}$, in tetrahedral covalent bonds is usually either larger or smaller than the Si atom-radius r_{Si}, assuming that in the P(B)-Si system $r_{P(B)} = r_{Si} = 0.117$ nm, with $1\,nm = 10^{-9}m$, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_{Si}$ ($r_{d(a)} < r_{Si}$), respectively, due to the d(a)-size effect [42]. From the numerical results of the effective dielectric constant, $\varepsilon(r_{d(a)})$, obtained from such a deformation potential energy model [42], for $0.113(0.117) \leq r_{d(a)}$ in nm ≤ 0.163 (0.141), we can propose its simple approximate form as:

$$\varepsilon(r_{d(a)}) \simeq 11.4 \times \left(\frac{r_{Si}}{r_{d(a)}}\right)^{4.377\ (4.7)}, \quad (1a)$$

being accurate to within 10% (7%), respectively, equal to 11.4 as $r_{d(a)} = r_{Si}$, according to the absence of the impurity size effect, and decreased with increasing $r_{d(a)}$. This $r_{d(a)}$-effect thus affects the changes in all the energy-band-structure parameters, expressed in terms of $\varepsilon(r_{d(a)}$. In particular, the changes in the unperturbed intrinsic band gap at 0K, $E_{go}(r_{P(B)}) = 1.17$ eV, and effective d(a)-ionization energy in absolute values $E_{do(ao)}(r_{P(B)}) = 33.58$ meV, are obtained in an effective Bohr model, as [42]:

$$E_{gon(p)}(r_{d(a)}) - E_{go}(r_{P(B)}) = E_{do(ao)}(r_{d(a)}) - E_{do(ao)}(r_{P(B)}) = E_{do(ao)}(r_{P(B)}) \times \left[\left(\frac{\varepsilon(r_{P(B)})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] \quad (1b)$$

Therefore, with increasing $r_{d(a)}$, the effective dielectric constant $\varepsilon(r_{d(a)})$, determined above, decreases, implying that $E_{go}(r_{d(a)})$ and $E_{do(ao)}(r_d)$ increase, as observed in the following Table 1.

Table 1. Impurity size effects on the effective dielectric constant $\varepsilon(r_{d(a)})$, determined in Eq. (1a), the intrinsic band gap $E_{gin(p)}(r_{d(a)})$, determined in Equations (1a) and Eq. (A4), and the intrinsic carrier concentration $n_{in(p)}$, calculated using Eq. (A4) of the Appendix A, for the effective average number of equivalent conduction (valence)-band edges $g_{c(v)} = 6(2)$, respectively. Here, T=300K.

Donor	P	Te	Sb	Bi
r_d (nm)	0.117	0.140	0.145	0.160
$\varepsilon(r_d)$	11.4	5.20	4.46	2.90
$E_{gin}(T, r_d)$	1.121 eV	1.269 eV	1.340 eV	1.724 eV
$n_{in}(T, r_d)$ in 10^{10} cm^{-3}	1.07	6.11×10^{-2}	1.55×10^{-2}	9.16×10^{-6}
Acceptor	B	Al	Ga	In
r_a (nm)	0.117	0.125	0.130	0.135
$\varepsilon(r_a)$	11.4	8.35	6.95	5.82
$E_{gip}(T, r_a)$	1.121 eV	1.150 eV	1.178 eV	1.216 eV
$n_{ip}(T, r_a)$ in 10^{10} cm^{-3}	1.07	6.12×10^{-1}	3.56×10^{-1}	1.69×10^{-1}

In summary, the effects of $N_{d(a)}$-heavy doping and $r_{d(a)}$- impurity size given in the HD[d(a)-Si]ER, and those of $N_{a(d)}$-low doping in the LD[a(d)-Si]BR, affect all the minority-carrier transport properties, given in the Appendix A, B and C, and in the following equations.

2.2. Total minority-carrier saturation current density at 300K

The total carrier-minority saturation current density is defined by:

$$J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}, \quad (2)$$

where $J_{Bp(n)o}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-Si]BR, being determined in Eq. (C1) of the Appendix C, and $J_{En(p)o}$ is the minority-hole saturation-current density injected into the HD[d(a)-Si]ER, being developed and determined from I and II, now reported in the following.

In the non-uniformly and heavily doped emitter region of d(a)-Si devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in the HD[d(a)-Si]ER-width W:

$$\rho_{d(a)}(x) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]^{-\left(\frac{x}{W}\right)^2}, \quad 0 \le x \le W,$$

$$N_{d(a)o}(W) \equiv 7.9 \times 10^{17}\ (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2\ (1) 10^{-7}\ \text{cm}}\right)^{1.066\ (0.5)}\right\}\ (\text{cm}^{-3}), \quad (3)$$

where $\rho_{d(a)}(x = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(x = W) = N_{d(a)o}(W)$, decreasing with increasing W [1, 2, 13]. Further, the "effective doping density" is defined by:

$$N_{d(a)eff.}(x, r_{d(a)}) \equiv \rho_{d(a)}(x)/\exp\left[\frac{\Delta E_{ga\,n(p)}(\rho_{d(a)}(x), r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)eff.}(x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{ga\,n(p)}(N_{d(a)}, r_{d(a)})}{k_B T}\right]} \text{ and } N_{d(a)eff.}(x = W,\ r_{d(a)}) \equiv \frac{N_{d(a)o}(W)}{\exp\left[\frac{\Delta E_{ga\,n(p)}(N_{d(a)o}(W), r_{d(a)})}{k_B T}\right]}, \quad (4)$$

where $\Delta E_{ga\,n(p)}$ are determined in Equations (B4, B5) of the Appendix B.

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n^2_{in(p)}}{N_{d(a)eff.}(x,\ r_{d(a)})}, \quad (5)$$

where $n^2_{in(p)}$ is determined in (A5) of the Appendix A and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(x)[n(x)]$ and $p_o(x)[n_o(x)]$, by [22, 25]:

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)])}{p_o(x)[n_o(x)]}, \quad (6)$$

which must verify the two following boundary conditions proposed by Shockley as [6]:

$$u(x = 0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}, \quad (7)$$

$$u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1. \quad (8)$$

Here, $n_{I(II)}(V)$ is a photovoltaic conversion factor determined in Equations (27, 28), $S\left(\frac{cm}{s}\right)$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(x)$.

Further, as developed in I and II, from the Fick's law for minority hole (electron)-diffusion equations [8, 12]:

$$J_{h(e)}(x) = \frac{-e(+e) \times n_i^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e) n^2_{in(p)} D_{h(e)}(x)}{N_{d(a)eff.}(x)} \times \frac{du(x)}{dx}, \quad (9)$$

where $N_{d(a)eff.}$ is given in Eq. (4), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (C3, C2, C6) of the Appendix C, and from the minority-hole (electron) continuity equation [8, 12]:

$$\frac{dJ_{h(e)}(x)}{dx} = -e(+e) \times n^2_{i\,n(p)} \times \frac{u(x)}{F_{h(e)}(x) \times L^2_{h(e)}} = -e(+e) \times n^2_{i\,n(p)} \times \frac{u(x)}{N_{d(a)eff.}(x) \times \tau_{h(e)E}}, \quad (10)$$

where $L_{h(e)}$ and $\tau_{h(e)E}$ are defined respectively in Equations (C7, C8) of the Appendix C, one finally obtains the following second-order differential equation as [22]:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0. \quad (11)$$

Then, taking into account the two boundary conditions (7, 8), one thus gets the general solution of this Eq. (11), as [22]:

$$u(x) = \frac{\sinh(P(x)) + I(W,S)\times\cosh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), I(W,S) = \frac{D_{h(e)}(N_o(W))}{S\times L_{h(e)}(N_o(W))}. \quad (12)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, determined in Eq. (29). Further, since $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$, $C = 10^{-17}\ (cm^4/s)$, for the crystalline Si, being an empirical parameter, chosen for each crystalline semiconductor, P(x) is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}),\ 0 \le x \le W, P(x = W) \equiv (\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}) \times W \equiv \frac{W}{L_{h(e)eff.}} = \frac{L_{h(e)}}{L_{h(e)eff.}} \times \frac{W}{L_{h(e)}}, \quad (13)$$

where $L_{h(e)eff.}$ is the effective minority-hole (electron) diffusion length. Further, from Eq. (9, 13), the minority-hole (electron) current density injected into the HD[d(a)-Si]ER is found to be determined by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S)\ [J_{Epo}(x, W, N_a, r_a, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \quad (14)$$

where $J_{En(p)o}$ is the saturation minority-hole (electron) current density,

$$J_{En(p)o}(x, W, N_{d(a)}, r_{d(a)}, S) = \frac{e n_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S)\times\sinh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))}. \quad (15)$$

Here, the intrinsic carrier concentration $n_{i\,n(p)}$ is computed by Eq. (A5) of the Appendix A, and the effective doping density $N_{d(a)eff.}$ is determined in Eq. (4), the minority-hole (electron) diffusion coefficient $D_{e(h)}$ and minority-hole (electron) diffusion length $L_{h(e)}$ are given respectively in Equations (C2, C3, C7) of the Appendix C, and the factor $I(W, S)$ is determined by:

$$I(W, S) = \frac{D_{h(e)}(N_{d(a)o}(W))}{S \times L_{h(e)}(N_{d(a)o}(W))}, \quad (16)$$

where $N_{d(a)o}(W)$ is determined in Eq. (3).

Further, one remarks that: (i) from Equations (12, 14-16) one obtains: $u(x = 0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS\times p_o(x=0)[n_o(x=0)]}$, which is just the first boundary condition given in Eq. (7), and then, (ii) Eq.

(12) yields: $u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1$, being the second boundary condition given in Eq. (8).

In the following, we will denote $P(W)$ and $I(W, S)$ by P and I, for a simplicity. So, Eq. (15) gives:

$$J_{En(p)o}\big(x = 0, W, N_{d(a)}, r_{d(a)}, S\big) = \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{1}{\sinh(P) + I \times \cosh(P)}, \tag{17}$$

$$J_{En(p)o}\big(x = W, W, N_{d(a)}, r_{d(a)}, S\big) = \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)}. \tag{18}$$

Thus, from Equations (14, 17, 18), one gets

$$\frac{J_{h(e)}\big(x=0, W, N_{d(a)}, r_{d(a)}, S, V\big)}{J_{h(e)}\big(x=W, W, N_{d(a)}, r_{d(a)}, S, V\big)} \equiv \frac{J_{En(p)o}\big(x=0, W, N_{d(a)}, r_{d(a)}, S\big)}{J_{En(p)o}\big(x=W, W, N_{d(a)}, r_{d(a)}, S\big)} = \frac{1}{\cosh(P) + I \times \sinh(P)}. \tag{19}$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by [22]:

$Q_{h(e)eff.}(x = W, N_{d(a)}, r_{d(a)}) \equiv \int_0^W +e(-e) \times u(x) \times p_o(x)[n_o(x)] \times \frac{\tau_{h(e)E}(N_{d(a)}, r_{d(a)})}{\tau_{h(e)E}(\rho_{d(a)}(x), r_{d(a)})} dx$, and the effective minority-hole transit time by: $\tau_{teff.}(x = W, W, N_{d(a)}, r_{d(a)}, S) \equiv Q_{h(e)\,eff.}(x = W, N_{d(a)}, r_{d(a)}) / J_{En(p)o}\big(x = W, W, N_{d(a)}, r_{d(a)}, S\big)$, one can define, from Equations (10, 19), the reduced effective minority-hole transit time:

$$\frac{\tau_{teff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{h(e)E}} \equiv 1 - \frac{J_{En(p)o}\big(x=0, W, N_{d(a)}, r_{d(a)}, S\big)}{J_{En(p)o}\big(x=W, W, N_{d(a)}, r_{d(a)}, S\big)} = 1 - \frac{1}{\cosh(P) + I \times \sinh(P)}. \tag{20}$$

Now, some important results can be obtained and discussed below.

As $P \ll 1$ (or $W \ll L_{h,eff.}$) and $S \to \infty$, $I \equiv I(W, S) = \frac{D_h(N_o(W))}{S \times L_h(N_o(W))} \to 0$, from Eq. (20), one has: $\frac{\tau_{t,eff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{h(e)E}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (18), one obtains:

$$J_{En(p)o}\big(x = W, N_{d(a)}, r_{d(a)}, S \to \infty\big) \to \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{1}{P(W)}, \tag{21a}$$

and then, as $P \gg 1$ (or $W \gg L_{h,eff.}$) and $S \to 0$, $I \equiv I(W, S) = \frac{D_h(N_o(W))}{S \times L_h(N_o(W))} \to \infty$, from Eq. (20), one has: $\frac{\tau_{teff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)}{\tau_{h(e)E}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (18), one gets:

$$J_{En(p)o}\big(x = W, N_{d(a)}, r_{d(a)}, S \to 0\big) \to \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \tanh(P). \tag{21b}$$

In summary, in the $n^+(p^+) - p(n)$ junction solar cells, the dark carrier-minority saturation current density J_o, defined in Eq. (2), is now replaced by $J_{oI(II)}$, for a good presentation, and rewritten by:

$$J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \equiv J_{En(p)o}(W, N_{d(a)}, r_{d(a)}, S) + J_{Bp(n)o}(N_{a(d)}, r_{a(d)}), \quad (22)$$

where $J_{En(p)o}$ and $J_{Bp(n)o}$ are determined respectively in Equations (18) and (C1) of the Appendix C.

Then, in the following, in the $n^+(p^+) - p(n)$ junction solar cells, and for physical conditions as:

$W = 0.0044\ (0.000206)\mu m, N_{P(B)} = 10^{20}\ cm^{-3}, S = 10^{50}\ \frac{cm}{s}, N_{B(P)} = 10^{16}\ cm^{-3}$,

we propose, at given $V_{ocI1(2)}$ and $V_{ocII1(2)}$, the experimental results of the short circuit current density $J_{scI(II)}$ and the fill factor $F_{I(II)}$, in order to formulate our treatment method of two fixe experimental points. Then, for the $n^+ - p$ junction [1, 2, 23, 27, 28],

$$V_{ocI1(2)} = 624\ (740)\ mV,\ J_{scI1(2)} = 36.3\ (41.8)\ mA/cm^2,\ F_{I1(2)} = 80.1\ (82.7)\ \%, \text{ and} \quad (23)$$

for the $p^+ - n$ junction [1, 2, 30],

$$V_{ocII1(2)} = 639\ (738)\ mV,\ J_{scII1(2)} = 39.3\ (42.6)\ mA/cm^2,\ F_{II1(2)} = 78.9\ (84.9)\ \%. \quad (24)$$

3. Photovoltaic conversion effect at 300K

As defined and developed in I and II, the net current density J, at T=300 K and for the infinite shunt resistance, expressed as a function of the applied voltage V, flowing through the $n^+(p^+) - p(n)$ junction of silicon solar cells, is defined by [1, 2, 5-10]:

$$J(V) \equiv J_{ph.}(V) - J_{oI(II)} \times \left(e^{X_{I(II)}(V)} - 1\right),\ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V)\times V_T},\ \ V_T \equiv \frac{k_B T}{e} = 25.85\ mV, \quad (25)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor (**PVCF**), noting that as $V = V_{oc}$, $J(V) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, for $V_{oc} \geq V_{ocI(II)1}$. Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \quad (26)$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$ is the PVCF, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc})\times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, given in I and II, and (ii) the function $\left(e^{X_{I(II)}(V_{oc})}-1\right)$ or the PVCF $n_{I(II)}$, representing the photovoltaic conversion effect, thus converts the light, represented by $J_{scI(II)}$, into the electricity, by $J_{oI(II)}$.

Further, from Equations (22, 26), we obtain for the $n^+ - p$ junction:

$n_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}, J_{scI1(2)}\right) \equiv \frac{V_{ocI1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scI1(2)}}{J_{oI}}+1\right)} \equiv n_{I1(2)}\left(V_{ocI1(2)}, J_{scI1(2)}\right)$, and then,

$$n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = n_{I1}(V_{ocI1}, J_{scI1}) + n_{I2}(V_{ocI2}, J_{scI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.1248}, \quad (27)$$

being valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$, and then, for the $p^+ - n$ junction:

$n_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}, J_{scII1(2)}\right) \equiv \frac{V_{ocII1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scII1(2)}}{J_{oII}}+1\right)} \equiv n_{II1(2)}\left(V_{ocII1(2)}, J_{scII1(2)}\right)$,

and then,

$$n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = n_{II1}(V_{ocI1}, J_{scI1}) + n_{II2}(V_{ocII2}, J_{scII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.0939664}, \quad (28)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Therefore, from Equations (23, 24, 27, 28), one obtains, $n_{I1(II1)} = 1.2344$ (1.45827) at $V_{ocI1(II1)} = 624\ (639)$ mV, and $n_{I2(II2)} = 1.4534$ (1.67622) at $V_{ocI2(II2)} = 740\ (738)$ mV, respectively, for $n^+(p^+) - p(n)$ junction solar cells.

Thus, X_I defined from Eq. (26) now becomes for the $n^+ - p$ junction:

$X_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \equiv \frac{V_{oc}}{n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \times V_T}$, and therefore, we can determine the values of the fill factors $F_{I1(2)}$ at $V_{oc} = V_{ocI1(2)}$ by [1, 2]:

$F_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}\right) = \frac{X_I\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}\right) - \ln\left[X_I\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}\right) + 0.72\ (0.72)\right]}{X_I\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}\right) + 1.1\ (0.472)} \equiv F_{I1(2)}\left(V_{oc} = V_{ocI1(2)}\right)$, for a presentation simplicity, and further, the fill factor F_I can be computed by:

$$F_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = F_{I1}(V_{ocI1}) + F_{I2}(V_{ocI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{2.0559}, \quad (29)$$

which is valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Then, also from Eq. (26), we can define for the $p^+ - n$ junction:

$X_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) \equiv \frac{V_{oc}}{n_{II}(W,N_a,r_a,S, N_d,r_d,V_{oc})\times V_T}$, where $n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc})$ is determined in Eq. (28). Therefore, we can determine the values of the fill factors $F_{II1(2)}$ at $V_{oc} = V_{ocII1(2)}$ as:

$F_{II1(2)}(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}) = \frac{X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(2)}) - \ln[X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(2)}) + 0.72\ (0.00)]}{X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(2)}) + 0.895\ (0.00)} \equiv F_{II1(2)}(V_{ocII1(2)})$, for a presentation simplicity, and further, the fill factor F_{II} is determined by:

$$F_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = F_{II1}(V_{ocII1}) + F_{II2}(V_{ocII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.41011}, \quad (30)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Further, in the $n^+(p^+) - p(n)$ junction solar cells, and for physical conditions:

$W = 0.0044\ (0.000206)\mu m, N_{P(B)} = 10^{20}\ cm^{-3}, r_{P(B)}, S = 10^{50} \frac{cm}{s}, N_{B(P)} = 10^{16}\ cm^{-3}, r_{B(P)}$, Equations (29, 30) give: $F_{I1(2)}$=80.01% (82.7%) at $V_{ocI1(2)} = 624\ (740)$ for the $n^+ - p$ junction, and $F_{II1(2)}$=78.9% (84.9%) at $V_{ocII1(2)} = 639\ (738)\ mV$ for the $p^+ - n$ junction, respectively, being in perfect agreement with the data given in Equations (23, 24).

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{I(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}) \equiv \frac{J_{scI(II)} \times V_{oc} \times F_{I(II)}}{P_{in.}}, \quad (31)$$

where $J_{scI(II)}$ and $F_{I(II)}$ are determined respectively in Equations (26, 29, 30), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$) [1, 2, 26-29].

4. Numerical results and concluding remarks

Here, we will respectively consider the two following cases: (4.1) the absence of the impurity size effect (**ISE**) such as the transparent $THD[P(B) - Si]ER - LD[B(P) - Si]BR-$ cases, because of $r_{d(a)} = r_{P(B)} = r_{Si} = 0.117\ nm$, and (4.2) the presence of the ISE such as the $THD[S(Tl) - Si]ER - LD[Tl(S) - Si]BR-$ cases, because of $r_{d(a)} = r_{S(Tl)} = 0.1628\ (0.1410) nm > r_{Si}$.

4.1. $\mathbf{HD[P(B) - Si]ER - LD[B(P) - Si]BR}$ −cases

(4.1a) $\boldsymbol{HD(P - Si)ER - LD(B - Si)BR -}$ ***case***. Here, we have $r_{P(PB)} = r_{Si} = 0.117\ nm$, according also to the absence of ISE, and we propose the usual physical conditions as:

$W = 15\ \mu m,\ N_P = 5 \times 10^{20}\ cm^{-3}, S = 100\ (cm/s),$ and $N_B = 10^{16}(5 \times 10^{17}, 10^{18}) cm^{-3}$. (32)

Then, from Equations (12, 13, 18, 20, 26,27,29,31) and (C7, C8) of the Appendix C, on obtains:

$P = \frac{W}{L_{heff.}} = 1.19 \times 10^{-3},\ L_h = 0.37\ cm, I(W,S) = 0,\ \tau_{hE} = 40.4\ ms,\ \frac{\tau_{teff.}}{\tau_{hE}} = 7.09 \times 10^{-7} \ll 1,$

suggesting the hightly transparent HTER condition, and from Eq. (18), $J_{Eno} = 1.54 \times 10^{-13} \left(\frac{A}{cm^2}\right)$. Further, in the LD[B-Si]-BR and for $N_B = 10^{16}(5 \times 10^{17}, 10^{18})\ cm^{-3} < N_{Cr.}(r_B) = 4.06 \times 10^{18}\ cm^{-3}$, according respectively to the non-degenerate case: $\frac{-E_{Fp}}{k_B T} = -8.04\ (-4.12, -3.42) < -1$, one gets from Eq. (C1) of the Appendix C: $J_{Bpo} = 6.09\ (0.51, 0.33) \times 10^{-13} \left(\frac{A}{cm^2}\right)$. Therefore, one obtains respectively: $J_{oI} = 7.63\ (2.05, 1.87) \times 10^{-13} \left(\frac{A}{cm^2}\right)$, and from the following Table 2, for example, at $V_{oc} = 715\ mV$, n_I=1.11 (1.06, 1.05) and η_I= 27.68 (28.21, 28.25)%, meaning that, with increasing N_B, both J_{oI} and n_I decrease, while η_I increases, being new results.

Table 2. With the physical conditions given in Eq. (32) and for $N_B = 10^{16}(5 \times 10^{17}, 10^{18})\ cm^{-3}$ given in the BR, our numerical results of n_I, J_{scI}, F_I, and η_I, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_B, the function n_I decreases, while the functions: J_{scI}, F_I, and η_I increase, being new results.

$V_{oc}(mV)$	n_I	$J_{scI}(\frac{mA}{cm^2})$	$F_I(\%)$	$\eta_I(\%)$
750	1.17 (1.12, 1.11)	41.86 (42.12, 42.14)	86.32 (86.97, 87.01)	27.10 (27.47, 27.50)
740	1.16 (1.10, 1.09)	43.09 (43.44, 43.46)	85.82 (86.47, 86.51)	27.37 (27.80, 27.82)
738	1.15 (1.09, 1.09)	43.33 (43.69, 43.72)	85.73 (86.38, 86.42)	27.41 (27.85, 27.88)
737	1.15 (1.09, 1.08)	43.44 (43.82, 43.84)	85.68 (86.33, 86.37)	27.43 (27.88, 27.91)
718	1.12 (1.06, 1.06)	45.41 (45.92, 45.95)	84.88 (85.52, 85.57)	27.67 (28.20, 28.23)
715	1.11 (1.06, 1.05)	45.67 (46.19, 46.23)	84.77 (85.41, 85.45)	**27.68 (28.21, 28.25)**
706	1.10 (1.04, 1.04)	46.34 (46.91, 46.95)	84.45 (85.10, 85.14)	27.63 (28.18, 28.22)
705	1.10 (1.04, 1.04)	46.41 (46.98, 47.02)	84.42 (85.06, 85.10)	27.62 (28.17, 28.21)
702	1.09 (1.04, 1.03)	46.58 (47.17, 47.21)	84.32 (84.97, 85.01)	27.57 (28.14, 28.17)
700	1.09 (1.03, 1.03)	46.69 (47.28, 47.33)	84.26 (84.90, 84.95)	27.54 (28.10, 28.14)
695	1.08 (1.03, 1.02)	46.90 (47.52, 47.56)	84.12 (84.76, 84.80)	27.42 (27.99, 28.03)
680	1.06 (1.00, 1.00)	47.09 (47.73, 47.77)	83.74 (84.38, 84.42)	26.82 (27.38, 27.42)

667	1.04 (0.99, 0.98)	46.56 (47.17, 47.21)	83.49 (84.12, 84.16)	25.93 (26.47, 26.50)
665	1.04 (0.98, 0.98)	46.41 (47.01, 47.05)	83.45 (84.09, 84.13)	25.76 (26.29, 26.32)
655	1.02 (0.97, 0.97)	45.34 (45.86, 45.90)	83.32 (83.95, 83.99)	24.74 (25.22, 25.25)
643	1.00 (0.95, 0.95)	43.18 (43.58, 43.61)	83.20 (83.84, 83.88)	23.10 (23.49, 23.52)
632	0.99 (0.94, 0.94)	40.06 (40.27, 40.28)	83.15 (83.78, 83.82)	21.05 (21.32, 21.34)
624	0.98 (0.93, 0.93)	36.30 (36.30, 36.30)	83.14 (83.77, 83.81)	18.83 (18.97, 18.98)

(4.1b) $\boldsymbol{HD(B-Si)ER-LD(P-Si)BR-}$ ***case***. Here, we have $r_{B(P)} = r_{Si} = 0.117$ nm , according also to the absence of ISE, and we propose the usual physical conditions $[2, 13, 18, 20, 26]$,

$$W = 15\ \mu m,\ N_B = 5\times10^{20}\ cm^{-3}, S = 100\ (cm/s),\ \text{and}\ N_P = 10^{16}(5\times10^{17}, 10^{18})cm^{-3}. \quad (33)$$

Then, from Equations $(12, 13, 18, 20, 26, 28, 30, 31)$ and (C7, C8) of the Appendix C, one obtains:

$P = \frac{W}{L_{e,eff.}} = 8.89\times10^{-3}$, $L_e = 0.24$ cm, $I(W,S) = 0$, $\tau_{eE} = 0.017$ s, and $\frac{\tau_{t,eff.}}{\tau_{eE}} = 3.95\times10^{-5} \ll 1$, suggesting the HTER – condition, and from Eq. (18), $J_{Epo} = 2.06\times10^{-14}\ \left(\frac{A}{cm^2}\right)$. Further, in the LD[P-Si]-BR and for $N_P = 10^{16}(5\times10^{17}, 10^{18})\ cm^{-3} < N_{Cr.}(r_P) = 3.52\times10^{18}\ cm^{-3}$, according respectively to the non-degenerate case: $\frac{E_{Fn}}{k_BT} = -7.96(-4.04, -3.34) < -1$, one gets from Eq. (C1) of the Appendix C: $J_{Bno} = 7.27\ (0.92, 0.60)\times10^{-13}\ \left(\frac{A}{cm^2}\right)$. Therefore, one obtains: $J_{oII} = 7.47\ (1.12, 0.81)\times10^{-13}\ \left(\frac{A}{cm^2}\right)$, and from the following Table 3, for example, at $V_{oc} = 749\ (742, 742)$ mV, $n_{II} = 1.169\ (1.075, 1.062)$, η_{II} = 29.82 (30.02, 30.06)%, suggesting that, with increasing N_P, both J_{oII} and n_{II} decrease, while η_{II} increases, being also new results.

Table 3. With the physical conditions given in Eq. (33) and for $N_P = 10^{16}(5\times10^{17}, 10^{18})\ cm^{-3}$ given in the BR, our numerical results of n_{II}, J_{scII}, F_{II}, and η_{II}, are computed by using Equations (28, 26, 30, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_P, the function n_{II} decreases, while the functions J_{scII}, F_{II}, and η_{II} increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	FF(%)	η(%)
755	1.18 (1.09, 1.08)	42.30 (42.50, 42.54)	91.70 (92.61, 92.76)	29.81 (29.95, 29.98)
749	1.17 (1.09, 1.07)	42.92 (43.18, 43.22)	91.14 (92.04, 92.19)	**29.82** (30.00, 30.03)
739	1.15 (1.07, 1.06)	43.89 (44.23, 44.29)	90.22 (91.12, 91.26)	29.79 (**30.02, 30.06**)
733	1.14 (1.06, 1.05)	44.43 (44.81, 44.88)	89.69 (90.58, 90.73)	29.73 (29.99, 30.04)

726	1.13 (1.05, 1.04)	45.00 (45.44, 45.51)	89.09 (89.98, 90.12)	29.62 (29.91, 29.96)
723	1.13 (1.05, 1.03)	45.22 (45.68, 45.76)	88.83 (89.72, 89.86)	29.56 (29.87, 29.92)
712	1.11 (1.03, 1.02)	45.92 (46.44, 46.53)	87.94 (88.82, 88.96)	29.27 (29.60, 29.66)
700	1.09 (1.01, 1.00)	46.41 (47.00, 47.09)	87.03 (87.90, 88.04)	28.78 (29.14, 29.20)
687	1.07 (0.99, 0.98)	46.54 (47.13, 47.23)	86.12 (86.98, 87.12)	28.03 (28.39, 28.45)
680	1.06 (0.98, 0.97)	46.39 (46.97, 47.07)	85.67 (86.53, 86.67)	27.51 (27.86, 27.92)
670	1.04 (0.97, 0.96)	45.53 (46.36, 46.45)	85.08 (85.94, 86.07)	26.59 (26.91, 26.96)
660	1.03 (0.95, 0.94)	44.76 (45.20, 45.28)	84.56 (85.42, 85.55)	25.43 (25.69, 25.73)
655	1.02 (0.95, 0.94)	43.98 (44.36, 44.43)	84.33 (85.19, 85.33)	24.74 (24.95, 24.99)
650	1.01 (0.94, 0.93)	42.99 (43.28, 43.33)	84.14 (84.99, 85.13)	23.94 (24.10, 24.13)
645	1.01 (0.94, 0.92)	41.70 (41.89, 41.92)	83.98 (84.83, 84.96)	22.99 (23.10, 23.12)
640	1.00 (0.93, 0.92)	39.87 (39.91, 39.92)	83.86 (84.72, 84.85)	21.79 (21.81, 21.82)
639	1.00 (0.93, 0.92)	39.30 (39.30, 39.30)	83.85 (84.71, 84.84)	21.44 (21.44, 21.44)

4.2. $\mathbf{HD[Bi(In) - Si]ER - LD[In(Bi) - Si]BR - cases}$

(4.2a) $\boldsymbol{HD[Bi - Si]ER - LD[In - Si]BR}$ ***–case***. Here, we have $r_{Bi(In)} = 0.160\ (0.135)\text{nm} > r_{Si} = 0.117\ \text{ nm}$, according to the presence of ISE, and we propose the usual physical conditions:

$$W = 15\ \mu m,\ \ N_{Bi} = 5 \times 10^{20}\ cm^{-3}, S\ = 100\ (cm/s\ ),\ \text{and}\ \ N_{In} = 10^{17}(10^{18}, 5 \times 10^{18})\ cm^{-3}. \quad (34)$$

Then, from Equations (12, 13, 18, 20, 26,27,29,31) and (C7, C8) of the Appendix C , one obtains:

$P = \frac{W}{L_{heff.}} = 2 \times 10^{-4}$, $L_h = 8.28 \times 10^5$ cm, $I = 0$, $\tau_{hE} = 3.15 \times 10^{12}$ s, $\frac{\tau_{teff.}}{\tau_{hE}} = 2.02 \times 10^{-8} \ll 1$, suggesting the HTER-condition, and $J_{Eno} = 6.6 \times 10^{-23}\ \left(\frac{A}{cm^2}\right)$. Further, in the LD(In-Si)-BR and $N_{In} = 10^{17}(10^{18}, 5 \times 10^{18})\ cm^{-3}$, according respectively to the non-degenerate condition: $\frac{-E_{Fp}}{k_B T} =- 5.74\ (-3.42, -1.77) <- 1$, as that given in Eq. (A6) of the Appendix A, one gets, from Eq. (C1) of the Appendix C: $J_{Bpo} = 1.91(0.42, 0.22) \times 10^{-15}\ \left(\frac{A}{cm^2}\right)$. Therefore, one obtains: $J_{oI} = 1.91(0.42, 0.22) \times 10^{-15}\ \left(\frac{A}{cm^2}\right) = J_{Bpo}$, and from the following Table 4, for example, at $V_{oc} = 703$ mV, n_I=0.88 (0.84, 0.82) and η_I= 30.09 (30.70, **31**)%, respectively, noting that, with increasing N_{Tl}, both J_{oI} and n_I decrease, while η_I increases, being new results.

Table 4. With the physical conditions given in Eq. (34) and for $N_{In} = 10^{17}(10^{18},\ 5 \times 10^{18})\ cm^{-3}$ given in the BR, our numerical results of n_I, J_{scI}, F_I, and η_I, are computed by using Equations (27, 26, 29, 31),

respectively. Here, on notes that, for a given V_{oc} and with increasing N_{TI}, the function n_I decreases, while the functions J_{scI}, F_I, and η_I increase, being new results.

V_{oc}(mV)	n_I	$J_{sc}(\frac{mA}{cm^2})$	FF(%)	η(%)
750	0.94 (0.90, 0.88)	43.09 (43.44, 43.54)	88.92 (89.44, 89.66)	28.73 (29.12, 29.29)
740	0.93 (0.88, 0.87)	44.70 (45.11, 45.30)	88.41 (88.93, 89.15)	29.24 (29.69, 29.88)
738	0.93 (0.88, 0.86)	45.00 (45.44, 45.63)	88.31 (88.83, 89.05)	29.33 (29.79, 29.99)
737	0.93 (0.88, 0.86)	45.16 (45.60, 45.80)	88.26 (88.78, 89.01)	29.37 (29.84, 30.04)
718	0.90 (0.86, 0.84)	47.76 (48.36, 48.64)	87.44 (87.96, 88.18)	29.98 (30.54, 30.79)
715	0.90 (0.85, 0.84)	48.10 (48.73, 49.02)	87.33 (87.84, 88.06)	30.03 (30.61, 30.86)
705	0.88 (0.84, 0.82)	49.09 (49.78, 50.10)	86.97 (87.49, 87.70)	30.10 (30.70, 30.97)
703	0.88 (0.84, 0.82)	49.25 (49.96, 50.28)	86.90 (87.42, 87.64)	**30.09 (30.70, 31.00)**
702	0.88 (0.84, 0.82)	49.33 (50.04, 50.36)	86.87 (87.39, 87.61)	30.08 (30.70, 30.97)
700	0.88 (0.83, 0.82)	49.47 (50.19, 50.52)	86.81 (87.32, 87.54)	30.06 (30.68, 30.96)
695	0.87 (0.83, 0.81)	49.77 (50.51, 50.85)	86.66 (87.17, 87.39)	29.97 (30.60, 30.88)
680	0.85 (0.81, 0.79)	50.05 (50.82, 51.17)	86.27 (86.78, 87.00)	29.36 (29.99, 30.27)
667	0.83 (0.80, 0.78)	49.38 (50.12, 50.45)	86.01 (86.52, 86.74)	28.33 (28.92, 29.19)
665	0.83 (0.79, 0.78)	49.19 (49.91, 50.24)	85.98 (8649, 86.71)	28.12 (28.71, 28.97)
655	0.82 (0.78, 0.77)	47.79 (48.43, 48.72)	85.84 (86.35, 86.56)	26.87 (27.39, 27.62)
643	0.81 (0.77, 0.75)	45.01 (45.49, 45.70)	85.72 (86.23, 86.45)	24.81 (25.22, 25.40)
632	0.80 (0.76, 0.74)	41.02 (41.27, 41.38)	85.67 (86.18, 86.39)	22.21 (22.47, 22.59)
624	0.79 (0.75, 0.74)	36.30 (36.30, 36.30)	85.66 (86.16, 86.38)	19.40 (19.52, 19.57)

(4.2b) $HD(In-Si)ER-LD(Bi-Si)BR$ −case. Here, we have $r_{In(Bi)} = 0.135(0.160)\text{nm} > r_{Si} = 0.117$ nm, according to the presence of ISE, and we propose the usual physical conditions:

$$W = 15\ \mu m,\ N_{In} = 5\times 10^{20}\ cm^{-3}, S = 100\ (cm/s), N_{Bi} = 10^{17}(10^{18}, 5\times 10^{18})\ cm^{-3}. \quad (35)$$

Then, from Equations (12, 13, 18, 20, 26,28,30,31) and (C7, C8) of the Appendix C) , on obtains: $P = \frac{W}{L_{eeff.}} = 2.61\times 10^{-3}$, $L_{eE} = 0.575$ cm, $I = 1.29\times 10^{-67}$, $\tau_{eE} = 0.151$ s, $\frac{\tau_{teff.}}{\tau_{eE}} = 3.41\times 10^{-6} \ll 1$, according to the HTER − condition, and $J_{Epo} = 1.75\times 10^{-15}\left(\frac{A}{cm^2}\right)$. Further, in the LD[Bi-Si]-BR and for $N_{Bi} = 10^{17}(10^{18}, 5\times 10^{18})\ cm^{-3}$, according respectively to the non-degenerate case: $\frac{E_{Fn}}{k_BT} =-5.64(-3.33,-1.67) <-1$, one gets: $J_{Bno} =$

$5.68(1.54,\ 0.688) \times 10^{-14} \left(\frac{A}{cm^2}\right)$. Therefore, one obtains: $J_{oII} = 5.85(1.71, 0.86\ ) \times 10^{-14} \left(\frac{A}{cm^2}\right) \simeq J_{Bno}$, and from the following Table 5, for example, n_{II}=1.04 (0.99, 0.97) and η_{II} = 30.37 (30.55, 30.65)% at $V_{oc} = 733$ mV, respectively, noting that, with increasing N_{Bi}, both J_{oII} and n_{II} decrease, while η_{II} increases, being new results.

Table 5. With the physical conditions given in Eq. (35) and for $N_{Bi} = 10^{17}(10^{18},\ 5 \times 10^{18})\ cm^{-3}$ given in the BR, our numerical results of n_{II}, J_{scII}, F_{I}, and η_{II}, are computed by using Equations (28, 26, 30, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_S, the function n_{II} decreases, while the functions J_{scII}, F_{II}, and η_{II} increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	FF(%)	η(%)
743	1.05(1.00,0.98)	43.93(44.13,44.24)	91.77(92.28,92.55)	30.38(30.53,30.61)
741	1.05(1.00,0.98)	44.14(44.35,44.49)	91.59(92.10,92.37)	30.39(30.54,30.63)
739	1.04(1.00,0.98)	44.35(44.57,44.69)	91.41(91.91,92.18)	30.39(30.55,30.64)
734	1.04(0.99,0.97)	44.85(45.10,45.24)	90.96(91.46,91.73)	30.37(30.55,30.65)
733	1.04(0.99,0.97)	44.95(45.20,45.34)	90.87(91.38,91.64)	**30.37(30.55,30.65)**
723	1.02(0.98,0.95)	45.84(46.14,46.31)	90.00(90.51,90.77)	30.26(30.47,30.58)
712	1.00(0.96,0.94)	46.63(46.97,47.16)	89.10(89.60,89.86)	30.01(30.24,30.37)
700	0.99(0.94,0.92)	47.19(47.56,47.78)	88.18(88.67,88.94)	29.55(29.80,29.93)
687	0.97(0.93,0.90)	47.34(47.72,47.94)	87.26(87.75,88.01)	28.79(29.04,29.17)
680	0.96(0.92,0.90)	47.17(47.55,47.76)	86.81(87.30,87.55)	28.25(28.49,28.62)
670	0.95(0.90,0.88)	46.54(46.89,47.09)	86.21(86.70,86.95)	27.28(27.49,27.61)
660	0.93(0.89.0.87)	45.36(45.65,45.81)	85.69(86.17,86.43)	26.03(26.20,26.30)
655	0.93(0.89,0.86)	44.49(44.74,44.88)	85.46(85.95,86.20)	25.27(25.42,25.50)
650	0.92(0.88,0.86)	43.39(43.58,43.68)	85.26(85.75,86.00)	24.40(24.51,24.58)
645	0.91(0.87,0.85)	41.95(42.08,42.14)	85.10(85.58,85.84)	23.36(23.44,23.48)
640	0.91(0.87,0.85)	39.93(39.96,39.97)	84.99(85.47,85.72)	22.03(22.06,22.07)
639	0.91(0.87,0.85)	39.30(39.30,39.30)	84.98(85.46,85.71)	21.65(21.66,21.66)

In conclusion, our values of limiting highest efficiency, obtained in Tables 2-5, are reported, as:

$\boldsymbol{\eta_{I(II)}}$ **= 28.25 (30.06) %**, obtained in Tables 2 and 3 for the hightly transparent $HD[P(B) - Si]ER - LD[B(P) - Si]BR -$ cases, with $E_{gi}(r_{P(B)}) = 1.12(1.12)$ eV , $S = 100$ (cm/s), $\tau_{h(e)E} = 40.4$ ms (0.017 s) and $W = 15$ μm, (36)

$\mathbf{\eta_{I(II)}}$ = **30.06 (31.00) %**, obtained in those Tables 4 and 5 for the hightly transparent $\mathrm{HD[Bi(In)-Si]ER-LD[In(Bi)-Si]BR}$ −cases, with $E_{gi}(r_{Bi(In)}) = 1.724\ (1.216)$ eV, $S = 100\ (\mathrm{cm/s})$, and $W = 15\ \mu m$. (37)

Our values of $\mathbf{\eta_{I(II)}}$ = **28.25 (30.06) %**, given in Eq. (36), can be compared respectively with other limiting η-results equal to: **29.43%** for a 110 μm thick solar cell made of intrinsic silicon, being obtained by Richter et al. [26], and **30%** for $E_{gi}(r_{P(B)}) = 1.1$ eV, being obtained by Shockley and Queisser [6].

Finally, our values of $\mathbf{\eta_{I(II)}}$ = **30.06 (31.00) %**, given in Eq. (37) can also be compared with other limiting result ($\mathbf{\eta = 31}$%) for physical conditions: $S = 100\ \mathrm{cm/s}$ and $W = 15\ \mu m$.

Acknowledgments: We thank Drs A. Pivet, C.T. Huynh-Pivet, C.V. Huynh, A.L. Pivet and I. Pivet for their continuous interest in this work.

Appendix

Appendix A. Fermi Energy

In the n(p)-type semiconductor, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11×10^{-4} [39, 40], is now summarized in the following.

First of all, we define the reduced electron density by:

$$u \equiv \frac{N_{d(a)}}{N_{c(v)}},\ N_c(T, r_d) = 12 \times \left(\frac{m_c(T,r_d)\times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (\mathrm{cm^{-3}}),\ N_v(T) = 4 \times \left(\frac{m_v(T)\times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (\mathrm{cm^{-3}}). \quad \text{(A1)}$$

Here, $N_{c(v)}$ is the conduction (valence)-band density of states, respectively, $m_c(T, r_d)$ is the effective mass of the electron in n-type Si can be defined by [31, 32]:

$$m_c(T, r_d) = 0.3216 \times m_o \times \left(\frac{E_{go}(r_d)}{E_{gi}(T,r_d)}\right)^{2/3}, \quad \text{(A2)}$$

where m_o being the electron rest mass, the effective mass of the hole in the p-type Si yields [31, 32]:

$$m_v(T) = g_v^{-2/3} \times \left(\frac{0.443587+0.3609528\times10^{-2}T+0.1173515\times10^{-3}T^2+0.1263218\times10^{-5}T^3+0.3025581\times10^{-8}T^4}{1+0.4683382\times10^{-2}T+0.2286895\times10^{-3}T^2+0.7469271\times10^{-6}T^3+0.1727481\times10^{-8}T^4}\right)^{2/3}, \quad \text{(A3)}$$

which gives $m_v(T = 0\ K) = m_{vo} = 0.3664 \times m_o$, and finally, $E_{gin(p)}(T, r_{d(a)})$ is the intrinsic band gap in the silicon (Si), due to the T-dependent carrier-lattice interaction-effect, by [1, 2, 33, 34]:

$$E_{gin(p)}(T, r_{d(a)}) \simeq E_{gon(p)}(r_{d(a)}) - 0.071\ (eV) \times \left\{\left[1 + (\frac{2T}{440.6913\ K})^{2.201}\right]^{\frac{1}{2.201}} - 1\right\}, \quad (A4)$$

being due to the d(a)-size effect are given in Table 1.

Furthermore, in the n(p)-type Si, one can define the intrinsic carrier concentration $n_{in(p)}$ by:

$$n_{i\,n(p)}^{2}(T, r_{d(a)}) \equiv N_c(T, r_d) \times N_v(T) \times \exp\left(\frac{-E_{gin(p)}(T, r_{d(a)})}{k_B T}\right). \quad (A5)$$

Then, denoting the reduced Fermi energy in the n(p)-type semiconductor, respectively by: $\frac{E_{Fn}(u)}{k_B T} (\frac{-E_{Fp}(u)}{k_B T})$, we found with a precision of the order of 10^{-7} [39], as:

$$\frac{E_{Fn}(u)}{k_B T} (\frac{-E_{Fp}(u)}{k_B T}) = \frac{G(u) + Au^B F(u)}{1 + Au^B}, A = 0.0005372 \text{ and } B = 4.82842262 \quad (A6)$$

where

$F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [(3\sqrt{\pi}/4) \times u]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$

and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$,

noting that: (i) $\frac{E_{Fn}(u \gg 1)}{k_B T} (\frac{-E_{Fp}(u \gg 1)}{k_B T}) > 1$, according to the HD[d(a)-Si]ER-case (i.e., the degenerate case), Eq. (A6) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u \ll 1)}{k_B T} (\frac{-E_{Fp}(u \ll 1)}{k_B T}) < -1$, to the LD[a(d)-Si]BR-case (i.e., the non-degenerate case), Eq. (A6) is reduced to the function G(u), respectively. Then, Eq. (A6) can be applied to the following cases as:

(i) in the HD[P(B)-Si]ER-case, for $N_{P(B)} = 10^{20}(10^{20})\ cm^{-3}$, we get: $\frac{E_{Fn}(u \gg 1)}{k_B T} (\frac{-E_{Fp}(u \gg 1)}{k_B T}) = 2.6\ (2.39) > 1$, and in the HD[Bi(In)-Si]ER-case, for $N_{Bi(In)} = 3(1) \times 10^{20} cm^{-3}$ we respectively get: $\frac{E_{Fn}(u \gg 1)}{k_B T} (\frac{-E_{Fp}(u \gg 1)}{k_B T}) = 6.1\ (2.40) > 1$, respectively, and

(ii) in the LD[B(P)-Si]BR-case, for $N_{B(P)} = 10^{16}(5 \times 10^{17},\ 10^{18})\ cm^{-3}$, we respectively get: $\frac{-E_{Fp}(u \ll 1)}{k_B T} = -8.04\ (-4.12,\ -3.42) < -1$ and $\frac{E_{Fn}(u \ll 1)}{k_B T} = -7.96\ (-4.04,\ -3.34) < -1$, and in the LD[In(Bi)-Si]BR-case, for $N_{In(Bi)} = 10^{17}(10^{18},\ 5 \times 10^{18})\ cm^{-3}$, we obtain: $\frac{-E_{Fp}(u \ll 1)}{k_B T} = -5.74\ (-3.42,\ -1.77) < -1$ and $\frac{E_{Fn}(u \ll 1)}{k_B T} = -5.64\ (-3.33, -1.67) < -1$, respectively. Those numerical results thus confirm the choice of the limiting $N_{a(d)}$-values, as those given in Tables 2-5.

Appendix B. Approximate forms for band gap narrowing and apparent band gap narrowing

First of all, in the n(p)-type Si, we define the effective Wigner-Seitz radius r_s characteristic of the interactions by [1, 2]

$$r_{sn} \equiv r_s(N_d, T, r_d) = 1.1723 \times 10^8 \times \left(\frac{6}{N_d}\right)^{1/3} \times \frac{m_c(T, r_d)}{\varepsilon(r_d)} \quad \text{(B1)}$$

and

$$r_{sp} \equiv r_s(N_a, T, r_a) = 1.1723 \times 10^8 \times \left(\frac{2}{N_a}\right)^{1/3} \times \frac{m_v(T)}{\varepsilon(r_a)}, \quad \text{(B2)}$$

where $m_c(T, r_d)$ and $m_v(T)$ are given in (A2) and (A3). Therefore, the correlation energy of an effective electron gas, $E_c(r_s)$, is given by [1, 2, 42]:

$$E_{cn(cp)}\left(N_{d(a)}, T, r_{d(a)}\right) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - \ln(2)]}{\pi^2}\right) \times \ln\left(r_{sn(sp)}\right) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}. \quad \text{(B3)}$$

Then, in the n-type heavily doped Si, the BGN is found to be given from I as

$$\Delta E_{gn}(N_d, r_d) \simeq a_1 \times \frac{\varepsilon(r_P)}{\varepsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_P)}{\varepsilon(r_d)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sn}) \times r_{sn}]\right) + a_3 \times \left[\frac{\varepsilon(r_P)}{\varepsilon(r_d)}\right]^{5/4} \times \sqrt{\frac{m_v(T)}{m_c(T, r_d)}} \times \left[\frac{m_c(T, r_d)}{m_c(T, r_P)}\right]^{\frac{1}{4}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_P)}{\varepsilon(r_d)}} \times \sqrt{\frac{m_c(T, r_P)}{m_c(T, r_d)}} \times N_r^{1/2} \times \left[1 + \sqrt{\frac{m_c(T, r_d)}{m_c(T, r_P)}}\right] + a_5 \times \left[\frac{\varepsilon(r_P)}{\varepsilon(r_d)}\right]^{\frac{3}{2}} \times \sqrt{\frac{m_c(T, r_d)}{m_c(T, r_P)}} \times N_r^{\frac{1}{6}}, \; N_r \equiv \left(\frac{N_d}{9.999 \times 10^{17}\ \text{cm}^{-3}}\right), \quad \text{(B4)}$$

where $a_1 = 3.8 \times 10^{-3}(\text{eV})$, $a_2 = 6.5 \times 10^{-4}(\text{eV})$, $a_3 = 2.8 \times 10^{-3}(\text{eV})$, $a_4 = 5.597 \times 10^{-3}(\text{eV})$ and $a_5 = 8.1 \times 10^{-4}(\text{eV})$, and in the p-type heavily doped Si, from II, one has

$$\Delta E_{gp}(N_a, r_a) \simeq a_1 \times \frac{\varepsilon(r_B)}{\varepsilon(r_a)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_B)}{\varepsilon(r_a)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sp}) \times r_{sp}]\right) + a_3 \times \left[\frac{\varepsilon(r_B)}{\varepsilon(r_a)}\right]^{5/4} \times \sqrt{\frac{m_c(T, r_P)}{m_v(T)}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_B)}{\varepsilon(r_a)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon(r_B)}{\varepsilon(r_a)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, \; N_r \equiv \left(\frac{N_a}{9.999 \times 10^{17}\ \text{cm}^{-3}}\right), \quad \text{(B5)}$$

where $a_1 = 3.15 \times 10^{-3}(\text{eV})$, $a_2 = 5.41 \times 10^{-4}(\text{eV})$, $a_3 = 2.32 \times 10^{-3}(\text{eV})$, $a_4 = 4.12 \times 10^{-3}(\text{eV})$ and $a_5 = 9.80 \times 10^{-5}(\text{eV})$.

Further, in the donor (acceptor)-Si, we define the effective intrinsic carrier concentration $n_{ien(p)}$, by

$$n_{i\,en(p)}^2(N_{d(a)}, r_{d(a)}) \equiv N_{d(a)} \times p_o(n_o) \equiv n_{i\,n(p)}^2 \times \exp\left[\frac{\Delta E_{gan(p)}}{k_B T}\right], \quad \text{(B6)}$$

where we can define the "effective doping density" by: $N_{d(a)eff.} \equiv N_{d(a)}/\exp\left[\frac{\Delta E_{ga\,n(p)}}{k_B T}\right]$ so that $N_{d(a)eff.} \times p_o(n_o) \equiv n_{i\,n(p)}^2$ [8], and also the apparent band gap narrowing (**ABGN**), $\Delta E_{ga\,n(p)}$, as

$$\Delta E_{ga\,n(p)} \equiv \Delta E_{g\,n(p)} + k_B T \times \ln\left(\frac{N_{d(a)}}{\mathbf{N_{c(v)}}}\right) - E_{Fn}\left(\frac{N_d}{N_c}\right)[-E_{Fp}\left(\frac{N_a}{N_v}\right)], \quad \text{(B7)}$$

where $N_{c(v)}$ is defined in Eq. (A1), the Fermi energy is determined in Eq. (A6).

Appendix C. Minority-carrier transport parameters

Here, the minority-electron (hole) saturation current density injected into the LD[a(d)-Si]BR, with an acceptor density equal to $N_{a(d)}$, is given in I and II by [1, 7]:

$$J_{Bp(n)o}(N_{a(d)}, r_{a(d)}) = \frac{e \times n_i^2(r_{d(a)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{e(h)B}(N_{a(d)})}}}{N_{a(d)}}, \quad \text{(C1)}$$

where $n_{i\,n(p)}^2(r_{d(a)})$ is determined in (A5), $D_{e(h)}(N_{a(d)}, r_{a(d)})$ is the minority-hole (electron) diffusion coefficient:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[92 + \frac{1360-92}{1+\left(\frac{N_a}{1.3\times10^{17}\,cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a)}{\varepsilon(r_B)}\right)^2 (cm^2 V^{-1} s^{-1}), \quad \text{(C2)}$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[130 + \frac{500-130}{1+\left(\frac{N_d}{8\times10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d)}{\varepsilon(r_P)}\right)^2 (cm^2 V^{-1} s^{-1}), \quad \text{(C3)}$$

and $\tau_{h(e)B}(N_{d(a)})$ is the minority-hole (electron) lifetime in the base region:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{2.5\times10^{-3}} + 3\times10^{-13}\times N_a + 1.83\times10^{-31}\times N_a^2. \quad \text{(C4)}$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{2.5\times10^{-3}} + 11.76\times10^{-13}\times N_d + 2.78\times10^{-31}\times N_d^2, \quad \text{(C5)}$$

Further, from (A6), (B4)-(B7)), in the HD[d(a)-Si]ER, we can define the following minority-hole(electron) transport parameter $F_{h(e)}$ as [8, 22, 25]:

$$F_{h(e)}\,(N_{d(a)}, r_{d(a)}) \equiv \frac{n_{i\,n(p)}^2(r_{d(a)})}{p_o(n_o)\times D_{h(e)}} = \frac{N_{d(a)eff.}}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}\times \exp\left[\frac{\Delta E_{g\,an(p)}}{k_B T}\right]} (cm^{-5}\times s), \quad \text{(C6)}$$

Furthermore, the minority-hole (electron) diffusion length, $L_{h(e)}(N_{d(a)}, r_{d(a)})$ and the minority-hole(electron) lifetime $\tau_{h(e)E}$ in the HD[d(a)-Si]ER can be determined by

$$L_{h(e)}^{-2}(N_{d(a)}, r_{d(a)}) = [\tau_{h(e)E}\times D_{h(e)}]^{-1} = (C\times F_{n(p)})^2 = \left(C\times\frac{N_{d(a)eff.}}{D_{h(e)}}\right)^2 = \left(C\times\frac{n_{i\,n(p)}^2(r_{d(a)})}{p_o(n_o)\times D_{h(e)}}\right)^2, \quad \text{(C7)}$$

where the constant $C[=10^{-17}\,(cm^4/s)]$ was chosen in I and II, and then, $\tau_{h(e)E}$ can be computed by:

$$\tau_{h(e)E} = \frac{1}{D_{h(e)}\times(C\times F_{n(p)})^2}. \quad \text{(C8)}$$

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