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30.76%(42.73)%-New Limiting Highest Efficiencies obtained in the $n^+(p^+) - p(n)$ Crystalline GaAs Junction Solar Cells at T=300 K, Due to The Effects of Heavy (Low) Doping and Impurity Size

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Abstract:

In the $n^+(p^+) - p(n)$ crystalline GaAs-junction solar cells, by basing on a same treatment method, and for a same heavy (low) doping effect, as those given in our recent paper (**RP**) [1], but using now a new expression, obtained for the relative dielectric constant $\varepsilon(r_{d(a)})$, determined exactly in the effective Bohr model, as that given in Eq. (1c), representing the donor (acceptor) d(a)-radius $r_{d(a)}$ – effect, or the $\varepsilon(r_{d(a)})$ – effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as observed in Tables 2 and 3, we finally get in our present paper, for highest values of $r_{d(a)}$, the new limiting highest efficiencies, η=30.76% (42.73%)$<\eta_{RP}$ =31.474% (44.359%), coming from the fact that: $\varepsilon(r_{d(a)}) > \varepsilon_{RP}(r_{d(a)})$, $\varepsilon_{RP}(r_{d(a)})$ being our inaccurate and simple formula, proposed in RP, and also reported in Eq. (1d), for a comparison. Finally, our new limiting highest efficiencies, **η=30.76% (42.73%)**, can also be compared with other limiting η-results, such as: **29.1%** for GaAs-thin film

cell, and **45.7%** and **44.4%**, respectively for GaInP/GaAs/GaInAs/GaInAs -and-InGaP/GaAs/InGaAs multijunction cells, obtained by Green et al. [3].

Keywords: donor (acceptor)-size effect; heavily doped emitter region; photovoltaic conversion factor; open circuit voltage; photovoltaic conversion efficiency

1. Introduction

In our recent paper (RP) [1], by basing on: (**i**) the heavy doping and impurity size effects, which affect the total carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{En(p)o}$ and $J_{Bp(n)o}$ being the carrier-minority saturation current densities, injected respectively into the heavily doped donor (acceptor)-GaAs emitter-lightly doped acceptor (donor)-Si base-regions, HD[d(a)- GaAs]ER-LD[a(d)- GaAs]BR, of $n^+(p^+) - p(n)$ junction solar cells, **(ii)** an effective Gaussian donor (acceptor)-density profile $\rho_{d(a)}$ to determine $J_{En(p)o}$ [1, 2, 13, 18-20, 22], and **(iii)** the use of two fixed experimental points, we investigated the photovoltaic conversion factor $n_{I(II)}$, the short circuit current density $J_{scI(II)}$, the fill factor $F_{I(II)}$, and finally the efficiency $\eta_{I(II)}$ [1- 45]. These physical quantities were expressed as functions of the open circuit voltage V_{oc}, and of various parameters such as: the emitter thickness W, high donor (acceptor) density $N_{d(a)}$, surface recombination velocity S, given in the HD[d(a)- GaAs]ER, and low acceptor (donor) density $N_{a(d)}$, given in the LD[a(d)- GaAs]BR.

Then, in our present paper, by basing on a same treatment method, and for a same heavy (low) doping effect, as those given in RP, but using now a new expression, obtained for the relative dielectric constant $\varepsilon(r_{d(a)})$, determined exactly in the effective Bohr model, as that given in Eq. (1c), which represents the donor (acceptor) d(a)-radius $r_{d(a)}-$effect or the $\varepsilon(r_{d(a)})-$effect, suggesting further that, for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, as showed in Table 1, according to the increase in photovoltaic efficiency η, as observed in Tables 2 and 3, we finally get in our present paper, for highest values of $r_{d(a)}$, the new limiting highest efficiencies, η=30.76% (42.73%) $< \eta_{RP} = 31.474\%$ (44.359%), according to: $\varepsilon(r_{d(a)}) > \varepsilon_{RP}(r_{d(a)})$, $\varepsilon_{RP}(r_{d(a)})$ being our inaccurate and simple formula, proposed in RP, and also reported in Eq. (1d), for a comparison.

Finally, our new limiting highest efficiencies, **η=30.76% (42.73%)**, can also be compared with other limiting η -results, such as: **29.1%** for GaAs-thin film cell, and **45.7%** and **44.4%**,

respectively for GaInP/GaAs/GaInAs/GaInAs -and- InGaP/GaAs/InGaAs multijunction cells, being obtained by Green et al. [3].

In Section 2, all the results energy-band-structure parameters for d(a)- GaAs systems are reported in Table 1, and the expressions for $J_{En(p)o}$ are also reported, so that we can determine the total (or dark) carrier-minority saturation current density $J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}$, $J_{Bp(n)o}$ being determined in Eq. (C1) of the Appendix C. In Section 3, the photovoltaic effect is investigated. Finally, some numerical results and concluding remarks are given and discussed in Section 4.

2. Energy-Band-Structure Parameters and dark minority-carrier saturation current density, due to impurity-size and heavy doping effects

Now, we present the effects of donor (acceptor) [d(a)]-size and heavy doping, taken on the energy-band-structure parameters, and investigate the minority-carrier saturation current densities, as follows.

2.1. Effect of d(a)-size

In d(a)-GaAs-systems at T=0 K, since the d(a)-radius $r_{d(a)}$, in tetrahedral covalent bonds is usually either larger or smaller than the As(Ga) atom-radii $r_{As(Ga)}$, a local mechanical strain (or deformation potential energy) is induced, according to a compression (dilation) for $r_{d(a)} > r_{As(Ga)}$ ($r_{d(a)} < r_{As(Ga)} = r_{do(ao)} = 0.118\ (0.126)$ nm), respectively, due to the d(a)-size effect [42]. Then, we have shown [8] that this $r_{d(a)}$-effect affects the changes in all the energy-band-structure parameters, expressed in terms of the relative dielectric constant $\varepsilon(r_{d(a)})$, as given in the following.

First, we note that in the GaAs [8] the relative dielectric constant of the intrinsic silicon is equal to: $\varepsilon(r_{As(Ga)}) = 13.13$, the relative effective electron (hole) mass in conduction (valence) bands yield: $(m_c/m_o) = 0.066$ and $(m_v/m_o) = \frac{0.082+0.5}{2} = 0.291$, the unperturbed intrinsic band gap at 0K, $E_{go}(r_{do(ao)} = r_{As(Ga)}) = 1.52$ eV, the effective donor (acceptor)-ionization energy at $r_{do(ao)} = r_{As(Ga)}$ in absolute values: $E_{do}(r_{do}) = \frac{13600\times(m_c/m_o)}{(\varepsilon(r_{As(Ga)}))^2}$ meV $= 5.2$ meV, and $E_{ao}(r_{ao}) = \frac{13600\times(m_v/m_o)}{(\varepsilon(r_{As(Ga)}))^2}$ meV $= 23$ meV, and the isothermal bulk modulus are defined, for the n(p)-type GaAs, by: $B_n \equiv \frac{E_{do}}{(4\pi/3)\times(r_{As})^3} = 1.212\times10^8\ (N/m^2)$, and finally, $B_p \equiv \frac{E_{ao}}{(4\pi/3)\times(r_{Ga})^3} = 4.389\times10^8\ (N/m^2)$.

Therefore, at $r_{d(a)} = r_{do(ao)}$, the boundary conditions are found to be, for the impurity-atom volume V, $V_{do(ao)} = (4\pi/3) \times \left(r_{do(ro)}\right)^3$, the pressure p, $p_o = 0$, and the deformation potential energy or the strain energy σ, $\sigma_o = 0$. Further, the two important equations [42], needed to determine the σ-variation $\Delta\sigma \equiv \sigma\text{-}\sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, in the n(p)-type GaAs, by an integration, one gets:

$$(\Delta\sigma)_{n(p)} = B_{n(p)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (1a)$$

Furthermore, we also shown [42] that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$) , the compression (dilatation) corresponding the repulsive (attractive) force increases (decreases) the energy gap $E_{gn(gp)}\left(r_{d(a)}\right)$ and the effective donor(acceptor)-ionization energy $E_{d(a)}\left(r_{d(a)}\right)$ in absolute values, obtained in the effective Bohr model, which is represented by: $+(-)\ (\Delta\sigma)_{n(p)}$, respectively. That gives:

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{As(Ga)})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$E_{gn(gp)} - E_{go} = E_{d(a)} - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon(r_{As(Ga)})}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\ (\Delta\sigma)_{n(p)}, \text{ for } r_{d(a)} \leq r_{do(ao)}. \quad (1b)$$

Then, from Equations (1a, 1b), the exact expression of relative dielectric constant $\varepsilon(r_{d(a)})$, is given by:

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{As(Ga)})}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon(r_{As(Ga)}), \text{ for } r_{d(a)} \geq r_{do(ao)}, \text{ and}$$

$$\varepsilon(r_{d(a)}) = \frac{\varepsilon(r_{As(Ga)})}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon(r_{As(Ga)}), \text{ for } r_{d(a)} \leq r_{do(ao)}, \quad (1c)$$

which can be compared with its approximated and simple form, proposed in our recent paper (RP) [1], by:

$$\varepsilon_{RP}(r_{d(a)}) \simeq 11.4 \times \left(\frac{r_{As(Ga)}}{r_{d(a)}}\right)^{4.377\ (4.7)}. \quad (1d)$$

Therefore, with increasing $r_{d(a)}$, the effective dielectric constant $\varepsilon(r_{d(a)})$, determined in Eq. (1c), decreases, implying that $E_{go}\left(r_{d(a)}\right)$ and $E_{do(ao)}(r_d)$, given in Eq. (1b), increase, as observed in the following Table 1, in which we also remark that, for a given $r_{d(a)}$, $\varepsilon_{RP}(r_{d(a)}) < \varepsilon(r_{d(a)})$. This remark further explains that the values of limiting highest efficiencies, given for the $n^+(p^+) - p(n)$ crystalline GaAs-junction solar cells, obtained in our RP, are found to

be high, 31.474% (44.359%), compared respectively with the present ones, 30.76% (42.73%), as those showed in Tables 2 and 3.

Table 1. Impurity size effects, taken on the effective dielectric constant $\varepsilon(r_{d(a)})$, being determined in Eq. (1c), the effective donor(acceptor)-ionization energy, $E_{d(a)}(r_{d(a)})$, in absolute values, and the band gap, $E_{gn(p)}(r_{d(a)})$, at T=0K, determined both in Eq. (1b), and finally, the intrinsic band gap, $E_{gin(p)}(T = 300K, r_{d(a)})$ and the intrinsic carrier concentration $n_{in(p)}(T = 300K, r_{d(a)})$, determined respectively in Equations (A4) and (A5) of the Appendix A

Donor	P	As	Te	Sb	Sn
r_d (nm) [8]	0.110	0.118	0.132	0.136	0.140
$\varepsilon(r_d)$	13.4	13.13 [8]	12.33	11.86	11.33
$E_d(r_d)$ in meV	5.0	5.2	5.91	6.38	7.00
$E_{gn}(r_d)$ in meV	1519.8	1520 [8]	1520.7	1521.2	1521.8
$E_{gin}(300K, r_d)$ in meV	1423.3	1423.5	1424.2	1424.7	1425.3
$n_{in}(300K, r_d)$in 10^6 cm^{-3}	1.44	1.43	1.41	1.40	1.38
Acceptor	B	Ga	Al	Mg	In
r_a (nm) [8]	0.088	0.126	0.126	0.140	0.144
$\varepsilon(r_a)$	24.38	13.13 [8]	13.13 [8]	12.42	11.99
$E_a(r_a)$ in meV	6.66	23	23	25.7	27.5
$E_{gp}(r_a)$ in meV	1503.7	1520 [8]	1520 [8]	1522.7	1524.5
$E_{gip}(300K, r_a)$ in meV	1407.2	1423.5	1423.5	1426.2	1428
$n_{ip}(300K, r_a)$ in 10^6 cm^{-3}	1.97	1.43	1.43	1.36	1.31

Comparison between our numerical results, obtained in present paper and recent paper **(RP)** [1]; $\varepsilon(r_{d(a)}) \gg [\boldsymbol{\varepsilon_{RP}(r_{d(a)})}]$

Donor	As	Te	Sb
r_d (nm) [$\mathbf{r_d}$ **(nm)**]	0.118 [**0.118**]	0.132 [**0.132**]	0.136 [**0.136**]
$\varepsilon(r_d)$ [$\boldsymbol{\varepsilon_{RP}(r_d)}$]	13.13 [**12.85**]	12.33 [**7.87**]	11.86 [**6.91**]
Acceptor	Ga	Al	In
r_a (nm) [$\mathbf{r_a}$ **(nm)**]	0.126 [**0.126**]	0.126 [**0.126**]	0.144 [**0.144**]
$\varepsilon(r_a)$ [$\boldsymbol{\varepsilon_{RP}(r_a)}$]	13.13 [**12.85**]	13.13 [**12.85**]	11.99 [**6.86**]

In summary, those effects of $N_{d(a)}$-heavy doping and $r_{d(a)}$- impurity size [or the $\varepsilon(r_{d(a)})$ −effect], given in the HD[d(a)-GaAs]ER, and those of $N_{a(d)}$-low doping in the LD[a(d)-GaAs]BR, affect all

the minority-carrier transport properties, as those given in the Appendix A, B and C, and also in the following equations.

2.2. Total minority-carrier saturation current density at 300K

The total carrier-minority saturation current density is defined by:

$$J_{oI(II)} \equiv J_{En(p)o} + J_{Bp(n)o}, \tag{2}$$

where $J_{Bp(n)o}$ is the minority-electron (hole) saturation current density injected into the LD[a(d)-GaAs]BR, being determined in Eq. (C1) of the Appendix C, and $J_{En(p)o}$ is the minority-hole saturation-current density injected into the HD[d(a)-GaAs]ER.

In the non-uniformly and heavily doped emitter region of d(a)-GaAs devices, the effective Gaussian d(a)-density profile or the d(a) (majority-e(h)) density, is defined in the HD[d(a)-GaAs]ER-width W:

$$\rho_{d(a)}(x) = N_{d(a)} \times \exp\left\{-\left(\frac{x}{W}\right)^2 \times \ln\left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]\right\} \equiv N_{d(a)} \times \left[\frac{N_{d(a)}}{N_{d(a)o}(W)}\right]^{-\left(\frac{x}{W}\right)^2}, \quad 0 \le x \le W,$$

$$N_{d(a)o}(W) \equiv 7.9 \times 10^{17}\ (2 \times 10^5) \times \exp\left\{-\left(\frac{W}{184.2\ (1)10^{-7}\ cm}\right)^{1.066\ (0.5)}\right\}\ (cm^{-3}), \tag{3}$$

where $\rho_{d(a)}(x = 0) = N_{d(a)}$ is the surface d(a)-density, and at the emitter-base junction, $\rho_{d(a)}(x = W) = N_{d(a)o}(W)$, decreasing with increasing W [1, 2, 13]. Further, the "effective doping density" is defined by:

$$N_{d(a)eff.}(x, r_{d(a)}) \equiv \rho_{d(a)}(x)/\exp\left[\frac{\Delta E_{ga\ n(p)}(\rho_{d(a)}(x), r_{d(a)})}{k_B T}\right],$$

$$N_{d(a)eff.}(x = 0, r_{d(a)}) \equiv \frac{N_{d(a)}}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)}, r_{d(a)})}{k_B T}\right]} \text{ and } N_{d(a)eff.}(x = W,\ r_{d(a)}) \equiv \frac{N_{d(a)o}(W)}{\exp\left[\frac{\Delta E_{ga\ n(p)}(N_{d(a)o}(W), r_{d(a)})}{k_B T}\right]}, \tag{4}$$

where $\Delta E_{ga\ n(p)}$ are determined in Equations (B4, B5) of the Appendix B.

Then, under low-level injection, in the absence of external generation, and for the steady-state case, we can define the minority-h(e) density by:

$$p_o(x)[n_o(x)] \equiv \frac{n_{in(p)}^2}{N_{d(a)eff.}(x,\ r_{d(a)})}, \tag{5}$$

where $n_{in(p)}^2$ is determined in (A5) of the Appendix A and a normalized excess minority-h(e) density u(x) or a relative deviation between $p(x)[n(x)]$ and $p_o(x)[n_o(x)]$, by [22, 25]:

$$u(x) \equiv \frac{p(x)[n(x)] - p_o(x)[n_o(x)])}{p_o(x)[n_o(x)]}, \tag{6}$$

which must verify the two following boundary conditions proposed by Shockley as [6]:

$$u(x=0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}, \tag{7}$$

$$u(x=W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1. \tag{8}$$

Here, $n_{I(II)}(V)$ is a photovoltaic conversion factor determined in Equations (27, 28), S ($\frac{cm}{s}$) is the surface recombination velocity at the emitter contact, V is the applied voltage, $V_T \equiv (k_B T/e)$ is the thermal voltage, and the minority-hole (electron) current density $J_{h(e)}(x)$.

Further, as developed in RP, from the Fick's law for minority hole (electron)-diffusion equations [8, 12]:

$$J_{h(e)}(x) = \frac{-e(+e) \times n_i^2}{F_{h(e)}(x)} \times \frac{du(x)}{dx} = \frac{-e(+e) n_{in(p)}^2 D_{h(e)}(x)}{N_{d(a)eff.}(x)} \times \frac{du(x)}{dx}, \tag{9}$$

where $N_{d(a)eff.}$ is given in Eq. (4), $D_{h(e)}$ and $F_{h(e)}$ are determined respectively in Equations (C3, C2, C6) of the Appendix C, and from the minority-hole (electron) continuity equation [8, 12]:

$$\frac{dJ_{h(e)}(x)}{dx} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{F_{h(e)}(x) \times L_{h(e)}^2} = -e(+e) \times n_{i\,n(p)}^2 \times \frac{u(x)}{N_{d(a)eff.}(x) \times \tau_{h(e)E}}, \tag{10}$$

where $L_{h(e)}$ and $\tau_{h(e)E}$ are defined respectively in Equations (C7, C8) of the Appendix C, one finally obtains the following second-order differential equation as [22]:

$$\frac{d^2u(x)}{dx^2} - \frac{dF_{h(e)}(x)}{dx} \times \frac{du(x)}{dx} - \frac{u(x)}{L_{h(e)}^2(x)} = 0. \tag{11}$$

Then, taking into account the two boundary conditions (7, 8), one thus gets the general solution of this Eq. (11), as [22]:

$$u(x) = \frac{\sinh(P(x)) + I(W,S) \times \cosh(P(x))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))} \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1\right), \; I(W,S) = \frac{D_{h(e)}(N_o(W))}{S \times L_{h(e)}(N_o(W))}. \tag{12}$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, determined in Eq. (29). Further, since $\frac{dP(x)}{dx} \equiv C \times F_{h(e)}(x) = \frac{1}{L_{h(e)}(x)}$, $C = 2.0893 \times 10^{-30}$ (cm^4/s), for the crystalline Si, being an empirical parameter, chosen for each crystalline semiconductor, P(x) is thus found to be defined by:

$$P(x) \equiv \int_0^x \frac{dx}{L_{h(e)}(x)}), \; 0 \le x \le W, \; P(x=W) \equiv (\frac{1}{W} \times \int_0^W \frac{dx}{L_{h(e)}(x)}) \times W \equiv \frac{W}{L_{h(e)eff.}} = \frac{L_{h(e)}}{L_{h(e)eff.}} \times \frac{W}{L_{h(e)}}, \tag{13}$$

where $L_{h(e)eff.}$ is the effective minority-hole (electron) diffusion length. Further, from Eq. (9, 13), the minority-hole (electron) current density injected into the HD[d(a)-GaAs]ER is found to be determined by:

$$J_{h(e)}(x, W, N_{d(a)}, r_{d(a)}, S, V) = -J_{Eno}(x, W, N_d, r_d, S)\,[J_{Epo}(x, W, N_a, r_a, S)] \times \left(\exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1\right), \tag{14}$$

where $J_{En(p)o}$ is the saturation minority-hole (electron) current density,

$$J_{En(p)o}\left(x, W, N_{d(a)}, r_{d(a)}, S\right) = \frac{en^2_{i\,n(p)} \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{\cosh(P(x)) + I(W,S) \times \sinh(P(x))}{\sinh(P(W)) + I(W,S) \times \cosh(P(W))}. \quad (15)$$

Here, the intrinsic carrier concentration $n_{i\,n(p)}$ is computed by Eq. (A5) of the Appendix A, and the effective doping density $N_{d(a)eff.}$ is determined in Eq. (4), the minority-hole (electron) diffusion coefficient $D_{e(h)}$ and minority-hole (electron) diffusion length $L_{h(e)}$ are given respectively in Equations (C2, C3, C7) of the Appendix C, and the factor $I(W,S)$ is determined by:

$$I(W,S) = \frac{D_{h(e)}(N_{d(a)o}(W))}{S \times L_{h(e)}(N_{d(a)o}(W))}, \quad (16)$$

where $N_{d(a)o}(W)$ is determined in Eq. (3).

Further, one remarks that: (i) from Equations (12, 14-16) one obtains: $u(x=0) \equiv \frac{-J_h(x=0)[J_e(x=0)]}{eS \times p_o(x=0)[n_o(x=0)]}$, which is just the first boundary condition given in Eq. (7), and then, (ii) Eq. (12) yields: $u(x = W) = \exp\left(\frac{V}{n_{I(II)}(V) \times V_T}\right) - 1$, being the second boundary condition given in Eq. (8).

In the following, we will denote $P(W)$ and $I(W,S)$ by P and I, for a simplicity. So, Eq. (15) gives:

$$J_{En(p)o}\left(x = 0, W, N_{d(a)}, r_{d(a)}, S\right) = \frac{en^2_{i\,n(p)} \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{1}{\sinh(P) + I \times \cosh(P)}, \quad (17)$$

$$J_{En(p)o}\left(x = W, W, N_{d(a)}, r_{d(a)}, S\right) = \frac{en^2_{i\,n(p)} \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{\cosh(P) + I \times \sinh(P)}{\sinh(P) + I \times \cosh(P)}. \quad (18)$$

Thus, from Equations (14, 17, 18), one gets

$$\frac{J_{h(e)}\left(x=0,W,N_{d(a)},r_{d(a)},S,V\right)}{J_{h(e)}\left(x=W,W,N_{d(a)},r_{d(a)},S,V\right)} \equiv \frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)} = \frac{1}{\cosh(P) + I \times \sinh(P)}. \quad (19)$$

Now, if defining the effective excess minority-hole (electron) charge storage in the emitter region by [22]:

$Q_{h(e)eff.}(x = W, N_{d(a)}, r_{d(a)}) \equiv \int_0^W +e(-e) \times u(x) \times p_o(x)[n_o(x)] \times \frac{\tau_{h(e)E}(N_{d(a)}, r_{d(a)})}{\tau_{h(e)E}(\rho_{d(a)}(x), r_{d(a)})} dx$, and the effective minority-hole transit time by: $\tau_{teff.}(x = W, W, N_{d(a)}, r_{d(a)}, S) \equiv Q_{h(e)\,eff.}(x = W, N_{d(a)}, r_{d(a)}) / J_{En(p)o}\left(x = W, W, N_{d(a)}, r_{d(a)}, S\right)$, one can define, from Equations (10, 19), the reduced effective minority-hole transit time:

$$\frac{\tau_{teff.}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)}{\tau_{h(e)E}} \equiv 1 - \frac{J_{En(p)o}\left(x=0,W,N_{d(a)},r_{d(a)},S\right)}{J_{En(p)o}\left(x=W,W,N_{d(a)},r_{d(a)},S\right)} = 1 - \frac{1}{\cosh(P) + I \times \sinh(P)}. \quad (20)$$

Now, some important results can be obtained and discussed below.

As $P \ll 1$ (or $W \ll L_{h,eff.}$) and $S \to \infty$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S \times L_h(N_o(W))} \to 0$, from Eq. (20), one has: $\frac{\tau_{t,eff.}(x=W,W,N_{d(a)},r_{d(a)},S)}{\tau_{h(e)E}} \to 0$, suggesting a completely transparent emitter region (CTER)-case, where, from Eq. (18), one obtains:

$$J_{En(p)o}(x = W, N_{d(a)}, r_{d(a)}, S \to \infty) \to \frac{en^2_{i\,n(p)} \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \frac{1}{P(W)}, \tag{21a}$$

and then, as $P \gg 1$ (or $W \gg L_{h,eff.}$) and $S \to 0$, $I \equiv I(W,S) = \frac{D_h(N_o(W))}{S \times L_h(N_o(W))} \to \infty$, from Eq. (20), one has: $\frac{\tau_{teff.}(x=W,W,N_{d(a)},r_{d(a)},S)}{\tau_{h(e)E}} \to 1$, suggesting a completely opaque emitter region (COER)-case, where, from Eq. (18), one gets:

$$J_{En(p)o}(x = W, N_{d(a)}, r_{d(a)}, S \to 0) \to \frac{en^2_{i\,n(p)} \times D_{h(e)}}{N_{d(a)eff.} \times L_{h(e)}} \times \tanh(P). \tag{21b}$$

In summary, in the $n^+(p^+) - p(n)$ junction solar cells, the dark carrier-minority saturation current density J_o, defined in Eq. (2), is now replaced by $J_{oI(II)}$, for a good presentation, and rewritten by:

$$J_{oI(II)}(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}) \equiv J_{En(p)o}(W, N_{d(a)}, r_{d(a)}, S) + J_{Bp(n)o}(N_{a(d)}, r_{a(d)}), \tag{22}$$

where $J_{En(p)o}$ and $J_{Bp(n)o}$ are determined respectively in Equations (18) and (C1) of the Appendix C.

3. Photovoltaic conversion effect at 300K

Here, in the $n^+(p^+) - p(n)$ junction solar cells, denoted respectively by I(II), and for physical conditions:

$W = 0.0044\ \mu m, N_{d\equiv As(a\equiv Ga)} = 10^{19}\ (10^{20}) cm^{-3}, r_{d\equiv As(a\equiv Ga)}, S = 10^{50}\ \frac{cm}{s}, N_{a\equiv Ga(d\equiv As)} = 10^{17}(10^{17})\ cm^{-3}, r_{a\equiv Ga(d\equiv As)},$ (23)

we propose, at given $V_{ocI1(2)}$ and $V_{ocII1(2)}$, the experimental results of the short circuit current density $J_{scI(II)}$, fill factor $F_{I(II)}$, and photovoltaic conversion factor $\eta_{I(II)}$, in order to formulate our following treatment method of two fixe experimental points [3, 4], for the $n^+ - p$ junction,

$V_{ocI1(I2)} = 980\ (1127.2)\ mV$, $J_{scI1(I2)} = 27.06\ (29.78)\ mA/cm^2$, $F_{I1(I2)} = 83.35\ (86.7)\ \%$, $\eta_{I1(I2)} = 22.07\ (29.1)\ \%$, and for the $p^+ - n$ junction,

$V_{ocII1(II2)} = 980\ (1030)\ mV$, $J_{scII1(II2)} = 24.2\ (29.8)\ mA/cm^2$, $F_{II1(II2)} = 76.4\ (86)\ \%$, $\eta_{II1(II2)} = 18.1\ (26.4)\ \%$. (24)

First of all, we define the net current density J at T=300 K, obtained for the infinite shunt resistance, and expressed as a function of the applied voltage V, flowing through the $n^+(p^+)-p(n)$ junction of GaAs solar cells, by [1, 2, 5-10]:

$$J(V) \equiv J_{ph.}(V) - J_{oI(II)} \times \left(e^{X_{I(II)}(V)} - 1\right),\ X_{I(II)}(V) \equiv \frac{V}{n_{I(II)}(V) \times V_T},\ \ V_T \equiv \frac{k_B T}{e} = 25.85\ \text{mV}, \tag{25}$$

where the function $n_{I(II)}(V)$ is the photovoltaic conversion factor, noting that as $V = V_{oc},\ J(V) = 0$, the photocurrent density is defined by: $J_{ph.}(V = V_{oc}) \equiv J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, for $V_{oc} \geq V_{ocI(II)1}$. Therefore, the photovoltaic conversion effect occurs, according to:

$$J_{scI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv J_{oI(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}\right) \times \left(e^{X_{I(II)}(V_{oc})} - 1\right), \tag{26}$$

where $n_{I(II)}(V_{oc}) \equiv n_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right)$, and $X_{I(II)}(V_{oc}) \equiv \frac{V_{oc}}{n_{I(II)}(V_{oc}) \times V_T}$.

Here, one remarks that (i) for a given V_{oc}, both $n_{I(II)}$ and $J_{oI(II)}$ have the same variations, obtained in the same physical conditions, as observed in many cases, given in Ref. [1], and (ii) the function $\left(e^{X_{I(II)}(V_{oc})} - 1\right)$ or the PVCF $n_{I(II)}$, representing the photovoltaic conversion effect, thus converts the light, represented by $J_{scI(II)}$, into the electricity, by $J_{oI(II)}$.

Further, from Equations (22, 26), we obtain for the $n^+ - p$ junction:

$$n_{I1(2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(2)}, J_{scI1(2)}\right) \equiv \frac{V_{ocI1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scI1(2)}}{J_{oI}} + 1\right)} \equiv n_{I1(2)}(V_{ocI1(2)}, J_{scI1(2)}),$$

and we then propose:

$$n_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = n_{I1}(V_{ocI1}, J_{scI1}) + n_{I2}(V_{ocI2}, J_{scI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.0825}, \tag{27}$$

being valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Furthermore, for the $p^+ - n$ junction,

$$n_{II1(2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(2)}, J_{scII1(2)}\right) \equiv \frac{V_{ocII1(2)}}{V_T} \times \frac{1}{\ln\left(\frac{J_{scII1(2)}}{J_{oII}} + 1\right)} \equiv n_{II1(2)}(V_{ocII1(2)}, J_{scII1(2)}),$$

and then,

$$n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = n_{II1}(V_{ocI1}, J_{scI1}) + n_{II2}(V_{ocII2}, J_{scII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{1.0523}, \tag{28}$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Therefore, from Equations (23, 24, 27, 28), one obtains, $n_{I1(II1)} = 0.9505\ (0.93323)$ at $V_{ocI1(II1)} = 980\ (980)$ mV, and $n_{I2(II2)} = 1.0906\ (0.97584)$ at $V_{ocI2(II2)} = 1127.2\ (1030)$ mV, respectively, for $n^+(p^+) - p(n)$ junction solar cells.

Thus, X_I defined from Eq. (26) now becomes for the $n^+ - p$ junction:

$X_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) \equiv \frac{V_{oc}}{n_I(W,N_d,r_d,S,N_a,r_a,V_{oc})\times V_T}$, and therefore, we can determine the values of the fill factors $F_{I1(I2)}$ at $V_{oc} = V_{ocI1(I2)}$ by [1, 2]:

$F_{I1(I2)}\left(W, N_d, r_d, S, N_a, r_a, V_{ocI1(I2)}\right) = \frac{X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)}\right)-\ln\left[X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)}\right)+0.72\right]}{X_I\left(W,N_d,r_d,S,N_a,r_a,V_{ocI1(I2)}\right)+3.38\ (1.75)} \equiv F_{I1(I2)}\left(V_{oc} = V_{ocI1(I2)}\right)$, for a presentation simplicity, and further, the fill factor F_I can be computed by:

$$F_I(W, N_d, r_d, S, N_a, r_a, V_{oc}) = F_{I1}(V_{ocI1}) + F_{I2}(V_{ocI2}) \times \left(\frac{V_{oc}}{V_{ocI1}} - 1\right)^{1.716}, \quad (29)$$

which is valid for any values of $(W, N_d, r_d, S, N_a, r_a, V_{oc} \geq V_{ocI1})$.

Then, also from Eq. (26), we can define for the $p^+ - n$ junction:

$X_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) \equiv \frac{V_{oc}}{n_{II}(W,N_a,r_a,S,N_d,r_d,V_{oc})\times V_T}$, where $n_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc})$ is determined in Eq. (28). Therefore, we can determine the values of the fill factors $F_{II1(II2)}$ at $V_{oc} = V_{ocII1(II2)}$ as:

$F_{II1(II2)}\left(W, N_a, r_a, S, N_d, r_d, V_{ocII1(II2)}\right) = \frac{X_{II}\left(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)}\right)-\ln\left[X_{II}\left(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)}\right)+0.72\right]}{X_{II}\left(W,N_a,r_a,S,N_d,r_d,V_{ocII1(II2)}\right)+7.675\ (2.3125)} \equiv F_{II1(2)}\left(V_{ocII1(II2)}\right)$, for a presentation simplicity, and further, the fill factor F_{II} is determined by:

$$F_{II}(W, N_a, r_a, S, N_d, r_d, V_{oc}) = F_{II1}(V_{ocII1}) + F_{II2}(V_{ocII2}) \times \left(\frac{V_{oc}}{V_{ocII1}} - 1\right)^{0.73688}, \quad (30)$$

being valid for any values of $(W, N_a, r_a, S, N_d, r_d, V_{oc} \geq V_{ocII1})$.

Then, with physical conditions given in Eq. (23), our numerical calculation shows that we obtain the same values of $J_{scI1(I2)}$ and $F_{I1(I2)}$ at $V_{ocI1(I2)} = 980\ (1127.2)$ mV, and $J_{scII1(II2)}$ and $F_{II1(II2)}$ at $V_{ocII1(II2)} = 980\ (1030)$ mV , as those given in Eq. (24).

Finally, the efficiency $\eta_{I(II)}$ can be defined in the $n^+(p^+) - p(n)$ junction solar cells, by:

$$\eta_{I(II)}\left(W, N_{d(a)}, r_{d(a)}, S, N_{a(d)}, r_{a(d)}, V_{oc}\right) \equiv \frac{J_{scI(II)}\times V_{oc}\times F_{I(II)}}{P_{in.}}, \quad (31)$$

where, $J_{scI(II)}$ and $F_{I(II)}$ are determined respectively in Equations (26, 29, 30), being assumed to be obtained at 1 sun illumination or at AM1.5G spectrum ($P_{in.} = 0.100\ \frac{W}{cm^2}$) [1, 2, 26-29]. Then, from Equations (31, 24), we get the numerical results of η, by using this assumption: $P_{in.} = 0.100\ \frac{W}{cm^2}$, and their relative errors in absolute values (RE), calculated by using the experimental results of $\eta_{I1(I2)}$ and $\eta_{II1(II2)}$ given in Eq. (24),

-for the $n^+ - p$ junction at $V_{ocI1(I2)} = 980\ (1127.2)$ mV, $\eta_{I1(I2)} = 22.18\ (29.25)$ % , with

RE=$4.4\times10^{-3}(5.2\times10^{-3})$, and

-for the p^+-n junction at $V_{ocII1(II2)}=980\ (1030)$ mV , $\eta_{II1(II2)}=18.21\ (26.53)\ \%$, with RE=$6.3\times10^{-3}(4.86\times10^{-3})$.

4. Numerical results and concluding remarks

We will respectively consider the two following cases, given in 8 $n^+(p^+)-p(n)$ junction solar cells: P(B)-B(P), As(Ga)-Ga(As), Sb(Mg)-Mg(Sb), Sn(In)-In(Sn), respectively, in the following.

4.1. HD [(P; As; Sb; Sn) − GaAs] ER − LD[(B; Ga; Mg; In) − GaAs)] BR −cases

Here, for those 4 $(n^+-p)-$ junctions: $[(P-B),(As-Ga),(Sb-Mg),(Sn-In)]$, respectively, we propose the following physical conditions as:

$$W=15\ \mu m,\ N_d=10^{19}\ cm^{-3}, S=100\ (cm/s),\ \text{and}\ N_a=10^{18}\ cm^{-3}. \quad (32)$$

Then, from Eq. (20) , one respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}}=(0,0,0,0)$, suggesting the completely transparent condition, and from Eq. (18), $J_{Eno}=(2.00, 1.96, 1.80, 1.73)\times10^{-23}\left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C: $J_{Bpo}=(2.22, 0.64, 0.54, 0.49)\times10^{-20}\left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively: $J_{oI}=(2.22, 0.64, 0.54, 0.49)\times10^{-20}\left(\frac{A}{cm^2}\right)=J_{Bpo}$, and from the following Table 2, for example, at $V_{oc}=1079$ mV, n_I= (0.998; 0.969; 0.965; 0.963) and η_I= (30.40; 30.70; 30.74; **30.76**) %, suggesting that, with increasing $r_{d(a)}$, or with decreasing $\varepsilon_{d(a)}$, due to the d(a)-size effect, both J_{oI} and n_I decrease, while η_I increases, being new obtained results.

Table 2. In the HD[(P; As; Sb; Sn)-GaAs] ER-LD[(B; Ga; Mg; In)-GaAs)] BR and for physical conditions given in Eq. (32), our numerical results of n_I, J_{scI}, F_I, and η_I, are computed by using Equations (27, 26, 29, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{d(a)}$, the function n_I decreases, while other functions J_{scI}, F_I, and η_I increase, being due to the impurity size $r_{d(a)}$-effect, suggesting thus our new obtained results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1130	1.047; 1.017; 1.013; 1.010	29.74; 29.81; 29.82; 29.83	87.7; 88.0; 88.1; 88.1	29.46; 29.66; 29.68; 29.70
1127.2	1.044; 1.014; 1.010; 1.008	29.94; 30.02; 30.03; 30.04	87.5; 87.9; 88.0; 88.0	29.55; 29.76; 29.78; 29.80
1082	1.000; 0.971; 0.968; 0.965	32.68; 32.85; 32.88; 32.89	86.0; 86.3; 86.4; 86.4	30.39; 30.69; 30.73; 30.75

1079	0.998; 0.969; 0.965; 0.963	32.81; 32.99; 33.01; 33.03	85.9; 86.2; 86.3; 86.3	**30.40; 30.70; 30.74; 30.76**
1072	0.991; 0.962; 0.958; 0.956	33.08; 33.27; 33.29; 33.31	85.7; 86.0; 86.1; 86.1	30.38; 30.69; 30.73; 30.75
1070	0.989; 0.960; 0.957; 0.954	33.14; 33.34; 33.36; 33.38	85.6; 86.0; 86.0; 86.0	30.36; 30.67; 30.71; 30.74
1000	0.926; 0.899; 0.895; 0.893	31.13; 31.26; 31.27; 31.28	84.3; 84.6; 85.0; 84.7	26.23; 26.46; 26.49; 26.50
990	0.918; 0.891; 0.888; 0.885	29.61; 29.69; 29.70; 29.71	84.2; 84.6; 84.6; 84.6	24.69; 24.86; 24.88; 24.89
980	0.910; 0.884; 0.880; 0.878	27.06; 27.06; 27.06; 27.06	84.2; 84.5; 84.6; 84.6	22.32; 22.42; 22.43; 22.44

4.2. HD $[(B;\ Ga;\ Mg;\ In) - \mathbf{GaAs}]$ ER $-$ LD$[(P;\ As;\ Sb;\ Sn) - \mathbf{GaAs}]$ BR $-$cases

Here, for those 4 $(p^+ - n) -$ junctions: $[(B - P), (Ga - As), (Mg - Sb), (In - Sn)]$, respectively, we propose the following physical conditions as:

$$W = 15\ \mu m,\ N_a = 10^{20}\ cm^{-3}, S\ = 100\ (cm/s\ ),\ and\ N_d = 10^{17}\ cm^{-3}. \quad (33)$$

Then, from Eq. (20) , one respectively obtains: $\frac{\tau_{teff.}}{\tau_{hE}} = (0, 0, 0, 0)$, suggesting the completely transparent condition, and from Eq. (18), $J_{Epo} = (2.75, 1.34, 1.22, 1.15) \times 10^{-23}\ \left(\frac{A}{cm^2}\right)$. Further, one respectively gets from Eq. (C1) of the Appendix C:

$J_{Bno} = (3.46, 3.36, 2.90, 2.71) \times 10^{-20}\left(\frac{A}{cm^2}\right)$. Then, from Eq. (22), one obtains respectively:

$J_{oII} = (3.46, 3.36, 2.90,\ 2.71) \times 10^{-20}\ \left(\frac{A}{cm^2}\right) = J_{Bno}$, and from the following Table 3, for example, at $V_{oc} = 1345\ mV$, n_{II}= (1.264; 1.263; 1.258; 1.256) and η_{II}= (42.67; 42.71; 42.72; **42.73**) %, meaning that, with increasing $r_{a(d)}$, or with decreasing $\varepsilon_{a(d)}$, due to the a(d)-size effect, both J_{oII} and n_{II} decrease, according to the increase in η_{II}, being also new obtained results.

Table 3. In the HD[(B; Ga; Mg; In)-GaAs] ER-LD[(P; As; Sb; Sn)-GaAs)] BR and for physical conditions given in Eq. (33), our numerical results of n_{II}, J_{scII}, F_{II}, and η_{II}, are computed by using Equations (28, 26, 30, 31), respectively. Here, on notes that, for a given V_{oc} and with increasing $r_{a(d)}$, the function n_{II} decreases, while other functions J_{scII}, F_{II}, and η_{II} increase, being due to the impurity size $r_{a(d)}$-effect, suggesting thus our new obtained results.

V_{oc}(mV)	n	$J_{sc}(\frac{mA}{cm^2})$	F(%)	η(%)
1380	1.298; 1.297; 1.293; 1.290	24.82; 24.82; 24.82; 24.82	121; 121; 121; 121	42.59; 42.63; 42.63; 42.63
1375	1.293; 1.292; 1.288; 1.286	25.01; 25.01; 25.01; 25.01	121; 121; 121; 121	42.61; 42.65; 42.65; 42.66
1366	1.284; 1.283; 1.279; 1.277	25.35; 25.35; 25.35; 25.35	120; 120; 120; 120	42.64; 42.68; 42.68; 42.69
1355	1.274; 1.273; 1.268; 1.266	25.76; 25.77; 25.77; 25.77	119; 119; 119; 119	42.66; 42.70; 42.71; 42.72

1345	1.264; 1.263; 1.258; 1.256	26.14; 26.14; 26.15; 26.15	118; 118; 118; 118	**42.67; 42.71; 42.72; 42.73**
1335	1.254; 1.253; 1.248; 1.246	26.52; 26.52; 26.52; 26.53	117; 117; 117; 117	42.66; 42.70; 42.71; 42.72
1030	0.964; 0.964; 0.960; 0.959	29.87; 29.87; 29.89; 29.90	86.2; 86.2; 86.3; 86.3	27.32; 27.35; 27.38; 27.39
1025	0.960; 0.959; 0;956; 0.954	29.53; 29.54; 29.56; 29.57	85.5; 85.5; 85.6; 85.6	26.66; 26.69; 26.72; 26.73
1000	0.939; 0.938; 0.935; 0.933	27.31; 27.32; 27.33; 27.33	81.5; 81.5; 81.6; 81.6	22.94; 22.97; 22.99; 22.99
980	0.922; 0.922; 0.919; 0.917	24.20; 24.20; 24.20; 24.20	76.6; 76.6; 76.7; 76.7	18.74; 18.76; 18.77; 18.77

In conclusion, our new limiting highest efficiency results: **30.76%** and **42.73%**, given in Tables 2 and 3, can also be compared respectively with other limiting η-results, such as: **29.1%** for GaAs-thin film cell, and **45.7%** and **44.4%**, respectively for GaInP/GaAs/GaInAs/GaInAs -and-InGaP/GaAs/InGaAs multijunction cells, being obtained by Green et al. [3].

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Appendix

Appendix A. Fermi Energy

In the n(p)-type Gas crystal, the Fermi energy $E_{Fn}(-E_{Fp})$, obtained for any T and donor density N, being investigated in our previous paper, with a precision of the order of 2.11×10^{-4} [39, 40], is now summarized in the following.

First of all, we define the reduced electron density by:

$$u\equiv\frac{N_{d(a)}}{N_{c(v)}},\ N_c(T)=2\times1\times\left(\frac{m_c\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}),\ N_v(T)=2\times1\times\left(\frac{m_v\times k_BT}{2\pi\hbar^2}\right)^{\frac{3}{2}}\ (cm^{-3}). \quad (A1)$$

Here, $N_{c(v)}$ is the conduction (valence)-band density of states, respectively, m_c is the effective mass of the electron in n-type GaAs can be defined by [8]:

$$m_c=0.066\times m_o, \quad (A2)$$

where $m_o=9.1096\times10^{-28}\ g$ is the electron rest mass, the averaged effective mass of the hole in the p-type GaAs yields [8]:

$$(m_v/m_o)=\frac{0.082+0.5}{2}=0.291, \quad (A3)$$

and finally, $E_{gin(p)}\left(T,r_{d(a)}\right)$ is the intrinsic band gap in the GaAs-crystal, due to the T-dependent carrier-lattice interaction-effect, by [1, 2, 33, 34]:

$$E_{gin(p)}(T, r_{d(a)}) = E_{gn(p)}(r_{d(a)}) - \frac{5.405\times10^{-4}\times T^2}{T+204}. \quad (A4)$$

Here, $E_{gn(p)}(r_{d(a)})$ is determined in Eq. (1b), due to the d(a)-size effect.

Furthermore, in the n(p)-type Si, one can define the intrinsic carrier concentration $n_{in(p)}$ by:

$$n^2_{i\,n(p)}(T, r_{d(a)}) \equiv N_c(T) \times N_v(T) \times \exp\left(\frac{-E_{gin(p)}(T, r_{d(a)})}{k_B T}\right). \quad (A5)$$

Then, denoting the reduced Fermi energy in the n(p)-type semiconductor, respectively, by $\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T})$,

being accurate to within 10^{-7} , we have [39] :

$$\frac{E_{Fn}(u)}{k_B T}(\frac{-E_{Fp}(u)}{k_B T}) = \frac{G(u)+Au^B F(u)}{1+Au^B}, A = 0.0005372 \text{ and } B = 4.82842262 \quad (A6)$$

where

$F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [(3\sqrt{\pi}/4) \times u]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$

and

$$G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du};\ d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0.$$

Here, one notes that: (i) $\frac{E_{Fn}(u\gg1)}{k_B T}(\frac{-E_{Fp}(u\gg1)}{k_B T}) > 1$, according to the HD[d(a)-GaAs]ER-case, or to the degenerate case, Eq. (A6) is reduced to the function F(u), and (ii) $\frac{E_{Fn}(u\ll1)}{k_B T}(\frac{-E_{Fp}(u\ll1)}{k_B T}) <- 1$, to the LD[a(d)-GaAs]BR-case, or to the non-degenerate case, Eq. (A6) is reduced to the function G(u), respectively.

(i) In the HD[d(a)-GaAs]ER-case for $N_{d(a)} = 10^{19}(10^{20})\ cm^{-3}$, we respectively get: $\frac{E_{Fn}}{k_B T}(\frac{-E_{Fp}}{k_B T}) = 10.04\ (10.56) > 1,$ according to degenerate conditions.

(ii) In the LD[a(d)-GaAs]BR-case and for $N_{a(d)} = 10^{18}(10^{17})\ cm^{-3}$, we respectively obtain: $\frac{-E_{Fp}}{k_B T}(\frac{E_{Fn}}{k_B T}) =- 1.3\ (-1.4) <- 1$, according to non-degenerate conditions. Thus, those limiting values of $N_{a(d)} = 10^{18}(10^{17})\ cm^{-3}$ can be used in the LD[a(d)-GaAs] BR-cases, respectively.

Appendix B. Approximate forms for band gap narrowing and apparent band gap narrowing

First of all, in the n(p)-type GaAs, we define the effective Wigner-Seitz radius r_s characteristic of the interactions by [1, 2]

$$r_{sn} \equiv r_s(N_d, r_d) = 1.1723 \times 10^8 \times \left(\frac{1}{N_d}\right)^{1/3} \times \frac{m_c}{\varepsilon(r_d)} \quad (B1)$$

and

$$r_{sp} \equiv r_s(N_a, r_a) = 1.1723 \times 10^8 \times \left(\frac{1}{N_a}\right)^{1/3} \times \frac{m_v}{\varepsilon(r_a)}, \tag{B2}$$

where m_c and m_v are given in (A2) and (A3). Therefore, the correlation energy of an effective electron gas, $E_{cn(cp)}\left(N_{d(a)}, r_{d(a)}\right)$, is given by [1, 2, 42]:

$$E_{cn(cp)}\left(N_{d(a)}, r_{d(a)}\right) = \frac{-0.87553}{0.0908+r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908+r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1+0.03847728 \times r_{sn(sp)}^{1.67378876}}. \tag{B3}$$

Then, in the n-type heavily doped GaAs, the band gap narrowing is found to be given as [1, 2]:

$$\Delta E_{gn}(N_d, r_d) \simeq a_1 \times \frac{\varepsilon(r_{As})}{\varepsilon(r_d)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{As})}{\varepsilon(r_d)} \times N_r^{\frac{1}{3}} \times (2.503 \times [-E_c(r_{sn}) \times r_{sn}]) + a_3 \times \left[\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}\right]^{5/4} \times \sqrt{\frac{m_v}{m_c}} \times N_r^{1/4} + a_4 \times \sqrt{\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}} \times N_r^{1/2} \times 2 + a_5 \times \left[\frac{\varepsilon(r_{As})}{\varepsilon(r_d)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, N_r \equiv \left(\frac{N_d}{9.999 \times 10^{17}\ cm^{-3}}\right), \tag{B4}$$

where $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$ and $a_5 = 8.1 \times 10^{-4}(eV)$, and in the p-type heavily doped GaAs, one has [1, 2]:

$$\Delta E_{gp}(N_a, r_a) \simeq a_1 \times \frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)} \times N_r^{1/3} + a_2 \times \frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times [-E_c(r_{sp}) \times r_{sp}]\right) + a_3 \times \left[\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}\right]^{5/4} \times \sqrt{\frac{m_c}{m_v}} \times N_r^{1/4} + 2a_4 \times \sqrt{\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}} \times N_r^{1/2} + a_5 \times \left[\frac{\varepsilon(r_{Ga})}{\varepsilon(r_a)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}}, \; N_r \equiv \left(\frac{N_a}{9.999 \times 10^{17}\ cm^{-3}}\right), \tag{B5}$$

where $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$ and $a_5 = 9.80 \times 10^{-5}(eV)$.

Further, in the donor (acceptor)-GaAs, we define the effective intrinsic carrier concentration $n_{ien(p)}$, by

$$n_{i\,en(p)}^2(N_{d(a)}, r_{d(a)}) \equiv N_{d(a)} \times p_o(n_o) \equiv n_{i\,n(p)}^2 \times \exp\left[\frac{\Delta E_{gan(p)}}{k_B T}\right], \tag{B6}$$

where we can define the "effective doping density" by: $N_{d(a)eff.} \equiv N_{d(a)} / \exp\left[\frac{\Delta E_{ga\,n(p)}}{k_B T}\right]$ so that $N_{d(a)eff.} \times p_o(n_o) \equiv n_{i\,n(p)}^2$ [8], and also the apparent band gap narrowing, $\Delta E_{ga\,n(p)}$, as

$$\Delta E_{ga\,n(p)} \equiv \Delta E_{g\,n(p)} + k_B T \times \ln\left(\frac{N_{d(a)}}{N_{c(v)}}\right) - E_{Fn}\left(\frac{N_d}{N_c}\right)[-E_{Fp}\left(\frac{N_a}{N_v}\right)], \tag{B7}$$

where $N_{c(v)}$ is defined in Eq. (A1), the Fermi energy is determined in Eq. (A6).

Appendix C. Minority-carrier transport parameters

Here, the minority-electron (hole) saturation current density injected into the LD[a(d)-GaAs]BR, with an acceptor density equal to $N_{a(d)}$, is given by [1, 2]:

$$J_{Bp(n)o}\left(N_{a(d)}, r_{a(d)}\right) = \frac{e \times n_i^2(r_{a(d)}) \times \sqrt{\frac{D_{e(h)}(N_{a(d)}, r_{a(d)})}{\tau_{e(h)B}(N_{a(d)})}}}{N_{a(d)}}, \quad \text{(C1)}$$

where $n^2_{i\,n(p)}(r_{d(a)})$ is determined in (A5), $D_{e(h)}(N_{a(d)}, r_{a(d)})$ is the minority-electron (hole) diffusion coefficient:

$$D_e(N_a, r_a) = \frac{k_B T}{e} \times \left[200 + \frac{8500-200}{1+\left(\frac{N_a}{1.3\times10^{17}\,cm^{-3}}\right)^{0.91}}\right] \times \left(\frac{\varepsilon(r_a)}{12.85}\right)^2 \left(cm^2V^{-1}s^{-1}\right), \quad \text{(C2)}$$

$$D_h(N_d, r_d) = \frac{k_B T}{e} \times \left[130 + \frac{400-130}{1+\left(\frac{N_d}{8\times10^{17}\,cm^{-3}}\right)^{1.25}}\right] \times \left(\frac{\varepsilon(r_d)}{12.85}\right)^2 \left(cm^2V^{-1}s^{-1}\right), \quad \text{(C3)}$$

and $\tau_{e(h)B}(N_{d(a)})$ is the minority-electron (hole) lifetime in the base region:

$$\tau_{eB}(N_a)^{-1} = \frac{1}{10^{-7}} + 3\times10^{-13}\times N_a + 1.83\times10^{-31}\times N_a^2. \quad \text{(C4)}$$

$$\tau_{hB}(N_d)^{-1} = \frac{1}{10^{-7}} + 11.76\times10^{-13}\times N_d + 2.78\times10^{-31}\times N_d^2, \quad \text{(C5)}$$

Further, from (A6), (B4)-(B7)), in the HD[d(a)-GaAs]ER, we can define the following minority-hole(electron) transport parameter $F_{h(e)}$ as [8, 22, 25]:

$$F_{h(e)}\ (N_{d(a)}, r_{d(a)}) \equiv \frac{n^2_{i\,n(p)}(r_{d(a)})}{p_o(n_o)\times D_{h(e)}} = \frac{N_{d(a)eff.}}{D_{h(e)}} \equiv \frac{N_{d(a)}}{D_{h(e)}\times\exp\left[\frac{\Delta E_{g\,an(p)}}{k_B T}\right]}\ (cm^{-5}\times s), \quad \text{(C6)}$$

Furthermore, the minority-hole (electron) diffusion length, $L_{h(e)}\left(N_{d(a)}, r_{d(a)}\right)$ and the minority-hole(electron) lifetime $\tau_{h(e)E}$ in the HD[d(a)-GaAs]ER can be determined by

$$L^{-2}_{h(e)}\left(N_{d(a)}, r_{d(a)}\right) = \left[\tau_{h(e)E}\times D_{h(e)}\right]^{-1} = \left(C\times F_{n(p)}\right)^2 = \left(C\times\frac{N_{d(a)eff.}}{D_{h(e)}}\right)^2 = \left(C\times\frac{n^2_{i\,n(p)}(r_{d(a)})}{p_o(n_o)\times D_{h(e)}}\right)^2, \quad \text{(C7)}$$

where the constant C is chosen to be $2.0893\times10^{-30}\ (cm^4/s)$, and then, $\tau_{h(e)E}$ can be computed by:

$$\tau_{h(e)E} = \frac{1}{D_{h(e)}\times\left(C\times F_{n(p)}\right)^2}. \quad \text{(C8)}$$

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