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31.474 (44.359) %- Limiting Highest Efficiencies obtained in the $n^+(p^+) - p(n)$ Crystalline GaAs Junction Solar Cells at T=300 K, Due to The Effects of Heavy (Low) Doping and Impurity Size

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Abstract:

In our recent works [1, 2], by basing on: **(1)** the effects of heavy(light) doping and donor (acceptor), d(a), size , which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{En(p)o}(J_{Bp(n)o})$, being injected respectively into the heavily doped donor (acceptor)-GaAs emitter-lightly doped acceptor (donor)-Si base regions, HD[d(a)-Si]ER-LD[a(d)- Si]BR, of $n^+(p^+) - p(n)$ junction solar cells, respectively, **(2)** an effective Gaussian donor-density profile to determine $J_{En(p)o}$, and **(3)** the use of two experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, short circuit current density $J_{scI(II)}$, fill factor $F_{I(II)}$, and finally efficiency $\eta_{I(II)}$. Then, the limiting highest efficiencies, 31% (30.65%), were obtained in $n^+(p^+) - p(n)$ junction solar cells at 300K.

In the present work, by basing on such a treatment method, and using the physical conditions such as: $W = 15\ \mu m$, $N_{Sb(In)} = 10^{19}\ (10^{20})\ cm^{-3}$ and $S = 100\ (cm/s)$, according to the highly transparent HD[Sb(In)-GaAs]ER, and then $N_{In(Sb)} = 10^{18}\ (10^{17}) cm^{-3}$ for LD[In(Sb)-

GaAs]BR, of $n^+(p^+) - p(n)$ junction solar cells, we get respectively the maximal values of $\eta_{I(II)}$, $\eta_{I(II)-max.}$ =31.474% (44.359%), as those observed in Tables 3 et 5, which can be compared with the result of $\eta = 29.1\,\%$, obtained for GaAs-thin film cell, and $\eta = 45.7\,\%$, for GaInP/GaAs/GaInAs/GaInAs multijunction cell, by Green et al. [3].

Keywords: donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; photovoltaic conversion efficiency

1. Introduction

In our recent works [1, 2], by basing on: **(i)** the heavy doping and impurity size effects, which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, where those $J_{En(p)o}$ ($J_{Bp(n)o}$) are injected respectively into the heavily doped donor (acceptor)-Si emitter-lightly doped acceptor (donor)-GaAs base-regions, HD[d(a)-Si]ER-LD[a(d)-Si]BR, of the $n^+(p^+) - p(n)$ junction solar cells, denoted by I(II), respectively, **(ii)** an effective Gaussian donor (acceptor)-density profile $\rho_{d(a)}$ to determine $J_{En(p)o}$[1, 2, 13, 18-20, 22] and **(iii)** the use of two fixed experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, the short circuit current density $J_{scI(II)}$, the fill factor $F_{I(II)}$, and finally the efficiency $\eta_{I(II)}$ [1- 45]. These physical quantities were expressed as functions of the open circuit voltage V_{oc}, and various parameters such as: the emitter thickness W, high donor (acceptor) density $N_{d(a)}$, surface recombination velocity S , given in the HD[d(a)-Si]ER, and low acceptor (donor) density $N_{a(d)}$, in the LD[a(d)-Si]BR.

Further, we remark [1, 2] that: (a) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations and strongly affect other ($J_{scI(II)}$, $F_{I(II)}$, $\eta_{I(II)}$)-results, and (b) for a given V_{oc}, and with decreasing S and increasing W, while both $n_{I(II)}$ and $J_{oI(II)}$ decrease from the completely transparent emitter region (CTER), as $S \to \infty$, to the completely opaque emitter-region (COER), as $S \to 0$, $J_{scI(II)}$, $F_{I(II)}$, and $\eta_{I(II)}$ therefore increase from the CTER-case to the COER-one.

Then, in our present work, we have used such a treatment method [1, 2] to determine the numerical results of $J_{scI(II)}$, $F_{I(II)}$, and $\eta_{I(II)}$, given in HD[d(a)-GaAs]ER- LD[a(d)-GaAs]BR of the $n^+(p^+) - p(n)$ junction solar cells, obtained in the CTER-cases, as those given in Tables 2-5. In particular, in the CTHD(Sb-GaAs)ER-LD(In-GaAs)BR of the $n^+ - p$ junction solar cell, the maximal value of η_{I}, is found to be given by: $\eta_{I-max.}$ =**31.474 %**, as observed in Table 3, and in the CTHD(In-GaAs)ER-LD(Sb-GaAs)BR of the $p^{+}-n$ junction solar cell, by: $\eta_{II-max.}$ =**44.359 %**, as obtained in Table 5.

In Section 2, all the results energy-band-structure parameters for d(a)- GaAs systems are reported in Table 1, and the expressions for $J_{En(p)o}$ are also reported, so that we can determine the total (or dark) carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, where $J_{Bp(n)o}$ is determined in Eq. (C1) of the Appendix C. In Section 3, the photovoltaic effect is presented. Finally, some numerical results and concluding remarks are given and discussed in Section 4.

2. Energy-Band-Structure Parameters and dark minority-carrier saturation current density, due to impurity-size and heavy doping effects

Here, we now present the effects of donor (acceptor) [d(a)]-size and heavy doping, taken on the energy-band-structure parameters and minority-carrier saturation current density, as follows.

2.1. Effect of d(a)-size

In d(a)-GaAs-systems at T=0 K, since the d(a)-radius $r_{d(a)}$, in tetrahedral covalent bonds [8], is usually either larger or smaller than the atom-radius $r_{As(Ga)} = 0.118\ (0.126)$ nm, $1\ \mathrm{nm} = 10^{-9}\mathrm{m}$, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_{As(Ga)}$ ($r_{d(a)} < r_{As(Ga)}$), respectively, due to the d(a)-size effect [42]. From the numerical results of the effective dielectric constant, $\varepsilon(r_{d(a)})$, obtained from such a deformation potential energy model [42], for 0.113(0.117) $nm \leq r_{d(a)} \leq$ 0.163 (0.144) nm, we can propose its simple approximate form as:

$$\varepsilon(r_{d(a)}) \simeq 12.85 \times \left(\frac{r_{As(Ga)}}{r_{d(a)}}\right)^{4.377\ (4.7)}, \tag{1a}$$

being accurate to within 10% (7%), respectively, and equal to 12.85 as $r_{d(a)} = r_{As(Ga)}$, according to the absence of the impurity size effect, and decreased (increased) with increasing (decreasing) $r_{d(a)}$. This $r_{d(a)}$ -effect thus affects the changes in all the energy-band-structure parameters, expressed in terms of $\varepsilon(r_{d(a)})$. In particular, the changes in the unperturbed intrinsic band gap at 0K, $E_{go}(r_{As(Ga)}) = 1.519$ eV, and effective d(a)-ionization energy in absolute values $E_{do(ao)}(r_{As(Ga)}) = 0.0055\ (0.0371)$ eV, are obtained in an effective Bohr model, as [42]:

$$E_{gon(p)}(r_{d(a)}) - E_{go}(r_{As(Ga)}) = E_{do(ao)}(r_{d(a)}) - E_{do(ao)}(r_{As(Ga)}) = E_{do(ao)}(r_{As(Ga)}) \times \left[\left(\frac{\varepsilon(r_{As(Ga)})}{\varepsilon(r_{d(a)})}\right)^{2} - 1\right]. \tag{1b}$$

Therefore, with increasing $r_{d(a)}$, the effective dielectric constant $\varepsilon(r_{d(a)})$, determined above, decreases, implying that $E_{go}(r_{d(a)})$ and $E_{do(ao)}(r_d)$ increase, as observed in the following Table 1.

Table 1. Impurity size effects on the effective dielectric constant $\varepsilon(r_{d(a)})$, determined in Eq. (1a), the intrinsic band gap $E_{gin(p)}(r_{d(a)})$, determined in Equations (1a) and Eq. (A4), and the intrinsic carrier concentration $n_{in(p)}$, calculated using Eq. (A4) of the Appendix A.

Donor	**As**	**Te**	**Sb**
r_d (nm) [8]	0.118	0.132	0.136
$\varepsilon(r_d)$	12.85	7.87	6.91
$E_{gin}(0K, r_d)$	1.519 eV	1.528 eV	1.533 eV
$E_{gin}(300K, r_d)$	1.422 eV	1.432 eV	1.436 eV
$n_{in}(300K, r_d)$ in 10^6 cm^{-3}	2.12	1.71	1.62
Acceptor	**Ga**	**Al**	**In**
r_a (nm) [8]	0.126	0.126	0.144
$\varepsilon(r_a)$	12.85	12.85	6.86
$E_{gip}(0K, r_a)$	1.519 eV	1.519 eV	1.612eV
$E_{gip}(300K, r_a)$	1.422 eV	1.422 eV	1.516 eV
$n_{ip}(300K, r_a)$ in 10^6 cm^{-3}	2.12	2.12	3.5×10^{-1}

In summary, the effects of $N_{d(a)}$-heavy doping and $r_{d(a)}$- impurity size given in the HD[d(a)-GaAs]ER, and those of $N_{a(d)}$-low doping in the LD[a(d)-GaAs]BR, affect all the minority-carrier transport properties, given in the Appendix A, B and C, and in the following equations.

2.2. Total minority-carrier saturation current density at 300K

The total carrier-minority saturation current density is defined by:

$$J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}, \tag{2}$$

where $J_{Bp(n)o}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-Si]BR, being determined in Eq. (C1) of the Appendix C, and $J_{En(p)o}$ is the minority-hole saturation-current density injected into the HD[d(a)-Si]ER.

In the non-uniformly and heavily doped emitter region of d(a)-Si devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in the HD[d(a)-Si]ER-width W:

$$\rho_{d(a)}(x) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]^{-\left(\frac{x}{W}\right)^2}, \quad 0 \le x \le W,$$

$$N_{d(a)o}(W) \equiv 7.9 \times 10^{17}\ (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2\ (1) 10^{-7}\ \mathrm{cm}}\right)^{1.066\ (0.5)}\right\}\ (\mathrm{cm}^{-3}), \tag{3}$$

where $\rho_{d(a)}(x = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(x = W) = N_{d(a)o}(W)$, decreasing with increasing W [1, 2, 13]. Further, the "effective doping density" is defined by:

$$N_{d(a)eff.}(x, r_{d(a)}) \equiv \rho_{d(a)}(x)/\exp\left[\frac{\Delta E_{ga\ n(p)}(\rho_{d(a)}(x), r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)eff.}(x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{ga\ n(p)}\left(N_{d(a)}, r_{d(a)}\right)}{k_B T}\right]} \text{ and } N_{d(a)eff.}(x = W,\ r_{d(a)}) \equiv \frac{N_{d(a)o}(W)}{\exp\left[\frac{\Delta E_{ga\ n(p)}\left(N_{d(a)o}(W), r_{d(a)}\right)}{k_B T}\right]}, \tag{4}$$

where $\Delta E_{ga\ n(p)}$ are determined in Equations (B4, B5) of the Appendix B.

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n_{in(p)}^2}{N_{d(a)eff.}(x,\ r_{d(a)})}, \tag{5}$$

where $n_{in(p)}^2$ is determined in (A5) of the Appendix A and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(x)[n(x)]$ and $p_o(x)[n_o(x)]$, by [22, 25]:

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)])}{p_o(x)[n_o(x)]}, \tag{6}$$

which must verify the two following boundary conditions proposed by Shockley as [6]:

$$u(x = 0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}, \tag{7}$$

$$u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1. \tag{8}$$

Here, $n_{I(II)}(V)$ is a photovoltaic conversion factor determined in Equations (27, 28), $S\ (\frac{cm}{s})$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(x)$.

Further, as developed in I and II, from the Fick's law for minority hole (electron)-diffusion equations [8, 12]:

$$J_{h(e)}(x) = \frac{-e(+e)\times n_i^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e)n_{in(p)}^2 D_{h(e)}(x)}{N_{d(a)eff.}(x)} \times \frac{du(x)}{dx}, \quad (9)$$

where $N_{d(a)eff.}$ is given in Eq. (4), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (C3, C2, C6) of the Appendix C, and from the minority-hole (electron) continuity equation [8, 12]:

$$\frac{dJ_{h(e)}(x)}{dx} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{F_{h(e)}(x)\times L_{h(e)}^2} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{N_{d(a)eff.}(x)\times \tau_{h(e)E}}, \quad (10)$$

where $L_{h(e)}$ and $\tau_{h(e)E}$ are defined respectively in Equations (C7, C8) of the Appendix C, one finally obtains the following second-order differential equation as [22]:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0. \quad (11)$$

Then, taking into account the two boundary conditions (7, 8), one thus gets the general solution of this Eq. (11), as [22]:

$$u(x) = \frac{\sinh(P(x)) + I(W,S)\times\cosh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \; I(W,S) = \frac{D_{h(e)}(N_o(W))}{S\times L_{h(e)}(N_o(W))}. \quad (12)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, determined in Eq. (29). Further, since $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$, $C = 2.0893\times 10^{-30}\ (cm^4/s)$, for the crystalline Si, being an empirical parameter, chosen for each crystalline semiconductor, P(x) is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}), \; 0 \le x \le W, \; P(x = W) \equiv (\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}) \times W \equiv \frac{W}{L_{h(e)eff.}} = \frac{L_{h(e)}}{L_{h(e)eff.}} \times \frac{W}{L_{h(e)}}, \quad (13)$$

where $L_{h(e)eff.}$ is the effective minority-hole (electron) diffusion length. Further, from Eq. (9, 13), the minority-hole (electron) current density injected into the HD[d(a)-GaAs]ER is found to be determined by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S)\ [J_{Epo}(x, W, N_a, r_a, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1\right), \quad (14)$$

where $J_{En(p)o}$ is the saturation minority-hole (electron) current density,

$$J_{En(p)o}(x, W, N_{d(a)}, r_{d(a)}, S) = \frac{e n_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S)\times\sinh(P(x))}{\sinh(P(W)) + I(W,S)\times\cosh(P(W))}. \quad (15)$$

Here, the intrinsic carrier concentration $n_{i\,n(p)}$ is computed by Eq. (A5) of the Appendix A, and the effective doping density $N_{d(a)eff.}$ is determined in Eq. (4), the minority-hole (electron) diffusion coefficient $D_{e(h)}$ and minority-hole (electron) diffusion length $L_{h(e)}$ are given respectively in Equations (C2, C3, C7) of the Appendix C, and the factor $I(W, S)$ is determined by:

$$\mathrm{I(W,S)} = \frac{\mathrm{D_{h(e)}(N_{d(a)o}(W))}}{\mathrm{S \times L_{h(e)}(N_{d(a)o}(W))}}, \tag{16}$$

where $\mathrm{N_{d(a)o}(W)}$ is determined in Eq. (3).

Further, one remarks that: (i) from Equations (12, 14-16) one obtains: $\mathrm{u(x=0)} \equiv \frac{\mathrm{-J_h(x=0)[J_e(x=0)]}}{\mathrm{eS \times p_o(x=0)[n_o(x=0)]}}$, which is just the first boundary condition given in Eq. (7), and then, (ii) Eq. (12) yields: $\mathrm{u(x=W)} = \exp\left(\frac{\mathrm{V}}{\mathrm{n_{I(II)}(V) \times V_T}}\right) - 1$, being the second boundary condition given in Eq. (8).

In the following, we will denote $\mathrm{P(W)}$ and $\mathrm{I(W,S)}$ by P and I, for a simplicity. So, Eq. (15) gives:

$$\mathrm{J_{En(p)o}(x=0, W, N_{d(a)}, r_{d(a)}, S)} = \frac{\mathrm{en^2_{i\,n(p)} \times D_{h(e)}}}{\mathrm{N_{d(a)eff.} \times L_{h(e)}}} \times \frac{1}{\mathrm{sinh(P) + I \times cosh(P)}}, \tag{17}$$

$$\mathrm{J_{En(p)o}(x=W, W, N_{d(a)}, r_{d(a)}, S)} = \frac{\mathrm{en^2_{i\,n(p)} \times D_{h(e)}}}{\mathrm{N_{d(a)eff.} \times L_{h(e)}}} \times \frac{\mathrm{cosh(P) + I \times sinh(P)}}{\mathrm{sinh(P) + I \times cosh(P)}}. \tag{18}$$

Thus, from Equations (14, 17, 18), one gets

$$\frac{\mathrm{J_{h(e)}(x=0, W, N_{d(a)}, r_{d(a)}, S, V)}}{\mathrm{J_{h(e)}(x=W, W, N_{d(a)}, r_{d(a)}, S, V)}} \equiv \frac{\mathrm{J_{En(p)o}(x=0, W, N_{d(a)}, r_{d(a)}, S)}}{\mathrm{J_{En(p)o}(x=W, W, N_{d(a)}, r_{d(a)}, S)}} = \frac{1}{\mathrm{cosh(P) + I \times sinh(P)}}. \tag{19}$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by [22]:

$\mathrm{Q_{h(e)eff.}(x=W, N_{d(a)}, r_{d(a)})} \equiv \int_0^{W} +\mathrm{e(-e) \times u(x) \times p_o(x)[n_o(x)]} \times \frac{\mathrm{\tau_{h(e)E}(N_{d(a)}, r_{d(a)})}}{\mathrm{\tau_{h(e)E}(\rho_{d(a)}(x), r_{d(a)})}} \mathrm{dx}$, and the effective minority-hole transit time by: $\mathrm{\tau_{teff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)} \equiv \mathrm{Q_{h(e)\,eff.}(x=W, N_{d(a)}, r_{d(a)}) / J_{En(p)o}(x=W, W, N_{d(a)}, r_{d(a)}, S)}$, one can define, from Equations (10, 19), the reduced effective minority-hole transit time:

$$\frac{\mathrm{\tau_{teff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)}}{\mathrm{\tau_{h(e)E}}} \equiv 1 - \frac{\mathrm{J_{En(p)o}(x=0, W, N_{d(a)}, r_{d(a)}, S)}}{\mathrm{J_{En(p)o}(x=W, W, N_{d(a)}, r_{d(a)}, S)}} = 1 - \frac{1}{\mathrm{cosh(P) + I \times sinh(P)}}. \tag{20}$$

Now, some important results can be obtained and discussed below.

As $\mathrm{P} \ll 1$ (or $\mathrm{W} \ll \mathrm{L_{h,eff.}}$) and $\mathrm{S} \to \infty$, $\mathrm{I} \equiv \mathrm{I(W,S)} = \frac{\mathrm{D_h(N_o(W))}}{\mathrm{S \times L_h(N_o(W))}} \to 0$, from Eq. (20), one has: $\frac{\mathrm{\tau_{t,eff.}(x=W, W, N_{d(a)}, r_{d(a)}, S)}}{\mathrm{\tau_{h(e)E}}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (18), one obtains:

$$\mathrm{J_{En(p)o}(x=W, N_{d(a)}, r_{d(a)}, S \to \infty)} \to \frac{\mathrm{en^2_{i\,n(p)} \times D_{h(e)}}}{\mathrm{N_{d(a)eff.} \times L_{h(e)}}} \times \frac{1}{\mathrm{P(W)}}, \tag{21a}$$

and then, as $P \gg 1$ (or $W \gg L_{h,eff.}$) and $S \to 0$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S \times L_h(N_o(W))} \to \infty$, from Eq. (20), one has: $\frac{\tau_{teff.}(x=W,W,N_{d(a)},r_{d(a)},S)}{\tau_{h(e)E}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (18), one gets:

$$J_{En(p)o}\left(x = W, N_{d(a)}, r_{d(a)}, S \to 0\right) \to \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \tanh(P). \quad (21b)$$

In summary, in the $n^+(p^+) - p(n)$ junction solar cells, the dark carrier-minority saturation current density J_o, defined in Eq. (2), is now replaced by $J_{oI(II)}$, for a good presentation, and rewritten by:

$$J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \equiv J_{En(p)o}(W, N_{d(a)}, r_{d(a)}, S) + J_{Bp(n)o}(N_{a(d)}, r_{a(d)}), \quad (22)$$

where $J_{En(p)o}$ and $J_{Bp(n)o}$ are determined respectively in Equations (18) and (C1) of the Appendix C.

3. Photovoltaic conversion effect at 300K

Here, in the $n^+(p^+) - p(n)$ junction solar cells, denoted respectively by I(II), and for physical conditions as:

$$W = 0.0044\ \mu m, N_{As(Ga)} = 10^{19}(10^{20})\ cm^{-3}, r_{As(Ga)}, S = 10^{50}\,\frac{cm}{s}, N_{Ga(As)} = 10^{17}\ cm^{-3}, r_{Ga(As)}, \quad (23)$$

we propose, at given $V_{ocI1(2)}$ and $V_{ocII1(2)}$, the experimental results of the short circuit current density $J_{scI(II)}$, fill factor $F_{I(II)}$, and photovoltaic conversion factor $\eta_{I(II)}$, in order to formulate our following treatment method of two fixe experimental points [3, 4], for the $n^+ - p$ junction,

$V_{ocI1(I2)} = 980\ (1127.2)\ mV$, $J_{scI1(I2)} = 27.06\ (29.78)\ mA/cm^2$, $F_{I1(I2)} = 83.35\ (86.7)\ \%$, $\eta_{I1(I2)} = 22.07\ (29.1)\ \%$, and for the $p^+ - n$ junction,

$V_{ocII1(II2)} = 980\ (1030)\ mV$, $J_{scII1(II2)} = 24.2\ (29.8)\ mA/cm^2$, $F_{II1(II2)} = 76.4\ (86)\ \%$, $\eta_{II1(II2)} = 18.1\ (26.4)\ \%$. (24)

First of all, we define the net current density J at T=300 K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage V, flowing through the $n^+(p^+) - p(n)$ junction of GaAs solar cells, by [1, 2, 5-10]:

$$J(V) \equiv J_{ph.}(V) - J_{oI(II)} \times \left(e^{X_{I(II)}(V)} - 1\right), \ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V) \times V_T}, \ V_T \equiv \frac{k_B T}{e} = 25.85\ mV, \quad (25)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, noting that as $V = V_{oc}$, $J(V) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, for $V_{oc} \geq V_{ocI(II)1}$. Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \quad (26)$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, given in Ref. [1], and (ii) the function $\left(e^{X_{I(II)}(V_{oc})} - 1\right)$ or the PVCF $n_{I(II)}$, representing the photovoltaic conversion effect, thus converts the light, represented by $J_{scI(II)}$, into the electricity, by $J_{oI(II)}$.

Further, from Equations (22, 26), we obtain for the $n^+ - p$ junction:

$$n_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}, J_{scI1(2)}\right) \equiv \frac{V_{ocI1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scI1(2)}}{J_{oI}} + 1\right)} \equiv n_{I1(2)}\left(V_{ocI1(2)}, J_{scI1(2)}\right),$$

and we then propose:

$$n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = n_{I1}(V_{ocI1}, J_{scI1}) + n_{I2}(V_{ocI2}, J_{scI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.1248}, \quad (27)$$

being valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Furthermore, for the $p^+ - n$ junction,

$$n_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}, J_{scII1(2)}\right) \equiv \frac{V_{ocII1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scII1(2)}}{J_{oII}} + 1\right)} \equiv n_{II1(2)}\left(V_{ocII1(2)}, J_{scII1(2)}\right),$$

and then,

$$n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = n_{II1}(V_{ocI1}, J_{scI1}) + n_{II2}(V_{ocII2}, J_{scII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.0939664}, \quad (28)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Therefore, from Equations (23, 24, 27, 28), one obtains, $n_{I1(II1)}$ =0.9701 (0.99492) at $V_{ocI1(II1)} = 980\ (980)$ mV, and $n_{I2(II2)}$ =1.1131 (1.04) at $V_{ocI2(II2)} = 1127.2\ (1030)$ mV, respectively, for $n^+(p^+) - p(n)$ junction solar cells.

Thus, X_I defined from Eq. (26) now becomes for the $n^+ - p$ junction:

$X_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \equiv \frac{V_{oc}}{n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \times V_T}$, and therefore, we can determine the values of the fill factors $F_{I1(I2)}$ at $V_{oc} = V_{ocI1(I2)}$ by [1, 2]:

$F_{I1(I2)}\big(W, N_d, r_d, S, N_a, r_a, V_{ocI1(I2)}\big) = \frac{X_I(W,N_d,r_d,S, N_a,r_a,V_{ocI1(I2)}) - \ln[X_I(W,N_d,r_d,S, N_a,r_a,V_{ocI1(I2)}) + 0.72\,]}{X_I(W,N_d,r_d,S, N_a,r_a,V_{ocI1(I2)}) + 3.385\ (1.758)} \equiv F_{I1(I2)}\big(V_{oc} = V_{ocI1(I2)}\big)$, for a presentation simplicity, and further, the fill factor F_I can be computed by:

$$F_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = F_{I1}(V_{ocI1}) + F_{I2}(V_{ocI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.716}, \tag{29}$$

which is valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Then, also from Eq. (26), we can define for the $p^+ - n$ junction:

$X_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) \equiv \frac{V_{oc}}{n_{II}(W,N_a,r_a,S, N_d,r_d,V_{oc}) \times V_T}$, where $n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc})$ is determined in Eq. (28). Therefore, we can determine the values of the fill factors $F_{II1(II2)}$ at $V_{oc} = V_{ocII1(II2)}$ as:

$F_{II1(II2)}\big(W, N_a, r_a, S, N_d, r_d, V_{ocII1(II2)}\big) = \frac{X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(II2)}) - \ln[X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(II2)}) + 0.72\,]}{X_{II}(W,N_a,r_a,S, N_d,r_d,V_{ocII1(II2)}) + 6.9795\ (1.9752)} \equiv F_{II1(2)}\big(V_{ocII1(II2)}\big)$, for a presentation simplicity, and further, the fill factor F_{II} is determined by:

$$F_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = F_{II1}(V_{ocII1}) + F_{II2}(V_{ocII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{0.73688}, \tag{30}$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Then, with physical conditions given in Eq. (23), our numerical calculation shows that we obtain the same values of $J_{scI1(I2)}$ and $F_{I1(I2)}$ at $V_{ocI1(I2)} = 980\ (1127.2)$ mV, and $J_{scII1(II2)}$ and $F_{II1(II2)}$ at $V_{ocII1(II2)} = 980\ (1030)$ mV , as those given in Eq. (24).

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{I(II)}\big(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\big) \equiv \frac{J_{scI(II)} \times V_{oc} \times F_{I(II)}}{P_{in.}}, \tag{31}$$

where, $J_{scI(II)}$ and $F_{I(II)}$ are determined respectively in Equations (26, 29, 30), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$) [1, 2, 26-29]. Then, from Equations (31, 24), we get the numerical results of η, by using this assumption: $P_{in.} = 0.100\ \frac{W}{cm^2}$, and their relative errors in absolute values (RE), calculated by using the experimental results of $\eta_{I1(I2)}$ and $\eta_{II1(II2)}$ given in Eq. (24),

-for the $n^+ - p$ junction at $V_{ocI1(I2)} = 980\ (1127.2)$ mV, $\eta_{I1(I2)} = 22.10\ (29.10)\ \%$, with RE=$10^{-3}(1.8 \times 10^{-4})$, and

-for the $p^+ - n$ junction at $V_{ocII1(II2)} = 980\ (1030)\ \mathrm{mV}$, $\eta_{II1(II2)} = 18.13\ (26.41)\ \%$, with RE=$1.1 \times 10^{-3}(1.8 \times 10^{-4})$.

4. Numerical results and concluding remarks

In the following, we will respectively consider the two cases: the $\mathrm{HD[As(Sb) - GaAs]ER - LD[Ga(In) - GaAs]BR}$, and the $\mathrm{HD[Ga(In) - GaAs]ER - LD[As(Sb) - GaAs]BR}$.

4.1. $\mathbf{HD[As(Sb) - GaAs]ER - LD[Ga(In) - GaAs]BR - cases}$

(4.1a) $HD(As - GaAs)ER - LD(Ga - GaAs)BR - case.$

Here, there is the absence of the impurity size effect **(ISE)**, because of $r_{As(Ga)} = 0.118\ (0.126)\ \mathrm{nm}$, and we propose the usual physical conditions as:

$$W = 15\ \mu\mathrm{m},\ N_{As} = 10^{19}\ \mathrm{cm}^{-3}, S = 100\ (\mathrm{cm/s}),\ \text{and}\ N_{Ga} = 10^{17}(10^{18})\mathrm{cm}^{-3}. \qquad (32)$$

Then, from Equations (12, 13, 18, 20, 26,27,29,31) and (C7, C8) of the Appendix C, on obtains:

$P = \frac{W}{L_{heff.}} = 2.19 \times 10^{-14}$, $L_h = 2.64 \times 10^9\ \mathrm{cm}$, $I(W, S) = 0$, $\tau_{hE} = 1.9 \times 10^{18}\ \mathrm{s}$, $\frac{\tau_{teff.}}{\tau_{hE}} = 0$,

suggesting the completely transparent **CTHDER** − condition, and from Eq. (18), $J_{Eno} = 6.86 \times 10^{-23}\ \left(\frac{A}{cm^2}\right)$. Further, in the LD[B-Si]-BR and for $N_{Ga} = 10^{17}(10^{18})\ \mathrm{cm}^{-3}$, one gets from Eq. (C1) of the Appendix C: $J_{Bpo} = 2.551\ (0.136) \times 10^{-19}\ \left(\frac{A}{cm^2}\right)$. Therefore, one obtains respectively: $J_{oI} = 2.551\ (0.136) \times 10^{-19}\ \left(\frac{A}{cm^2}\right) = J_{Bpo}$, and from the following Table 2, for example, at $V_{oc} = 1078\ \mathrm{mV}$, n_I=1.038 (0.965) and the maximal values of η_I, $\boldsymbol{\eta_{I,max.}}$ **= 29.80 (30.52)%**, meaning that, with increasing N_{Ga}, both J_{oI} and n_I decrease, while η_I increases, being new results.

Table 2. In the HD(As-GaAs) ER-LD(Ga-GAAs) BR and for physical conditions given in Eq. (32), our numerical results of n, J_{sc}, F, and η, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_{Ga}, the function n_I decreases, while the functions J_{sc}, F, and η increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1130	1.112 (1.035)	29.60 (29.77)	86.86 (87.80)	29.05 (29.54)
1127.2	1.109 (1.032)	29.79 (29.98)	86.75 (87.69)	29.13 (29.63)
1079	1.039 (0.966)	33.05 (33.53)	84.52 (85.44)	29.80 (30.52)

1078	1.038 (0.965)	33.06 (33.54)	84.50 (85.42)	**29.80 (30.52)**
1077	1.037 (0.965)	33.08 (33.56)	84.47 (85.39)	29.79 (30.51)
1000	0.983 (0.915)	30.88 (31.18)	83.50 (84.42)	25.78 (26.32)
990	0.975 (0.907)	29.46 (29.64)	83.43 (84.34)	24.33 (24.75)
980	0.967 (0.900)	27.06 (27.06)	83.39 (84.31)	22.11 (22.36)

(4.1b) $HD(Sb-GaAs)ER-LD(In-GaAs)BR-case$.

Here, there is the presence of the ISE, since $r_{Sb(In)} = 0.136(0.144)\text{nm} > r_{As(Ga)} = 0.118\ (0.126)$ nm, and we propose the usual physical conditions:

$$W = 15\ \mu\text{m},\ N_{Sb} = 10^{19}\ \text{cm}^{-3}, S = 100\ (\text{cm/s}),\ \text{and}\ N_{In} = 10^{17}(10^{18})\ \text{cm}^{-3}. \quad (33)$$

Then, from Equations (12, 13, 18, 20, 26,27,29,31) and (C7, C8) of the Appendix C , one obtains:

$P = \frac{W}{L_{heff.}} = 1.94 \times 10^{-14},\ L_h = 3.17 \times 10^9$ cm, $I = 0,\ \tau_{hE} = 9.51 \times 10^{18}$ s, $\frac{\tau_{teff.}}{\tau_{hE}} = 0$,

suggesting the CTHDER-condition, and $J_{Eno} = 4.51 \times 10^{-23}\ \left(\frac{A}{cm^2}\right)$. Further , in the LD(In-Si)-BR and $N_{In} = 10^{17}(10^{18})\ \text{cm}^{-3}$, one gets, from Eq. (C1) of the Appendix C: $J_{Bpo} = 3.72(\ 0.368) \times 10^{-21}\ \left(\frac{A}{cm^2}\right)$, and therefore, $J_o = 3.76(\ 0.372) \times 10^{-21}\ \left(\frac{A}{cm^2}\right) \simeq J_{Bpo}$. Then, from the following Table 3, one notes that, for a given V_{oc} and with increasing N_{In}, both J_{oI} and n_I decrease, while η_I increases, being new results. In particular, for $N_{In} = 10^{17}(10^{18})\ \text{cm}^{-3}$ and at $V_{oc} = 1072$ mV, one gets: n_I=0.950 (0.894) and $\boldsymbol{\eta_{I,max.}}$**= 30.814 % (31.474) %**, respectively.

Table 3. In the HD(Sb-GaAs) ER-LD(In-GaAs) BR and for physical conditions given in Eq. (33), our numerical results of n, J_{sc}, F, and η, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_{In}, the function n decreases, while the functions J_{sc}, F, and η increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1130	1.004 (0.945)	29.85 (30.01)	88.18 (88.93)	29.739 (30.154)
1127.2	1.002 (0.942)	30.06 (30.23)	88.07 (88.82)	29.839 (30.269)
1072	0.950 (0.894)	33.35 (33.78)	86.19 (86.93)	**30.814 (31.474)**
1071	0.949 (0.893)	33.39 (33.81)	86.16 (86.90)	30.809 (31.470)

1070	0.948 (0.892)	33.42 (33.85)	86.13 (86.87)	30.802 (31.466)
1000	0.888 (0.835)	31.31 (31.60)	84.79 (85.51)	26.549 (27.021)
990	0.880 (0.828)	29.73 (29.90)	84.71 (85.44)	24.929 (25.291)
980	0.873 (0.821)	27.06 (27.06)	84.68 (85.40)	22.455 (22.647)

One remarks from Tables 2 and 3 that the obtained results of $\eta_{I,max.}$, being given in Table 3, are found to be very large compared with those given in Table 2, since the corresponding values of J_{oI} *and* n_I, obtained in Table 3, are very small compared with those obtained in Table 2.

4.2. $\mathbf{HD[Ga(In)} - \boldsymbol{GaAs}\mathbf{]ER - LD[As(Sb)} - \boldsymbol{GaAs}\mathbf{]BR}$ −cases

(4.2a) $HD[Ga - GaAs]ER - LD[As - GaAs]BR$ *−case.*

Here, we propose the usual physical conditions:

$$W = 15\ \mu\text{m},\ \ N_{Ga} = 10^{20}\ \text{cm}^{-3}, S = 100\ (\text{cm/s}),\ \text{and}\ \ N_{As} = 10^{16}(10^{17})\ \text{cm}^{-3}. \qquad (34)$$

Then, from Equations (12, 13, 18, 20, 26,27,29,31) and (C7, C8) of the Appendix C , one obtains:

$P = \frac{W}{L_{heff.}} = 3.99 \times 10^{-15},\ L_e = 3.77 \times 10^{11}\ \text{cm}, I = 0,\ \tau_{eE} = 5.77 \times 10^{19}\ \text{s},\ \frac{\tau_{teff.}}{\tau_{hE}} = 0,$ suggesting the CTHDER-condition, and $J_{Epo} = 3.77 \times 10^{-22}\ \left(\frac{A}{cm^2}\right)$. Further, one gets, from Eq. (C1) of the Appendix C: $J_{Bno} = 7.31(0.72) \times 10^{-19}\ \left(\frac{A}{cm^2}\right)$. Therefore, one obtains: $J_{oII} = 7.31(0.72) \times 10^{-19}\ \left(\frac{A}{cm^2}\right) = J_{Bpo}$, and from the following Table 4, for example, at $V_{oc} = 1375\ (1355)\ \text{mV}$, n_{II}=1.396 (1.296) and η_{II}= 43.76 (43.97)%, respectively, noting that, with increasing N_{As}, both J_{oII} and n_{II} decrease, while η_{II} increases, being new results.

Table 4. In the HD(Ga-GaAs) ER-LD(As-GaAs) BR and for physical conditions given in Eq. (34), our numerical results of n, J_{sc}, F, and η, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_{As}, the function n_I decreases, while the functions J_{sc}, F, and η increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1380	1.401 (1.321)	25.51 (25.50)	120.8 (122.2)	43.761 (43.899)
1375	1.396 (1.316)	25.69 (25.69)	120.4 (121.7)	**43.763 (43.922)**
1366	1.386 (1.307)	26.02 (26.04)	119.6 (121.0)	43.758 (43.953)

1361	1.381 (1.302)	26.20 (26.23)	119.2 (120.6)	43.750 (43.965)
1355	1.375 (1.296)	26.42 (26.46)	118.7 (120.1)	43.735 (43.974)
1338	1.357 (1.279)	27.03 (27.12)	117.3 (118.6)	43.659 (43.965)
1290	1.306 (1.231)	28.70 (28.92)	113.2 (114.5)	43.154 (43.633)
1030	1.042 (0.982)	29.79 (30.15)	85.97 (87.07)	27.251 (27.695)
1000	1.014 (0.955)	27.25 (27.44)	81.26 (82.33)	22.890 (23.147)
980	0.996 (0.939)	24.20 (24.20)	76.37 (77.41)	18.7432 (18.823)

(4.2b) $HD(In - GaAs)ER - LD(Sb - GaAs)BR$ *−case.*

Here, we propose the usual physical conditions:

$$W = 15\ \mu m,\ N_{In} = 10^{20}\ cm^{-3}, S = 100\ (cm/s), N_{Sb} = 10^{16}(10^{17})\ cm^{-3}. \quad (35)$$

Then, from Equations (12, 13, 18, 20, 26,28,30,31) and (C7, C8) of the Appendix C) , on obtains: $P = \frac{W}{L_{eeff.}} = 2.63 \times 10^{-15}$, $L_{eE} = 5.75 \times 10^{11}$ cm, $I = 2.7 \times 10^{-80}$, $\tau_{eE} = 7.2 \times 10^{20}$ s, $\frac{\tau_{teff.}}{\tau_{eE}} = 0$, corresponding to the CTHDER − condition, and $J_{Epo} = 1.56 \times 10^{-23} \left(\frac{A}{cm^2}\right)$. Further, in the LD[Bi-Si]-BR , one gets: $J_{Bno} = 2.29(0.22) \times 10^{-19} \left(\frac{A}{cm^2}\right)$. Therefore, one obtains : $J_{oII} = 2.29(0.22) \times 10^{-19} \left(\frac{A}{cm^2}\right) = J_{Bno}$, and from the following Table 5, for example, at $V_{oc} = 1361\ (1338)$ mV, one obtains: n_{II} =1.340 (1.243) and η_{II} = 43.928 (44.359)% , respectively, noting that, with increasing N_{Sb}, both J_{oII} and n_{II} decrease, while η_{II} increases, being new results.

Table 5. In the HD(In-GaAs) ER-LD(Sb-GaAs) BR and for physical conditions given in Eq. (35), our numerical results of n, J_{sc}, F, and η, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing N_{Sb}, the function n_I decreases, while the functions J_{sc}, F, and η increase, being new results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1366	1.345 (1.270)	26.03 (26.06)	120.3 (121.6)	43.926 (44.289)
1361	1.340 (1.265)	26.22 (26.25)	119.9 (121.2)	**43.928** (44.311)
1355	1.334 (1.259)	26.44 (26.49)	119.4 (120.7)	43.925 (44.333)

1338	1.316 (1.243)	27.07 (27.16)	118.0 (119.3)	43.883 (**44.359**)
1290	1.267 (1.196)	28.81 (29.03)	113.9 (151.1)	43.465 (44.118)
1030	1.011 (0.954)	29.97 (30.34)	86.54 (87.59)	27.523 (28.092)
1000	0.984 (0.929)	27.34 (2754)	81.81 (82.83)	23.062 (23.424)
980	0.967 (0.913)	24.20 (24.20)	76.90 (77.89)	18.819 (18.985)

In conclusion, by basing on such a treatment method, and using the physical conditions such as: $W = 15\ \mu m$, $N_{Sb(In)} = 10^{19}\ (10^{20})\ cm^{-3}$ and $S = 100\ (cm/s)$, according to the CTHD[Sb(In)-GaAs]ER, and then $N_{In(Sb)} = 10^{18}\ (10^{17}) cm^{-3}$ for LD[In(Sb)-GaAs]BR, of $n^{+}(p^{+}) - p(n)$ junction solar cells, we get respectively the maximal values (or limiting ones) of $\eta_{I(II)}$, $\eta_{I(II)-max.}$ =31.474% (44.359%), as those observed in Tables 3 et 5. They can also be compared with the other ones: $\eta = 29.1\ \%$, obtained for the GaAs-thin film cell, and $\eta = 45.7\ \%$ for GaInP/GaAs/GaInAs/GaInAs multijunction cell, by Green et al. [3].

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Appendix

Appendix A. Fermi Energy

In the n(p)-type semiconductor, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11×10^{-4} [39, 40], is now summarized in the following.

First of all, we define the reduced electron density by:

$$u \equiv \frac{N_{d(a)}}{N_{c(v)}},\ N_c(T, r_d) = 2 \times \left(\frac{m_c(T, r_d) \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}),\ N_v(T) = 2 \times \left(\frac{m_v(T) \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}). \quad \text{(A1)}$$

Here, $N_{c(v)}$ is the conduction (valence)-band density of states, respectively, $m_c(T, r_d)$ is the effective mass of the electron in n-type Si can be defined by [31, 32]:

$$m_c(T, r_{d(a)}) = 0.067 \times m_o \times \left(\frac{E_{go}(r_{d(a)})}{E_{gi}(T, r_{d(a)})}\right)^{2/3}, \quad \text{(A2)}$$

where $m_o = 9.1096 \times 10^{-28}$ (g) is the electron rest mass, the effective mass of the hole in the p-type Si yields [31, 32]:

$$m_v = 0.45 \times m_o, \tag{A3}$$

and $E_{gin(p)}(T, r_{d(a)})$ is the intrinsic band gap in the GaAs-semiconductor, due to the T-dependent carrier-lattice interaction-effect, by [1, 2, 33, 34]:

$$E_{gin(p)}(T, r_{d(a)}) \simeq E_{gon(p)}(r_{d(a)}) - \frac{5.405\times10^{-4}\times T^2}{T+204}. \tag{A4}$$

Here, $E_{gon(p)}(r_{d(a)})$ is determined in Eq. (1b), due to the d(a)-size effect.

Furthermore, in the n(p)-type Si, one can define the intrinsic carrier concentration $n_{in(p)}$ by:

$$n^2_{i\,n(p)}(T, r_{d(a)}) \equiv N_c(T, r_d) \times N_v(T) \times \exp\left(\frac{-E_{gin(p)}(T, r_{d(a)})}{k_B T}\right). \tag{A5}$$

Then, denoting the reduced Fermi energy in the n(p)-type semiconductor, respectively, by $\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T})$,

Being accurate to within 10^{-7} , we have [39] :

$$\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T}) = \frac{G(u)+Au^B F(u)}{1+Au^B}, A = 0.0005372 \text{ and } B = 4.82842262 \tag{A6}$$

where

$$F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}, \quad a = [(3\sqrt{\pi}/4) \times u]^{2/3}, \quad b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2 \quad \text{and } c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$$

and

$$G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}; \; d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0.$$

Here, one notes that: (i) $\frac{E_{Fn}(u\gg1)}{k_B T}(\frac{-E_{Fp}(u\gg1)}{k_B T}) > 1$, according to the HD[d(a)-GaAs]ER-case or to the degenerate case, Eq. (A6) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u\ll1)}{k_B T}(\frac{-E_{Fp}(u\ll1)}{k_B T}) <-1$, to the LD[a(d)-GaAs]BR-case or to the non-degenerate case, Eq. (A6) is reduced to the function G(u), respectively.

(i) In the HD[As(Sb)-GaAs]ER-case for $N_{As(Sb)} = 10^{19}$ cm^{-3}, we get: $\frac{E_{Fn}}{k_B T} = 9.49\ (9.53) > 1$, and in the HD[Ga(In)-GaAs]ER-case for $N_{Ga(In)} = 10^{20}$ cm^{-3}, we get: $\frac{-E_{Fp}}{k_B T} = 6.94\ (6.94) > 1$ according to degenerate conditions.

(ii) In the LD[Ga(In)-GaAs]BR-case for $N_{Ga(In)} = 10^{17}(10^{18})\ cm^{-3}$ we obtain: $\frac{-E_{Fp}}{k_B T} = -4.32\ (-1.98) < -1$, and in the LD[As(Sb)-GaAs]BR-case for $N_{As(Sb)} = 10^{16}(10^{17})\ cm^{-3}$ we get: $\frac{E_{Fn}}{k_B T} = -3.8\ (-1.5) < -1$, according to non-degenerate conditions. Those obtained results confirm the limiting values of $N_{a(d)}[= 10^{18}(10^{17})\ cm^{-3}]$, given in Tables 2-5, for the LD[a(d)-GaAs] BR, respectively.

Appendix B. Approximate forms for band gap narrowing and apparent band gap narrowing

First of all, in the n(p)-type Si, we define the effective Wigner-Seitz radius r_s characteristic of the interactions by [1, 2]

$$r_{sn} \equiv r_s(N_d, T, r_d) = 1.1723 \times 10^8 \times \left(\frac{1}{N_d}\right)^{1/3} \times \frac{m_c(T, r_d)}{\varepsilon(r_d)} \tag{B1}$$

and

$$r_{sp} \equiv r_s(N_a, T, r_a) = 1.1723 \times 10^8 \times \left(\frac{1}{N_a}\right)^{1/3} \times \frac{m_v(T)}{\varepsilon(r_a)}, \tag{B2}$$

where $m_c(T, r_d)$ and $m_v(T)$ are given in (A2) and (A3). Therefore, the correlation energy of an effective electron gas, $E_c(r_s)$, is given by [1, 2, 42]:

$$E_{cn(cp)}\left(N_{d(a)}, T, r_{d(a)}\right) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln\left(r_{sn(sp)}\right) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}. \tag{B3}$$

Then, in the n-type heavily doped GaAs, the band gap narrowing is found to be given as [1, 2]:

$$\Delta E_{gn}(N_d, r_d) \simeq a_1 \times \frac{\varepsilon(r_{As})}{\varepsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{As})}{\varepsilon(r_d)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_c(r_{sn}) \times r_{sn}]) + a_3 \times \left[\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}\right]^{5/4} \times \sqrt{\frac{m_v(T)}{m_c(T, r_d)}} \times \left[\frac{m_c(T, r_d)}{m_c(T, r_{As})}\right]^{\frac{1}{4}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}} \times \sqrt{\frac{m_c(T, r_{As})}{m_c(T, r_d)}} \times N_r^{1/2} \times \left[1 + \sqrt{\frac{m_c(T, r_d)}{m_c(T, r_{As})}}\right] + a_5 \times \left[\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}\right]^{\frac{3}{2}} \times \sqrt{\frac{m_c(T, r_d)}{m_c(T, r_{As})}} \times N_r^{\frac{1}{6}},\ N_r \equiv \left(\frac{N_d}{9.999 \times 10^{17}\ cm^{-3}}\right), \tag{B4}$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type heavily doped GaAs, one has [1, 2]:

$$\Delta E_{gp}(N_a, r_a) \simeq a_1 \times \frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sp}) \times r_{sp}]\right) + a_3 \times \left[\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}\right]^{5/4} \times \sqrt{\frac{m_c(T, r_{As})}{m_v(T)}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r \equiv \left(\frac{N_a}{9.999 \times 10^{17}\ cm^{-3}}\right), \tag{B5}$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$ and $a_5 = 9.80 \times 10^{-5}(eV)$.

Further, in the donor (acceptor)-GaAs, we define the effective intrinsic carrier concentration $n_{ien(p)}$, by

$$n^2_{i\,en(p)}(N_{d(a)}, r_{d(a)}) \equiv N_{d(a)} \times p_o(n_o) \equiv n^2_{i\,n(p)} \times \exp\left[\frac{\Delta E_{gan(p)}}{k_B T}\right], \tag{B6}$$

where we can define the "effective doping density" by: $N_{d(a)eff.} \equiv N_{d(a)}/\exp\left[\frac{\Delta E_{ga\,n(p)}}{k_B T}\right]$ so that $N_{d(a)eff.} \times p_o(n_o) \equiv n^2_{i\,n(p)}$ [8], and also the apparent band gap narrowing, $\Delta E_{ga\,n(p)}$, as

$$\Delta E_{ga\,n(p)} \equiv \Delta E_{g\,n(p)} + k_B T \times \ln\left(\frac{N_{d(a)}}{\mathbf{N_{c(v)}}}\right) - E_{Fn}(\frac{N_d}{N_c})[-E_{Fp}(\frac{N_a}{N_v})], \tag{B7}$$

where $N_{c(v)}$ is defined in Eq. (A1), the Fermi energy is determined in Eq. (A6).

Appendix C. Minority-carrier transport parameters

Here, the minority-electron (hole) saturation current density injected into the LD[a(d)-GaAs]BR, with an acceptor density equal to $N_{a(d)}$, is given by [1, 2]:

$$J_{Bp(n)o}\left(N_{a(d)}, r_{a(d)}\right) = \frac{e \times n_i^2(r_{d(a)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{e(h)B}(N_{a(d)})}}}{N_{a(d)}}, \tag{C1}$$

where $n^2_{i\,n(p)}(r_{d(a)})$ is determined in (A5), $D_{e(h)}(N_{a(d)}, r_{a(d)})$ is the minority-hole (electron) diffusion coefficient:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[200 + \frac{8500-200}{1+\left(\frac{N_a}{1.3\times10^{17}\,cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a)}{12.85}\right)^2 \left(cm^2V^{-1}s^{-1}\right), \tag{C2}$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[130 + \frac{400-130}{1+\left(\frac{N_d}{8\times10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d)}{12.85}\right)^2 \left(cm^2V^{-1}s^{-1}\right), \tag{C3}$$

and $\tau_{h(e)B}(N_{d(a)})$ is the minority-hole (electron) lifetime (s) in the base region:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2. \tag{C4}$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76 \times 10^{-13} \times N_d + 2.78 \times 10^{-31} \times N_d^2, \tag{C5}$$

Further, from (A6), (B4)-(B7)), in the HD[d(a)-GaAs]ER, we can define the following minority-hole(electron) transport parameter $F_{h(e)}$ as [8, 22, 25]:

$$F_{h(e)}\ (N_{d(a)}, r_{d(a)}) \equiv \frac{n^2_{i\,n(p)}(r_{d(a)})}{p_o(n_o)\times D_{h(e)}} = \frac{N_{d(a)eff.}}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}\times\exp\left[\frac{\Delta E_{g\,an(p)}}{k_B T}\right]}\ (cm^{-5} \times s), \tag{C6}$$

Furthermore, the minority-hole (electron) diffusion length, $L_{h(e)}\left(N_{d(a)}, r_{d(a)}\right)$ and the minority-hole(electron) lifetime $\tau_{h(e)E}$ in the HD[d(a)-GaAs]ER can be determined by

$$L_{h(e)}^{-2}(N_{d(a)}, r_{d(a)}) = [\tau_{h(e)E} \times D_{h(e)}]^{-1} = (C \times F_{n(p)})^2 = \left(C \times \frac{N_{d(a)eff.}}{D_{h(e)}}\right)^2 = \left(C \times \frac{n_{i\,n(p)}^2(r_{d(a)})}{p_o(n_o) \times D_{h(e)}}\right)^2, \quad \text{(C7)}$$

where the constant C is chosen to be 2.0893×10^{-30} (cm^4/s), and then, $\tau_{h(e)E}$ can be computed by:

$$\tau_{h(e)E} = \frac{1}{D_{h(e)} \times (C \times F_{n(p)})^2}. \quad \text{(C8)}$$

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