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28.68% (29.87%)- New Limiting Highest Efficiencies obtained in $n^+(p^+) - p(n)$ Crystalline Silicon (Si) Junction Solar Cells at T=300 K, Due to The Effects of Heavy (Low) Doping and Impurity Size

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Abstract:

In the $n^+(p^+) - p(n)$ crystalline Si-junction solar cells, by basing on a same treatment method, and for a same heavy (low) doping effect, as those given in our recent paper (**RP**) [1], but using now a new expression, obtained for the relative dielectric constant $\varepsilon(r_{d(a)})$, determined exactly in the effective Bohr model, as that given in Eq. (1c), representing the donor (acceptor) d(a)-radius $r_{d(a)}$ – effect or the $\varepsilon(r_{d(a)})$ – effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as observed in Tables 2 and 3, we finally get in our present paper, for highest values of $r_{d(a)}$, the new limiting highest efficiencies, η=28.68% (29.87%)$<\eta_{RP}$ =31% (30.65%), being due to $r_{d(a)}$ [8] $< r_{d(a),RP}$, according to: $\varepsilon(r_{d(a)}) > \varepsilon_{RP}(r_{d(a)})$, $\varepsilon_{RP}(r_{d(a)})$ being our inaccurate and simple formula, proposed in RP, and also reported in Eq. (1d), for a comparison. Finally, our new limiting highest efficiencies, **η=28.68% (29.87%)**, can also be compared with other limiting η-results, such as: **29.43%** [26], **30%**[6], and **31**% [3, 4].

Keywords: donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; photovoltaic conversion efficiency

1. Introduction

In our recent paper (RP) [1], by basing on: (**i**) the heavy doping and impurity size effects, which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{En(p)o}$ and $J_{Bp(n)o}$ being the carrier-minority saturation current densities, injected respectively into the heavily doped donor (acceptor)-Si emitter-lightly doped acceptor (donor)-Si base-regions, HD[d(a)-Si]ER-LD[a(d)-Si]BR, of $n^+(p^+) - p(n)$ junction solar cells, **(ii)** an effective Gaussian donor (acceptor)-density profile $\rho_{d(a)}$ to determine $J_{En(p)o}$ [1, 2, 13, 18-20, 22], and **(iii)** the use of two fixed experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, the short circuit current density $J_{scI(II)}$, the fill factor $F_{I(II)}$, and finally the efficiency $\eta_{I(II)}$ [1- 45]. These physical quantities were expressed as functions of the open circuit voltage V_{oc}, and of various parameters such as: the emitter thickness W, high donor (acceptor) density $N_{d(a)}$, surface recombination velocity S, given in the HD[d(a)-Si]ER, and low acceptor (donor) density $N_{a(d)}$, given in the LD[a(d)-Si]BR.

Then, in our present paper, by basing on a same treatment method, and for a same heavy (low) doping effect, as those given in RP, but using now a new expression, obtained for the relative dielectric constant $\varepsilon(r_{d(a)})$, determined exactly in the effective Bohr model, as that given in Eq. (1c), representing the donor (acceptor) d(a)-radius $r_{d(a)}$ −effect or the $\varepsilon(r_{d(a)})$ −effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as observed in Tables 2 and 3, we finally get in our present paper, for highest values of $r_{d(a)}$, the new limiting highest efficiencies, η=28.68% (29.87%) $< \eta_{RP} = 31\%$ (30.65%), being due to $r_{d(a)}$ [8] $< r_{d(a),RP}$, according to: $\varepsilon(r_{d(a)}) > \varepsilon_{RP}(r_{d(a)})$, $\varepsilon_{RP}(r_{d(a)})$ being our inaccurate and simple formula, proposed in RP, and also reported in Eq. (1d), for a comparison. Finally, our new limiting highest efficiencies, **η=28.68% (29.87%)**, can also be compared with other limiting η-results, such as: **29.43%** [26], **30%** [6], and **31**% [3, 4].

In Section 2, all the results energy-band-structure parameters for d(a)- Si systems are reported in Table 1, and the expressions for $J_{En(p)o}$ are also reported, so that we can determine the total (or dark) carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{Bp(n)o}$ being determined in Eq. (C1) of the Appendix C. In Section 3, the photovoltaic effect is investigated. Finally, some numerical results and concluding remarks are given and discussed in Section 4.

2. Energy-Band-Structure Parameters and dark minority-carrier saturation current density, due to impurity-size and heavy doping effects

2.1. Effect of d(a)-size

In d(a)-Si-systems at T=0 K, since the d(a)-radius $r_{d(a)}$, in tetrahedral covalent bonds is usually either larger or smaller than the Si atom-radius r_{Si}, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_{Si}$ ($r_{d(a)} < r_{Si} = 0.117\ \text{nm}$), respectively, due to the d(a)-size effect [42]. Further, we also suppose that there exist the donor (acceptor)-atoms, having their donor (acceptor)-radii $r_{do(ao)}$, so that $r_{do(ao)} = r_{Si} = 0.117\ \text{nm}$, $1\ \text{nm} = 10^{-9}\text{m}$, corresponding to the absence of impurity size effect. Then, we have shown [8] that this $r_{d(a)}$-effect affects the changes in all the energy-band-structure parameters, expressed in terms of the relative dielectric constant $\varepsilon(r_{d(a)})$, as given in the following.

First, we note that in the Si [8] the relative dielectric constant of the intrinsic silicon is equal to: $\varepsilon(r_{Si}) = 11.7$, the relative effective electron (hole) mass in conduction (valence) bands yield: $(m_c/m_o) = 0.2$ and $(m_v/m_o) = \frac{0.16+0.52}{2} = 0.34$, the unperturbed intrinsic band gap at 0K, $E_{go}(r_{Si}) = 1.17$ eV, the effective donor(acceptor)-ionization energy at $r_{do(ao)} = r_{Si}$ in absolute values:

$E_{do}(r_{do}) = \frac{13600\times(m_c/m_o)}{(\varepsilon(r_{Si}))^2}\ \text{meV} = 19.9\ \text{meV}$, and $E_{ao}(r_{ao}) = \frac{13600\times(m_v/m_o)}{(\varepsilon(r_{Si}))^2}\ \text{meV} = 33.8\ \text{meV}$, and the isothermal bulk modulus are defined, for the n(p)-type Si, by: $B_n \equiv \frac{E_{do}}{(4\pi/3)\times(r_{Si})^3} = 4.745 \times 10^8\ (\text{N/m}^2)$, and $B_p \equiv \frac{E_{ao}}{(4\pi/3)\times(r_{Si})^3} = 8.066 \times 10^8\ (\text{N/m}^2)$.

Therefore, at $r_{d(a)} = r_{do(ao)}$, the boundary conditions are found to be, for the impurity-atom volume V, $V_{do(ao)} = (4\pi/3) \times \left(r_{do(ro)}\right)^3$, the pressure p, $p_o = 0$, and the deformation potential energy or the strain energy σ, $\sigma_o = 0$.

Further, the two important equations [42], needed to determine the σ-variation $\Delta\sigma \equiv \sigma\text{-}\sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, in the n(p)-type Si, by an integration, one gets:

$$(\Delta\sigma)_{n(p)} = B_{n(p)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \qquad (1a)$$

Furthermore, at T=0K, we also shown [42] that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) corresponding the repulsive (attractive) force increases (decreases) the energy gap $E_{gn(gp)}\left(r_{d(a)}\right)$ and the effective donor(acceptor)-ionization energy $E_{d(a)}\left(r_{d(a)}\right)$ in

absolute values, obtained in the effective Bohr model, which is represented by: $+(-)\ (\Delta\sigma)_{n(p)}$, respectively. That gives:

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{Si})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{Si})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \leq r_{do(ao)}. \quad (1b)$$

Then, from Equations (1a, 1b), the exact expression of relative dielectric constant $\varepsilon(r_{d(a)})$, is given by:

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{Si})}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon(r_{Si}), \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{Si})}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon(r_{Si}), \text{ for } r_{d(a)} \leq r_{do(ao)}, \quad (1c)$$

which can be compared with its approximated and simple form, proposed in our recent paper (RP) [1], by:

$$\varepsilon_{RP}(r_{d(a)}) \simeq 11.4 \times \left(\frac{r_{Si}}{r_{d(a)}}\right)^{4.377\ (4.7)}. \quad (1d)$$

Therefore, with increasing $r_{d(a)}$, the effective dielectric constant $\varepsilon(r_{d(a)})$, determined in Eq. (1c), decreases, implying that $E_{go}(r_{d(a)})$ and $E_{do(ao)}(r_d)$, given in Eq. (1b), increase, as observed in the following Table 1, in which we also remark that, for a given $r_{d(a)}$, $\varepsilon_{RP}(r_{d(a)}) < \varepsilon(r_{d(a)})$. This remark further explains that the values of limiting highest efficiencies, given for the $n^+(p^+) - p(n)$ crystalline Si-junction solar cells, obtained in our RP, are found to be high, 31% (30.65%), compared respectively with the present ones, 28.68% (29.87%), as those showed in Tables 2 and 3.

Table 1. Impurity size effects, taken on the effective dielectric constant $\varepsilon(r_{d(a)})$, being determined in Eq. (1c), the effective donor(acceptor)-ionization energy, $E_{d(a)}(r_{d(a)})$, in absolute values, and the band gap, $E_{gn(p)}(r_{d(a)})$, at T=0K, determined both in Eq. (1b), and finally, the intrinsic band gap, $E_{gin(p)}(T = 300K, r_{d(a)})$ and the intrinsic carrier concentration $n_{in(p)}(T = 300K, r_{d(a)})$, determined respectively in Equations (A4) and (A5) of the Appendix A.

Donor	P	r_{do}	As	Te	Sb	Sn
r_d (nm) [8]	0.110	0.117	0.118	0.132	0.136	0.140
$\varepsilon(r_d)$	11.89	11.7 [8]	11.69	10.87	10.43	9.94

$E_d(r_d)$ in meV	19.2	19.9	19.99	23	25	28
$E_{gn}(r_d)$ in meV	1169	1170 [8]	1170.9	1173	1175	1178
$E_{gin}(300K, r_d)$ in meV	1123	1124	1124.1	1127	1129	1133
$n_{in}(300K, r_d)$ in 10^9 cm^{-3}	4.24	4.16	4.159	3.92	3.77	3.56
Acceptor	B	r_{ao}	Ga	Al	Mg	In
r_a (nm) [8]	0.088	0.117	0.126	0.126	0.140	0.144
$\varepsilon(r_a)$	16.4	11.7 [8]	11.39	11.39	9.95	9.43
$E_a(r_a)$ in meV	17.2	33.8	35.6	35.6	47	52
$E_{gp}(r_a)$ in meV	1153	1170 [8]	1172	1172	1183	1188
$E_{gip}(300K, r_a)$ in meV	1107	1124	1126	1126	1137	1143
$n_{ip}(300K, r_a)$ in 10^9 cm^{-3}	5.8	4.16	4	4	3.23	2.92

Comparison between our numerical results, obtained in present paper and recent paper (**RP)** [1]; $\varepsilon(r_{d(a)}) \gg [\boldsymbol{\varepsilon_{RP}(r_{d(a)})}]$

Donor	P	Te	Sb
r_d (nm) [$\mathbf{r_d}$ **(nm)**]	0.110 [**0.117**]	0.132 [**0.140**]	0.136 [**0.145**]
$\varepsilon(r_d)$ [$\boldsymbol{\varepsilon_{RP}(r_d)}$]	11.89 [**11.4**]	10.87 [**5.20**]	10.43 [**4.46**]
Acceptor	B	Ga	In
r_a (nm) [$\mathbf{r_a}$ **(nm)**]	0.088 [**0.117**]	0.126 [**0.130**]	0.144 [**0.135**]
$\varepsilon(r_a)$ [$\boldsymbol{\varepsilon_{RP}(r_a)}$]	16.4 [**11.4**]	11.39 [**6.95**]	9.43 [**5.82**]

In summary, the effects of $N_{d(a)}$-heavy doping and $r_{d(a)}$- impurity size, given in the HD[d(a)-Si]ER, and those of $N_{a(d)}$-low doping and $r_{a(d)}$- impurity size, given in the LD[a(d)-Si]BR, affect all the minority-carrier transport properties, as those given in the Appendix A, B and C, and also in the following equations.

2.2. Total minority-carrier saturation current density at 300K

The total carrier-minority saturation current density is defined by:

$$J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}, \tag{2}$$

where $J_{Bp(n)o}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-Si]BR, being determined in Eq. (C1) of the Appendix C, and $J_{En(p)o}$ is the minority-hole saturation-current density injected into the HD[d(a)-Si]ER, being developed and determined from I and II, now reported in the following.

In the non-uniformly and heavily doped emitter region of d(a)-Si devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in the HD[d(a)-Si]ER-width W:

$$\rho_{d(a)}(x) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]^{-\left(\frac{x}{W}\right)^2}, \quad 0 \le x \le W,$$

$$N_{d(a)o}(W) \equiv 7.9 \times 10^{17}\ (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2\ (1)10^{-7}\ cm}\right)^{1.066\ (0.5)}\right\}\ (cm^{-3}), \tag{3}$$

where $\rho_{d(a)}(x = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(x = W) = N_{d(a)o}(W)$, decreases with increasing W [1, 2, 13]. Further, the "effective doping density" is defined by:

$$N_{d(a)eff.}(x, r_{d(a)}) \equiv \rho_{d(a)}(x)/\exp\left[\frac{\Delta E_{ga\ n(p)}(\rho_{d(a)}(x), r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)eff.}(x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)}, r_{d(a)})}{k_B T}\right]} \text{ and } N_{d(a)eff.}(x = W,\ r_{d(a)}) \equiv \frac{N_{d(a)o}(W)}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)o}(W), r_{d(a)})}{k_B T}\right]}, \tag{4}$$

where $\Delta E_{ga\ n(p)}$ are determined in Equations (B4, B5) of the Appendix B.

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n_{in(p)}^2}{N_{d(a)eff.}(x,\ r_{d(a)})}, \tag{5}$$

where $n_{in(p)}^2$ is determined in (A5) of the Appendix A and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(x)[n(x)]$ and $p_o(x)[n_o(x)]$, by [22, 25]:

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)])}{p_o(x)[n_o(x)]}, \tag{6}$$

which must verify the two following boundary conditions proposed by Shockley as [6]:

$$u(x = 0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}, \tag{7}$$

$$u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1. \tag{8}$$

Here, $n_{I(II)}(V)$ is a photovoltaic conversion factor determined in Equations (27, 28), S $(\frac{cm}{s})$ is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(x)$.

Further, as developed in I, from the Fick's law for minority hole (electron)-diffusion equations [8, 12]:

$$J_{h(e)}(x) = \frac{-e(+e) \times n_i^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e) n_{in(p)}^2 D_{h(e)}(x)}{N_{d(a)eff.}(x)} \times \frac{du(x)}{dx}, \tag{9}$$

where $N_{d(a)eff.}$ is given in Eq. (4), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (C3, C2, C6) of the Appendix C, and from the minority-hole (electron) continuity equation [8, 12]:

$$\frac{dJ_{h(e)}(x)}{dx} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{F_{h(e)}(x) \times L_{h(e)}^2} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{N_{d(a)eff.}(x) \times \tau_{h(e)E}}, \quad (10)$$

where $L_{h(e)}$ and $\tau_{h(e)E}$ are defined respectively in Equations (C7, C8) of the Appendix C, one finally obtains the following second-order differential equation as [22]:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0. \quad (11)$$

Then, taking into account the two boundary conditions (10, 11), one thus gets the general solution of this Eq. (11), as [22]:

$$u(x) = \frac{\sinh(P(x)) + I(W,S) \times \cosh(P(x))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1\right), \; I(W,S) = \frac{D_{h(e)}(N_o(W))}{S \times L_{h(e)}(N_o(W))}. \quad (12)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, determined in Eq. (29). Further, since $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$, $C = 10^{-17}$ (cm^4/s), for the crystalline Si, being an empirical parameter, chosen for each crystalline semiconductor, P(x) is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}), \; 0 \le x \le W, \; P(x = W) \equiv \left(\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}\right) \times W \equiv \frac{W}{L_{h(e)eff.}} = \frac{L_{h(e)}}{L_{h(e)eff.}} \times \frac{W}{L_{h(e)}}, \quad (13)$$

where $L_{h(e)eff.}$ is the effective minority-hole (electron) diffusion length. Further, from Eq. (9, 13), the minority-hole (electron) current density injected into the HD[d(a)-Si]ER is found to be determined by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S)\,[J_{Epo}(x, W, N_a, r_a, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1\right), \quad (14)$$

where $J_{En(p)o}$ is the saturation minority-hole (electron) current density,

$$J_{En(p)o}(x, W, N_{d(a)}, r_{d(a)}, S) = \frac{en_{i\,n(p)}^2 \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S) \times \sinh(P(x))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))}. \quad (15)$$

Here, the intrinsic carrier concentration $n_{i\,n(p)}$ is computed by Eq. (A5) of the Appendix A, and the effective doping density $N_{d(a)eff.}$ is determined in Eq. (4), the minority-hole (electron) diffusion coefficient $D_{e(h)}$ and minority-hole (electron) diffusion length $L_{h(e)}$ are given respectively in Equations (C2, C3, C7) of the Appendix C, and the factor $I(W,S)$ is determined by:

$$I(W,S) = \frac{D_{h(e)}(N_{d(a)o}(W))}{S \times L_{h(e)}(N_{d(a)o}(W))}, \quad (16)$$

where $N_{d(a)o}(W)$ is determined in Eq. (3).

Further, one remarks that: (i) from Equations (12, 14-16) one obtains: $u(x=0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS\times p_o(x=0)[n_o(x=0)]}$, which is just the first boundary condition given in Eq. (7), and then, (ii) Eq. (12) yields: $u(x=W) = \exp\left(\frac{V}{n_{I(II)}(V)\times V_T}\right) - 1$, being the second boundary condition given in Eq. (8).

In the following, we will denote $P(W)$ and $I(W,S)$ by P and I, for a simplicity. So, Eq. (15) gives:

$$J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right) = \frac{en_{i\,n(p)}^2\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \frac{1}{\sinh(P)+I\times\cosh(P)}, \tag{17}$$

$$J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right) = \frac{en_{i\,n(p)}^2\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \frac{\cosh(P)+I\times\sinh(P)}{\sinh(P)+I\times\cosh(P)}. \tag{18}$$

Thus, from Equations (14, 17, 18), one gets

$$\frac{J_{h(e)}\left(x=0,W,N_{d(a)},r_{d(a)},S,V\right)}{J_{h(e)}\left(x=W,W,N_{d(a)},r_{d(a)},S,V\right)} \equiv \frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)} = \frac{1}{\cosh(P)+I\times\sinh(P)}. \tag{19}$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by [22]:

$Q_{h(e)eff.}(x=W,N_{d(a)},r_{d(a)}) \equiv \int_0^W +e(-e)\times u(x)\times p_o(x)[n_o(x)]\times\frac{\tau_{h(e)E}(N_{d(a)},r_{d(a)})}{\tau_{h(e)E}(\rho_{d(a)}(x),r_{d(a)})}dx$, and the effective minority-hole transit time by: $\tau_{teff.}(x=W,W,N_{d(a)},r_{d(a)},S) \equiv Q_{h(e)\,eff.}(x=W,N_{d(a)},r_{d(a)})/J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)$, one can define, from Equations (10, 19), the reduced effective minority-hole transit time:

$$\frac{\tau_{teff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}} \equiv 1 - \frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)} = 1 - \frac{1}{\cosh(P)+I\times\sinh(P)}. \tag{20}$$

Now, some important results can be obtained and discussed below.

As $P \ll 1$ (or $W \ll L_{h,eff.}$) and $S\to\infty$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S\times L_h(N_o(W))} \to 0$, from Eq. (20), one has: $\frac{\tau_{t,eff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (18), one obtains:

$$J_{En(p)o}\left(x=W,N_{d(a)},r_{d(a)},S\to\infty\right) \to \frac{en_{i\,n(p)}^2\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \frac{1}{P(W)}, \tag{21a}$$

and then, as $P \gg 1$ (or $W \gg L_{h,eff.}$) and $S\to 0$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S\times L_h(N_o(W))} \to \infty$, from Eq. (20), one has: $\frac{\tau_{teff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (18), one gets:

$$J_{En(p)o}\left(x=W,N_{d(a)},r_{d(a)},S\to 0\right) \to \frac{en_{i\,n(p)}^2\times D_{h(e)}}{N_{d(a)eff.}\times L_{h(e)}} \times \tanh(P). \tag{21b}$$

In summary, in the $n^+(p^+)-p(n)$ junction solar cells, the dark carrier-minority saturation current density J_o, defined in Eq. (2), is now replaced by $J_{oI(II)}$, for a good presentation, and rewritten by:

$$J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \equiv J_{En(p)o}(W, N_{d(a)}, r_{d(a)}, S) + J_{Bp(n)o}(N_{a(d)}, r_{a(d)}), \quad (22)$$

where $J_{En(p)o}$ and $J_{Bp(n)o}$ are determined respectively in Equations (18) and (C1) of the Appendix C. Then, in the following, in the $n^+(p^+)-p(n)$ junction solar cells, and for physical conditions as: $W = 0.0044\ (0.000206)\mu m, N_{d\equiv do(a\equiv ao)} = 10^{20}(10^{20})\ cm^{-3}, S = 10^{50}\frac{cm}{s}, N_{a\equiv ao(d\equiv do)} = 10^{16}(10^{16})\ cm^{-3}$, we propose, at given $V_{ocI1(2)}$ and $V_{ocII1(2)}$, the experimental results of the short circuit current density $J_{scI(II)}$ and the fill factor $F_{I(II)}$, in order to formulate our treatment method of two fixe experimental points. Then, for the n^+-p junction [1, 2, 23, 27, 28],

$$V_{ocI1(2)} = 624\ (740)\ mV,\ J_{scI1(2)} = 36.3\ (41.8)\ mA/cm^2,\ F_{I1(2)} = 80.1\ (82.7)\ \%, \text{ and} \quad (23)$$

for the p^+-n junction [1, 2, 30],

$$V_{ocII1(2)} = 639\ (738)\ mV,\ J_{scII1(2)} = 39.3\ (42.6)\ mA/cm^2,\ F_{II1(2)} = 78.9\ (84.9)\ \%. \quad (24)$$

3. Photovoltaic conversion effect at 300K

As defined and developed in I, the net current density J, at T=300 K and for the infinite shunt resistance, expressed as a function of the applied voltage V, flowing through the $n^+(p^+)-p(n)$ junction of silicon solar cells, is defined by [1, 2, 5-10]:

$$J(V) \equiv J_{ph.}(V) - J_{oI(II)} \times \left(e^{X_{I(II)}(V)} - 1\right),\ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V)\times V_T},\ V_T \equiv \frac{k_B T}{e} = 25.85\ mV, \quad (25)$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor (**PVCF**), noting that as $V = V_{oc}$, $J(V) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, for $V_{oc} \geq V_{ocI(II)1}$. Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \quad (26)$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$ is the PVCF, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc})\times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, given in I , and (ii) the function

$\left(e^{X_{I(II)}(V_{oc})}-1\right)$ or the PVCF $n_{I(II)}$, representing the photovoltaic conversion effect, thus converts the light, represented by $J_{scI(II)}$, into the electricity, by $J_{oI(II)}$.

Further, from Equations (22, 26), we obtain for the n^+-p junction:

$n_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}, J_{scI1(2)}\right) \equiv \frac{V_{ocI1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scI1(2)}}{J_{oI}}+1\right)} \equiv n_{I1(2)}(V_{ocI1(2)}, J_{scI1(2)})$, and

then,

$$n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = n_{I1}(V_{ocI1}, J_{scI1}) + n_{I2}(V_{ocI2}, J_{scI2}) \times \left(\frac{V_{oc}}{V_{ocI1}}-1\right)^{1.1216}, \quad (27)$$

being valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$, and then, for the p^+-n junction:

$n_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}, J_{scII1(2)}\right) \equiv \frac{V_{ocII1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scII1(2)}}{J_{oII}}+1\right)} \equiv n_{II1(2)}(V_{ocII1(2)}, J_{scII1(2)})$,

and then,

$$n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = n_{II1}(V_{ocI1}, J_{scI1}) + n_{II2}(V_{ocII2}, J_{scII2}) \times \left(\frac{V_{oc}}{V_{ocII1}}-1\right)^{1.09103}, \quad (28)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Therefore, from Equations (23, 24, 27, 28), one obtains, $n_{I1(II1)} = 1.0808$ (1.20469) at $V_{ocI1(II1)} = 624$ (639) mV, and $n_{I2(II2)} = 1.2737$ (1.38588) at $V_{ocI2(II2)} = 740$ (738) mV, respectively, for $n^+(p^+)-p(n)$ junction solar cells.

Thus, X_I defined from Eq. (26) now becomes for the n^+-p junction:

$X_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \equiv \frac{V_{oc}}{n_I(W,N_d,r_d,S,N_a,r_a,V_{oc})\times V_T}$, and therefore, we can determine the values of the fill factors $F_{I1(2)}$ at $V_{oc} = V_{ocI1(2)}$ by [1, 2]:

$F_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}\right) = \frac{X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(2)}\right)-\ln\left[X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(2)}\right)+0.72\ (0.72)\right]}{X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(2)}\right)+1.63\ (0.9)} \equiv F_{I1(2)}\left(V_{oc} = V_{ocI1(2)}\right)$, for a presentation simplicity, and further, the fill factor F_I can be computed by:

$$F_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = F_{I1}(V_{ocI1}) + F_{I2}(V_{ocI2}) \times \left(\frac{V_{oc}}{V_{ocI1}}-1\right)^{2.0559}, \quad (29)$$

which is valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Then, also from Eq. (26), we can define for the p^+-n junction:

$X_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) \equiv \frac{V_{oc}}{n_{II}(W,N_a,r_a,S,N_d,r_d,V_{oc})\times V_T}$, where $n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc})$ is determined in Eq. (28). Therefore, we can determine the values of the fill factors $F_{II1(2)}$ at $V_{oc} = V_{ocII1(2)}$ as:

$F_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}\right) = \frac{X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(2)})-\ln[X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(2)})+0.72\ (0.72)]}{X_{II}(W,N_a,r_a,S,N_d,r_d,V_{ocII1(2)})+1.6136\ (0.0595)} \equiv F_{II1(2)}\left(V_{ocII1(2)}\right)$, for a presentation simplicity, and further, the fill factor F_{II} is determined by:

$$F_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = F_{II1}(V_{ocII1}) + F_{II2}(V_{ocII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.4209}, \quad (30)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Numerically, Equations (29, 30) give: $F_{I1(2)}$=80.01% (82.7%**)** at $V_{ocI1(2)} = 624\ (740)$ for the $n^+ - p$ junction, and $F_{II1(2)}$ =78.9% (84.9%) at $V_{ocII1(2)} = 639\ (738)$ mV for the $p^+ - n$ junction, respectively, being in perfect agreement with the data given in Equations (23, 24).

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv \frac{J_{scI(II)}\times V_{oc}\times F_{I(II)}}{P_{in.}}, \quad (31)$$

where $J_{scI(II)}$ and $F_{I(II)}$ are determined respectively in Equations (26, 29, 30), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$) [1, 2, 26-29].

4. Numerical results and concluding remarks

We will respectively consider the two following cases, given in 8 $n^+(p^+) - p(n)$ junction solar cells: P(B)-B(P), As(Ga)-Ga(As), Sb(Mg)-Mg(Sb), Sn(In)-In(Sn), respectively, in the following.

4.1. HD $[(P;\ As;\ Sb;\ Sn) - Si]$ ER $-$ LD$[(B;\ Ga;\ Mg;\ In) - Si)]$ BR $-$cases

Here, for those 4 $(n^+ - p) -$ junctions: $[(P - B), (As - Ga), (Sb - Mg), (Sn - In)]$, respectively, we propose the following physical conditions as:

$$W = 15\ \mu m,\ N_d = 5 \times 10^{20}\ cm^{-3}, S\ = 100\ (cm/s\ ),\ and\ N_a = 10^{18}\ cm^{-3}. \quad (32)$$

Then, from Eq. (20) , on respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}} = (2.34,\ 2.07,\ 0.75,\ 0.48) \times 10^{-4} \ll 1$, suggesting the highly transparent condition, and from Eq. (18),

$J_{Eno} = (1.\ 3, 1.4, 1.8, 2.1) \times 10^{-15}\ \left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C:

$J_{Bpo} = (1.4, 0.5, 0.3, 0.2\ ) \times 10^{-14}\ \left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively:

$J_{oI} = (1.5,\ 0.6,\ 0.4,\ 0.4) \times 10^{-14}\ \left(\frac{A}{cm^2}\right)$, and from the following Table 2, for example, at $V_{oc} = 705\ mV$, n_I= (0.948, 0.918, 0.909, 0.906) and η_I= (28.17, 28.53, 28.64, **28.68**) %, meaning that, with increasing $r_{d(a)}$ or with decreasing $\varepsilon_{d(a)}$, which is due to the d(a)-size effect, both J_{oI} and n_I decrease, while η_I increases, being new obtained results.

Table 2. In the HD[(P; As; Sb; Sn)-**Si**] ER-LD[(B; Ga; Mg; In)-**Si)**] BR and for physical conditions given in Eq. (32), our numerical results of n_I, J_{scI}, F_I, and η_I, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{d(a)}$, the function n_I decreases, while other functions J_{scI}, F_I, and η_I increase, being due to the impurity size $r_{d(a)}$-effect, suggesting our new obtained results.

V_{oc}(mV)	n_I	$J_{scI}(\frac{mA}{cm^2})$	F_I(%)	η_I (%)
750	1.013; 0.981; 0.971; 0.969	41.65; 41.80; 41.85; 41.87	86.6; 87.0; 87.1; 87.1	27.04; 27.27; 27.34; 27.36
740	0.998; 0.967; 0.957; 0.954	43.11; 43.33; 43.39; 43.41	86.1; 86.5; 86.6; 86.6	27.46; 27.73; 27.81; 27.84
720	0.969; 0.939; 0.930; 0.927	45.71; 46.03; 46.12; 46.16	85.2; 85.6; 85.7; 85.8	28.04; 28.37; 28.47; 28.50
710	0.955; 0.925; 0.916; 0.913	46.75; 47.11; 47.23; 47.26	84.8; 85.2; 85.4; 85.4	28.16; 28.51; 28.62; 28.66
705	0.948; 0.918; 0.909; 0.906	47.19; 47.57; 47.69; 47.72	84.7; 85.1; 85.2; 85.2	**28.17; 28.53; 28.64; 28.68**
700	0.941; 0.911; 0.903; 0.900	47.56; 47.96; 48.08; 48.12	84.5; 84.9; 85.0; 85.1	28.13; 28.51; 28.62; 28.66
680	0.914; 0.885; 0.876; 0.874	48.22; 48.65; 48.78; 48.83	84.0; 84.4; 84.5; 84.5	27.54; 27.92; 28.03; 28.07
655	0.881; 0.853; 0.845; 0.843	46.42; 46.79; 46.90; 46.94	83.6; 84.0; 84.1; 84.1	25.41; 25.73; 25.83; 25.86
640	0.863; 0.836; 0.828; 0.825	43.24; 43.49; 43.56; 43.59	83.4; 83.8; 83.9; 84.0	23.09; 23.33; 23.40; 23.43
624	0.847; 0.820; 0.812; 0.810	36.30; 36.30; 36.30; 36.30	83.4; 83.8; 83.9; 83.9	18.89; 18.98; 19.00; 19.01

4.2. HD $[(B;\ Ga;\ Mg;\ In) - Si]$ ER $-$ LD$[(P;\ As;\ Sb;\ Sn) - Si]$ BR $-$cases

Here, for those 4 $(p^+ - n) -$ junctions: $[(B - P), (Ga - As), (Mg - Sb), (In - Sn)]$, respectively, we propose the following physical conditions as:

$$W = 15\ \mu m,\ N_a = 5 \times 10^{20}\ cm^{-3}, S\ = 100\ (cm/s\ ),\ and\ N_d = 10^{18}\ cm^{-3}. \quad (33)$$

Then, from Eq. (20) , on respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}} = (1.0, 0.97, 0.88, 0.81)$, suggesting the highly opaque condition, and from Eq. (18), $J_{Epo} = (5.35,\ 2.56,\ 1.69, 1.40) \times 10^{-17}\ \left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C:

$J_{Bno} = (\ 9.89, 9.75, 8.69, 8.28) \times 10^{-15}\left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively:

$J_{oII} = (9.94, 9.77, 8.71, 8.30) \times 10^{-15}\ \left(\frac{A}{cm^2}\right)$, and from the following Table 3, for example, at $V_{oc} = (731)\ mV$, n_{II} = (0.970, 0.970, 0.966, 0.964) and η_{II} = (29.74, 29.81, 29.85, **29.87**) %, meaning that, with increasing $r_{a(d)}$ or with decreasing $\varepsilon_{a(d)}$, which is due to the a(d)-size effect, both J_{oII} and n_{II} decrease, according to the increase in η_{II}, being new obtained results.

Table 3. In the HD[(B; Ga; Mg; In)-**Si**] ER-LD[(P; As; Sb; Sn)-**Si)**] BR and for physical conditions given in Eq. (33), our numerical results of n_{II}, J_{scII}, F_{II}, and η_{II}, are computed by using Equations (28, 26, 30, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{a(d)}$, the function n_{II} decreases, while other functions J_{scII}, F_{II}, and η_{II} increase, being due to the impurity size $r_{a(d)}$-effect, suggesting our new obtained results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
800	1.070; 1.069; 1.065; 1.063	35.80; 35.80; 35.80; 35.80	96.1; 96.1; 96.1; 96.2	28.13; 28.18; 28.19; 28.20
738	0.980; 0.980; 0.976; 0.974	43.86; 43.86; 43.88; 43.88	89.9; 89.9; 89.9; 90.0	29.72; 29.78; 29.83; 29.84
731	0.970; 0.970; 0.966; 0.964	44.61; 44.61; 44.63; 44.64	89.3; 89.3; 89.3; 89.3	**29.74; 29.81; 29.85; 29.87**
724	0.961; 0.960; 0.956; 0.955	45.29; 45.30; 45.32; 45.33	88.7; 88.7; 88.7; 88.7	29.71; 29.78; 29.82; 29.84
715	0.948; 0.948; 0.944; 0.942	46.06; 46.06; 46.09; 46.10	87.9; 87.9; 88.0; 88.0	29.59; 29.66; 29.70; 29.72
670	0.888; 0.888; 0.884; 0.883	46.56; 46.56; 46.59; 46.61	84.8; 84.8; 84.9; 84.9	27.06; 27.11; 27.16; 27.18
650	0.864; 0.863; 0.860; 0.858	43.46; 43.46; 43.48; 43.49	83.9; 83.9; 84.0; 84.0	24.24; 24.29; 24.32; 24.34
645	0.858; 0.858; 0.854; 0.853	42.01; 42.02; 42.03; 42.03	83.8; 83.8; 83.8; 83.8	23.21; 23.26; 23.29; 23.30
640	0.853; 0.852; 0.849; 0.848	39.95; 39.95; 39.95; 39.95	83.7; 83.7; 83.7; 83.7	21.87; 21.91; 21.93; 21.94
639	0.852; 0.852; 0.848; 0.847	39.30; 39.30; 39.30; 39.30	83.6; 83.6; 83.7; 83.7	21.48; 21.52; 21.54; 21.55

In conclusion, our new limiting highest efficiency results: **28.68%** and **29.87%**, given in Tables 2 and 3, can also be compared respectively with other limiting η-results equal to:

(i) **29.43%**, for a 110 μm thick solar cell made of intrinsic silicon, being obtained by Richter et al. [26],

(ii) **30%**, for $E_{gi}\left(r_{P(B)}\right) = 1.1$ eV, being investigated by Shockley and Queisser [6], and

(iii) **31**%, for physical conditions: $S = 100\ cm/s$ and $W = 15\ \mu m$, being obtained by Bhattacharya and John [3, 4].

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Appendix

Appendix A. Fermi Energy

In the n(p)-type Si crystal, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11×10^{-4} [39, 40], is now summarized in the following.

First of all, we define the reduced electron density by:

$$u\equiv\frac{N_{d(a)}}{N_{c(v)}},\ N_c(T,r_d)=2\times6\times\left(\frac{m_c\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}),\ N_v(T)=2\times2\times\left(\frac{m_v\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}). \tag{A.1}$$

Here, $N_{c(v)}$ is the conduction (valence)-band density of states, respectively, m_c is the effective mass of the electron in n-type Si can be defined by [8]:

$$m_c/m_o=0.2, \tag{A2}$$

where m_o is the electron rest mass, the averaged effective mass of the hole m_v, given in the p-type Si yields [8]:

$$m_v/m_o=\frac{0.16+0.52}{2}=0.34, \tag{A3}$$

and finally, $E_{gin(p)}(T,r_{d(a)})$ is the intrinsic band gap, given in the silicon (Si), due to the T-dependent carrier-lattice interaction-effect, by [1, 33, 34]:

$$E_{gin(p)}(T,r_{d(a)})=E_{gn(p)}(r_{d(a)})-0.071\ (eV)\times\left\{\left[1+\left(\frac{2T}{440.6913\ K}\right)^{2.201}\right]^{\frac{1}{2.201}}-1\right\}, \tag{A4}$$

where $E_{gn(p)}(r_{d(a)})$ is determined in Eq. (1b) and its numerical results are given in Table 1.

Furthermore, in the n(p)-type Si, one can define the intrinsic carrier concentration $n_{in(p)}$ by:

$$n^2_{i\,n(p)}(T,r_{d(a)})\equiv N_c(T,r_d)\times N_v(T)\times\exp\left(\frac{-E_{gin(p)}(T,r_{d(a)})}{k_BT}\right). \tag{A5}$$

Then, denoting the reduced Fermi energy in the n(p)-type semiconductor, respectively by:

$\frac{E_{Fn}(u)}{k_BT}\left(\frac{-E_{Fp}(u)}{k_BT}\right)$, we found with a precision of the order of 10^{-7} [39], as:

$$\frac{E_{Fn}(u)}{k_BT}\left(\frac{-E_{Fp}(u)}{k_BT}\right)=\frac{G(u)+Au^BF(u)}{1+Au^B},\ A=0.0005372\ \text{and}\ B=4.82842262 \tag{A6}$$

where

$$F(u)=au^{\frac{2}{3}}\left(1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}},\quad a=[(3\sqrt{\pi}/4)\times u]^{2/3},\quad b=\frac{1}{8}\left(\frac{\pi}{a}\right)^2\quad\text{and}\quad c=\frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$$

and

$$G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du};\ d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0,$$

noting that: (i) $\frac{E_{Fn}(u\gg1)}{k_BT}\left(\frac{-E_{Fp}(u\gg1)}{k_BT}\right) > 1$, according to the HD[d(a)-Si]ER-case (i.e., the degenerate case), Eq. (A6) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u\ll1)}{k_BT}\left(\frac{-E_{Fp}(u\ll1)}{k_BT}\right) <- 1$, to the LD[a(d)-Si]BR-case (i.e., the non-degenerate case), Eq. (A6) is now reduced to the function G(u), respectively. Then, Eq. (A6) can be applied to the following cases as:

(i) in the HD[d(a)-Si]ER-case, for $N_{d(a)} = 10^{20}(10^{20})\ cm^{-3}$, we respectively get: $\frac{E_{Fn}(u\gg1)}{k_BT}\left(\frac{-E_{Fp}(u\gg1)}{k_BT}\right) = 4.84\ (5.85) > 1$, according to degenerate conditions.

(ii) in the LD[a(d)-Si]BR-case, for $N_{a(d)} = 10^{18}(10^{18})\ cm^{-3}$, we respectively get: $\frac{-E_{Fp}(u\ll1)}{k_BT}\left(\frac{E_{Fn}(u\ll1)}{k_BT}\right) =- 2.26\ (-2.57) <- 1$, according to non-degenerate conditions. Thus, those limiting values of $N_{a(d)} = 10^{18}(10^{18})\ cm^{-3}$ can be used in the LD[a(d)-Si] BR-cases, respectively.

Appendix B. Approximate forms for band gap narrowing and apparent band gap narrowing

First of all, in the n(p)-type Si, we define the effective Wigner-Seitz radius r_s characteristic of the interactions by [1, 2]

$$r_{sn} \equiv r_s(N_d, r_d) = 1.1723 \times 10^8 \times \left(\frac{6}{N_d}\right)^{1/3} \times \frac{m_c}{\varepsilon(r_d)} \tag{B1}$$

and

$$r_{sp} \equiv r_s(N_a, r_a) = 1.1723 \times 10^8 \times \left(\frac{2}{N_a}\right)^{1/3} \times \frac{m_v}{\varepsilon(r_a)}. \tag{B2}$$

Therefore, the correlation energy of an effective electron gas, $E_c\left(r_{sn(sp)}\right)$, is given by [1, 2, 42]:

$$E_{cn(cp)}\left(N_{d(a)}, r_{d(a)}\right) = \frac{-0.87553}{0.0908+r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908+r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right)\times \ln\left(r_{sn(sp)}\right) - 0.093288}{1+0.03847728\times r_{sn(sp)}^{1.67378876}}. \tag{B3}$$

Then, in the n-type heavily doped Si, the BGN is found to be given as [1]:

$$\Delta E_{gn}(N_d, r_d) \simeq a_1 \times \frac{\varepsilon(r_{Si})}{\varepsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{Si})}{\varepsilon(r_d)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_c(r_{sn}) \times r_{sn}]) + a_3 \times \left[\frac{\varepsilon(r_{Si})}{\varepsilon(r_d)}\right]^{5/4} \times \sqrt{\frac{m_v}{m_c}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_{Si})}{\varepsilon(r_d)}} \times N_r^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon(r_{Si})}{\varepsilon(r_d)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r \equiv \left(\frac{N_d}{9.999\times10^{17}\ cm^{-3}}\right), \tag{B4}$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type heavily doped Si, one has [1]:

$$\Delta E_{gp}(N_a, r_a) \simeq a_1 \times \frac{\varepsilon(r_{Si})}{\varepsilon(r_a)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{Si})}{\varepsilon(r_a)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sp}) \times r_{sp}]\right) + a_3 \times \left[\frac{\varepsilon(r_{Si})}{\varepsilon(r_a)}\right]^{5/4} \times \sqrt{\frac{m_c}{m_v}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_{Si})}{\varepsilon(r_a)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon(r_{Si})}{\varepsilon(r_a)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, \; N_r \equiv \left(\frac{N_a}{9.999 \times 10^{17}\,cm^{-3}}\right), \quad (B5)$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$ and $a_5 = 9.80 \times 10^{-5}(eV)$.

Further, in the donor (acceptor)-Si, we define the effective intrinsic carrier concentration $n_{ien(p)}$, by

$$n_{i\,en(p)}^2(N_{d(a)}, r_{d(a)}) \equiv N_{d(a)} \times p_o(n_o) \equiv n_{i\,n(p)}^2 \times \exp\left[\frac{\Delta E_{gan(p)}}{k_B T}\right], \quad (B6)$$

where we can define the "effective doping density" by: $N_{d(a)eff.} \equiv N_{d(a)} / \exp\left[\frac{\Delta E_{ga\,n(p)}}{k_B T}\right]$ so that $N_{d(a)eff.} \times p_o(n_o) \equiv n_{i\,n(p)}^2$ [8], and also the apparent band gap narrowing (**ABGN**), $\Delta E_{ga\,n(p)}$, as

$$\Delta E_{ga\,n(p)} \equiv \Delta E_{g\,n(p)} + k_B T \times \ln\left(\frac{N_{d(a)}}{\mathbf{N_{c(v)}}}\right) - E_{Fn}\left(\frac{N_d}{N_c}\right)[-E_{Fp}\left(\frac{N_a}{N_v}\right)], \quad (B7)$$

where $N_{c(v)}$ is defined in Eq. (A1), the Fermi energy is determined in Eq. (A6).

Appendix C. Minority-carrier transport parameters

Here, the minority-electron (hole) saturation current density injected into the LD[a(d)-Si]BR, with an acceptor density equal to $N_{a(d)}$, is given in RP by [1, 7]:

$$J_{Bp(n)o}\left(N_{a(d)}, r_{a(d)}\right) = \frac{e \times n_i^2(r_{a(d)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{e(h)B}(N_{a(d)})}}}{N_{a(d)}}, \quad (C1)$$

where $n_{i\,n(p)}^2(r_{d(a)})$ is determined in (A5), $D_{e(h)}(N_{a(d)}, r_{a(d)})$ is the minority- electron (hole) diffusion coefficient:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[92 + \frac{1360 - 92}{1 + \left(\frac{N_a}{1.3 \times 10^{17} cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a)}{\varepsilon(r_B)}\right)^2 \left(cm^2 V^{-1} s^{-1}\right), \quad (C2)$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[130 + \frac{500 - 130}{1 + \left(\frac{N_d}{8 \times 10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d)}{\varepsilon(r_p)}\right)^2 \left(cm^2 V^{-1} s^{-1}\right), \quad (C3)$$

and $\tau_{e(h)B}(N_{d(a)})$ is the minority- electron (hole) lifetime in the base region:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{2.5 \times 10^{-3}} + 3 \times 10^{-13} \times N_a + 1.83 \times 10^{-31} \times N_a^2. \quad (C4)$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{2.5 \times 10^{-3}} + 11.76 \times 10^{-13} \times N_d + 2.78 \times 10^{-31} \times N_d^2, \quad (C5)$$

Further, from (A6), (B4)-(B7)), in the HD[d(a)-Si]ER, we can define the following minority-hole(electron) transport parameter $F_{h(e)}$ as [8, 22, 25]:

$$F_{h(e)}\ (N_{d(a)}, r_{d(a)}) \equiv \frac{n^2_{i\,n(p)}(r_{d(a)})}{p_o(n_o)\times D_{h(e)}} = \frac{N_{d(a)eff.}}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}\times \exp\left[\frac{\Delta E_{g\,an(p)}}{k_B T}\right]}\ (cm^{-5}\times s), \quad (C6)$$

Furthermore, the minority-hole (electron) diffusion length, $L_{h(e)}\left(N_{d(a)}, r_{d(a)}\right)$ and the minority-hole(electron) lifetime $\tau_{h(e)E}$ in the HD[d(a)-Si]ER can be determined by

$$L^{-2}_{h(e)}\left(N_{d(a)}, r_{d(a)}\right) = \left[\tau_{h(e)E}\times D_{h(e)}\right]^{-1} = \left(C\times F_{n(p)}\ \right)^2 = \left(C\times \frac{N_{d(a)eff.}}{D_{h(e)}}\right)^2 = \left(C\times \frac{n^2_{i\,n(p)}(r_{d(a)})}{p_o(n_o)\times D_{h(e)}}\right)^2, \quad (C7)$$

where the constant $C[=10^{-17}\ (cm^4/s)]$ was chosen in I and II, and then, $\tau_{h(e)E}$ can be computed by:

$$\tau_{h(e)E} = \frac{1}{D_{h(e)}\times\left(C\times F_{n(p)}\ \right)^2}. \quad (C8)$$

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